Brief introduction on radiative transfer

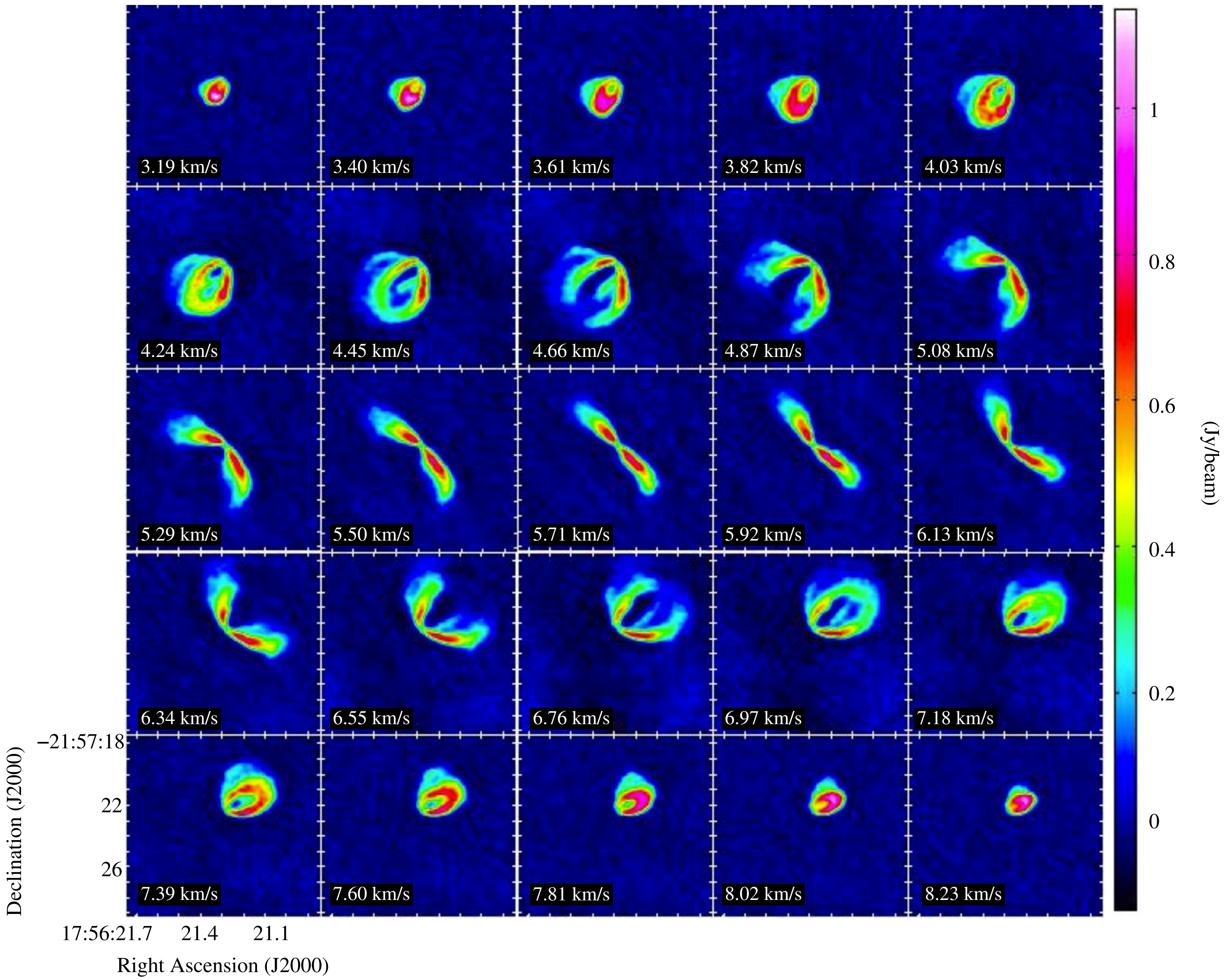
Christophe Pinte

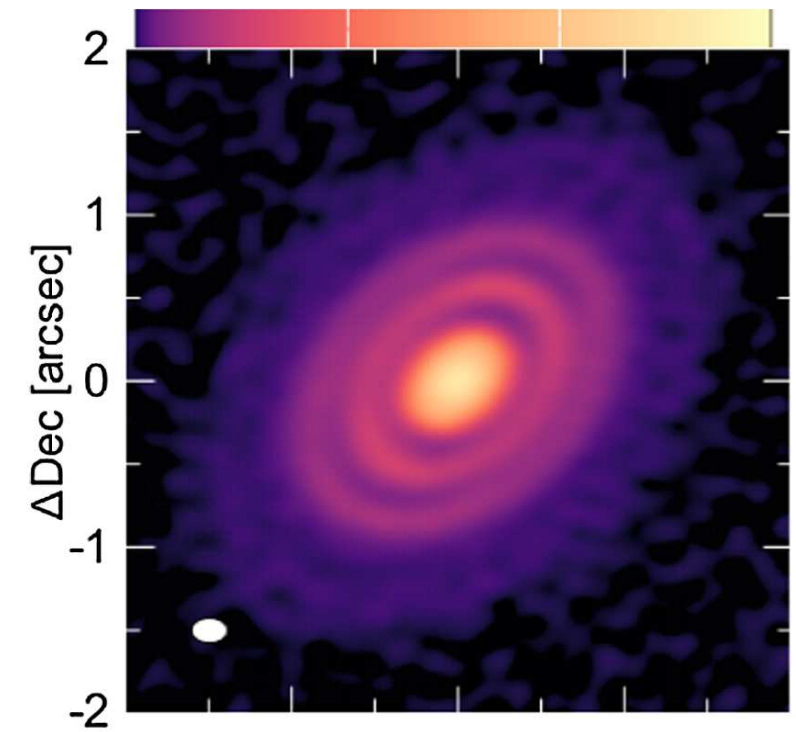
MONASH University

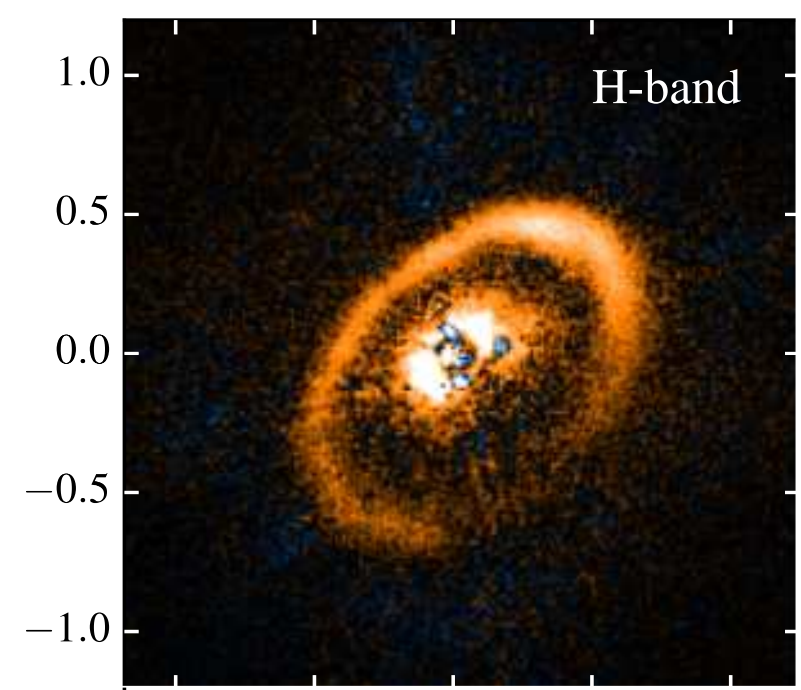
IPAG Institut de Planétologie et d'Astrophysique de Grenoble

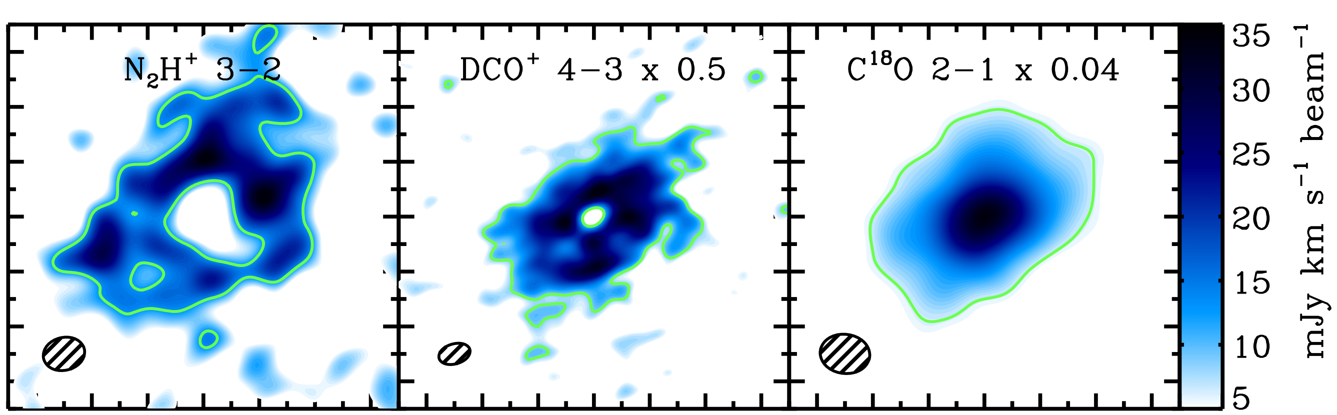
cnrs INSU Observer & comprendre

Radiative transfer as a diagnostic tool









Radiative transfer as a physical process

- Heating and cooling and energy transport
  - astrophysical objects cool by emitting radiation
  - inside the object: radiation can transport energy from one place to another
  - that same radiation is the radiation we observe with our telescopes

- Drives photo-chemistry
  Energetic photons can:
  – photoionize atoms, molecules
  – photodissociate molecules
  – charge dust grains

Two kind of radiative transfer models

- Post-processing, for comparison to observations:
  – Must be very accurate, and frequency dependent
  – Must include complex radiative physics (lines, dust, pola)
  – Must not necessarily be extremely fast

- In dynamic models:
  – Must be fast (RT=bottle neck)
  – Must be as parallellizable as hydrodynamics
  – High accuracy not feasible so far (not always necessary)
  – Using mean opacities, flux lim diffusion, simplex-style

The radiative transfer problem

Radiative Transfer is a 7-dimensional problem
(that's *one* of the reasons it is so hard and expensive to solve):

$$I(x, y, z, \theta, \phi, \nu, t) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

Usually: semi-steady-state:

$$I(x, y, z, \theta, \phi, \nu) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

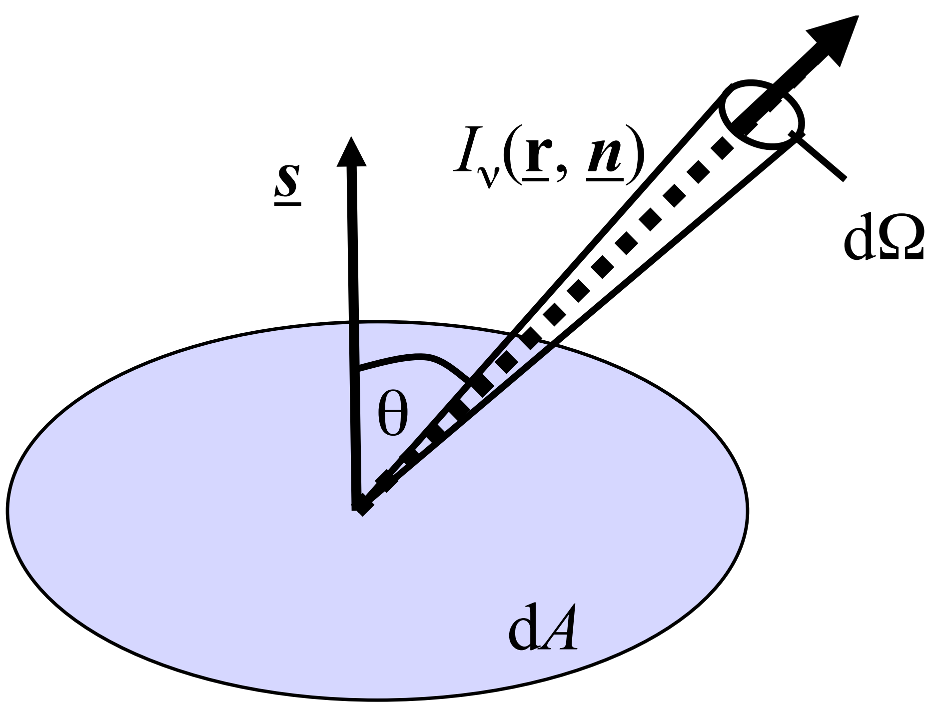
If the emission and extinction coefficients are known, you can reduce this to the Formal Transfer Equation along a single ray:

$$I(s, \nu) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

Specific intensity & flux

$$\mathrm{d}E_\lambda = I_\lambda \,\mathrm{d}A \,\mathrm{d}t \,\mathrm{d}\lambda \,\mathrm{d}\Omega$$

Units of I_λ: J/m^2/s/m/sr (ergs/cm^2/s/n/sr)
Function of position and direction



s is normal to dA

$$\lambda I_\lambda = \nu I_\nu$$

$$\mathrm{d}F_\lambda = I_\lambda \cos\theta \,\mathrm{d}\Omega$$

$$F_\lambda = \int_\Omega I_\lambda \cos\theta \,\mathrm{d}\Omega$$

Intensity is constant a long a ray

Key property : energy conservation

I_λ is independent of distance when no sources or sinks

$$\frac{dI_\lambda(s, \overrightarrow{n})}{ds} = 0 \qquad \Longrightarrow \qquad F_\lambda \propto \frac{1}{r^2}$$

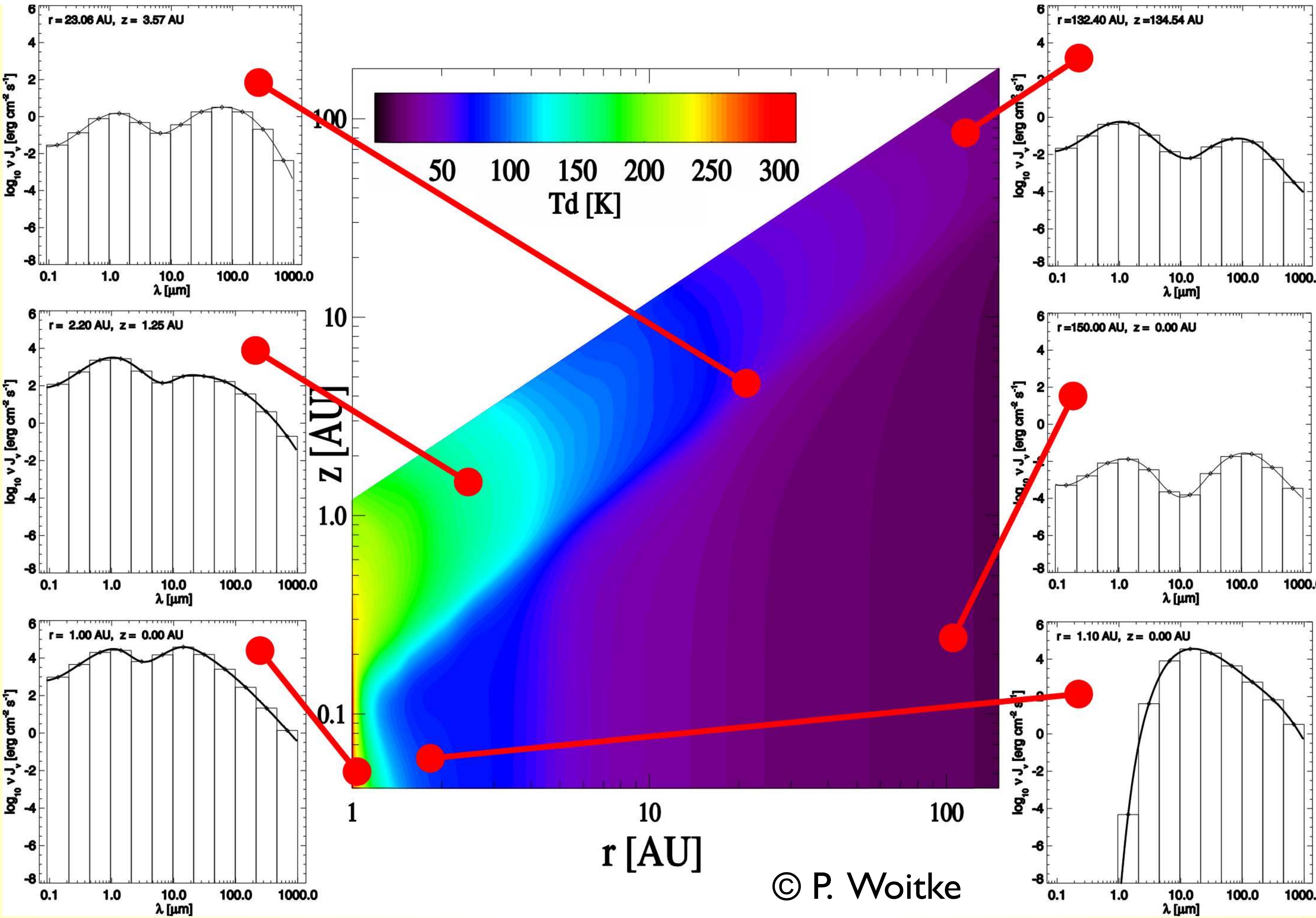
More generally: I_ν changes due to

- Scattering (directional change)
- Doppler-shift (frequency change)
- Absorption
- Emission

Mean intensity

$$J_\lambda = \frac{1}{4\pi}\int_\Omega I_\lambda \, \mathrm{d}\Omega = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\lambda \, \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

Same units as I_ν
Function of position
Determines heating, ionization, level populations, etc


r = 23.06 AU, z = 3.57 AU
r = 2.20 AU, z = 1.25 AU
r = 1.00 AU, z = 0.00 AU
r = 132.40 AU, z = 134.54 AU
r = 150.00 AU, z = 0.00 AU
r = 1.10 AU, z = 0.00 AU
$\log_{10} \nu J_\nu$ [erg cm^{-2} s^{-1}]
λ [μm]
50 100 150 200 250 300
Td [K]
z [AU]
r [AU]
© P. Woitke

Remark : moment of intensity

$$J_\lambda = \frac{1}{4\pi} \int_\Omega I_\lambda(\overrightarrow{n})\, \mathrm{d}\Omega$$

Mean intensity

$$\overrightarrow{H_\lambda} = \frac{1}{4\pi} \int_\Omega I_\lambda(\overrightarrow{n})\ \cos\theta\, \overrightarrow{n}\, \mathrm{d}\Omega$$

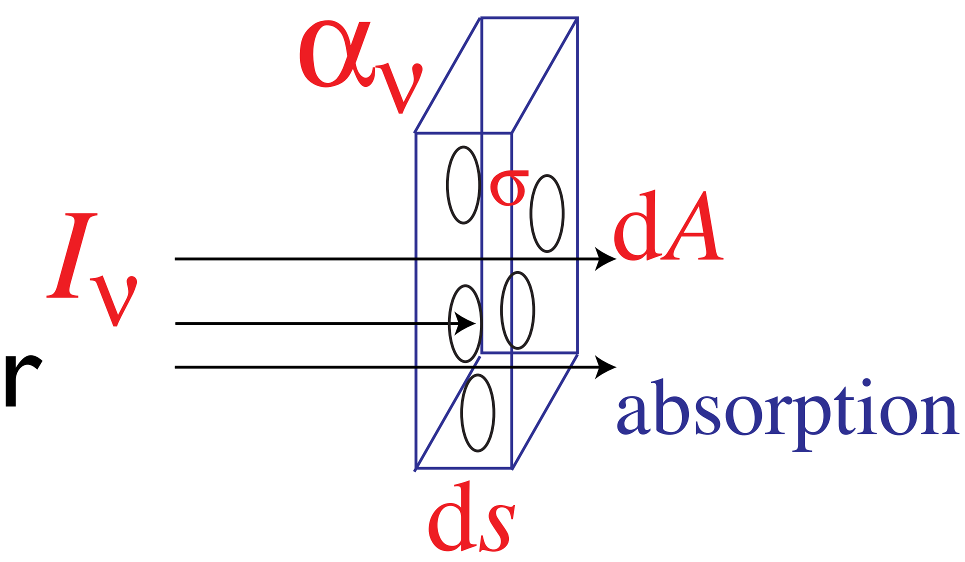
Flux

$$K_\lambda = \frac{1}{4\pi} \int_\Omega I_\lambda(\overrightarrow{n})\ \cos^2\theta\, \mathrm{d}\Omega$$

Radiation pressure

For homogenous and isotropic radiation $K_\lambda = \frac{1}{3} J_\lambda$

Extinction



Energy removed from beam
Defined per particule, per mass, per volume

$$\mathrm{d}I_\lambda(s,\overrightarrow{n}) = -n(s)\,\sigma_\lambda(s)\,I_\lambda(s,\overrightarrow{n})\,\mathrm{d}s$$

σ_λ = cross section [m^2]
n = particule density [m^3]

$$\mathrm{d}I_\lambda(s,\overrightarrow{n}) = -\alpha_\lambda(s)\,I_\lambda(s,\overrightarrow{n})\,\mathrm{d}s$$

α_λ: units of m^{-2}

$$\mathrm{d}I_\lambda(s,\overrightarrow{n}) = -\rho(s)\,\kappa_\lambda(s)\,I_\lambda(s,\overrightarrow{n})\,\mathrm{d}s$$

κ_λ: units of m^2.kg

ρ = density [kg.m^{-3}]

remark : stimulated emission if $\alpha_\lambda < 0$

Extinction

Opacity and optical depth :

$$\tau_\lambda(s_0, s_1) = \int_{s_0}^{s_1} \alpha_\lambda(s)\,\mathrm{d}s$$

Optically thick and thin medium :

$\tau_\lambda \gg 1$ and $\tau_\lambda \ll 1$

Mean free path :

$$l_\lambda = \frac{1}{\alpha_\lambda(s)}$$

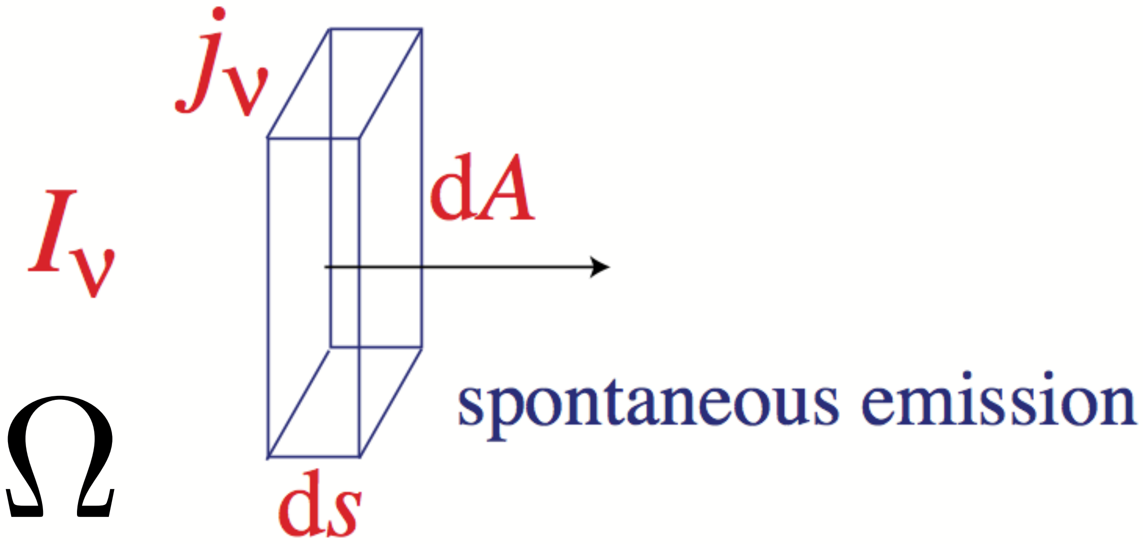
Physically, τ is the number of photon mean free paths

Radiative transfer equation with absorption

$$\frac{dI_\lambda(s, \overrightarrow{n})}{ds} = -\alpha_\lambda(s)\, I_\lambda(s, \overrightarrow{n})$$

$$I_\lambda(s, \overrightarrow{n}) = I_\lambda(s_0, \overrightarrow{n})\, e^{-\tau_\lambda(s_0, s)}$$

Emission

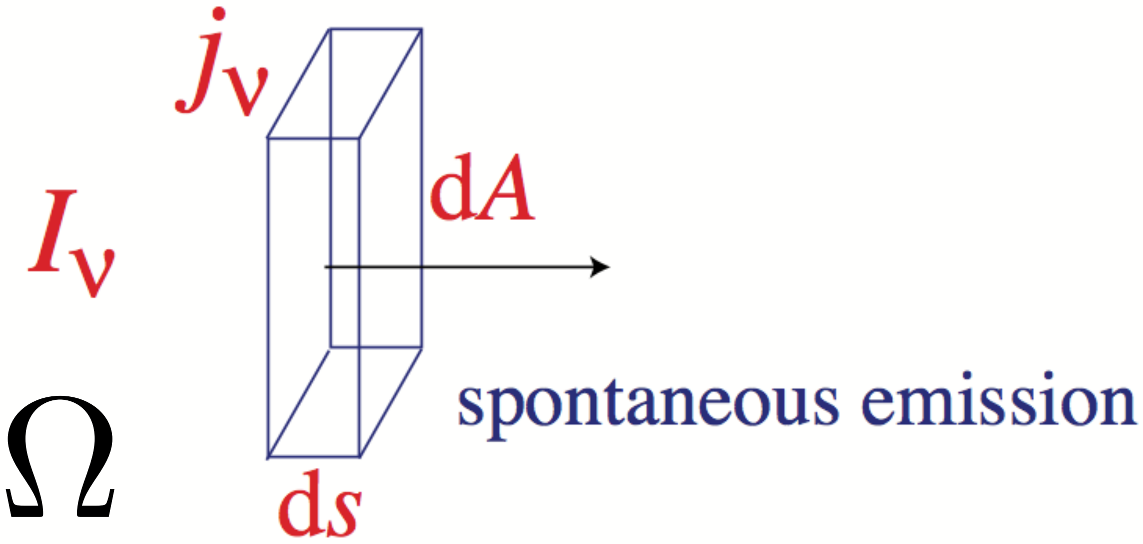


$$\mathrm{d}E_\lambda = j_\lambda \,\mathrm{d}V\,\mathrm{d}t\,\mathrm{d}\lambda\,\mathrm{d}\Omega$$

Energy, dE_λ, added:

- stimulated emission
- spontaneous emission
- thermal emission
- energy scattered into the beam

Emission



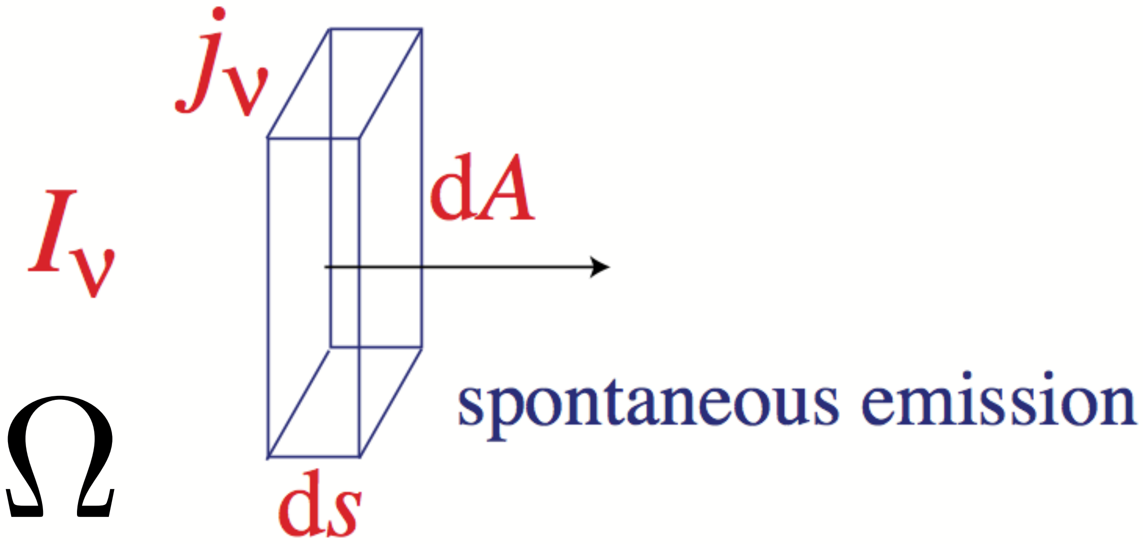
$$\mathrm{d}E_\lambda = j_\lambda \, \mathrm{d}V \, \mathrm{d}t \, \mathrm{d}\lambda \, \mathrm{d}\Omega$$

Energy, dE_λ, added:

- stimulated emission
- spontaneous emission
- thermal emission
- energy scattered into the beam

$$\frac{dI_\lambda(s, \overrightarrow{n})}{ds} = j_\lambda(s) - \alpha_\lambda(s) \, I_\lambda(s, \overrightarrow{n})$$

Emission



$$\mathrm{d}E_\lambda = j_\lambda \, \mathrm{d}V \, \mathrm{d}t \, \mathrm{d}\lambda \, \mathrm{d}\Omega$$

Energy, dEλ, added:

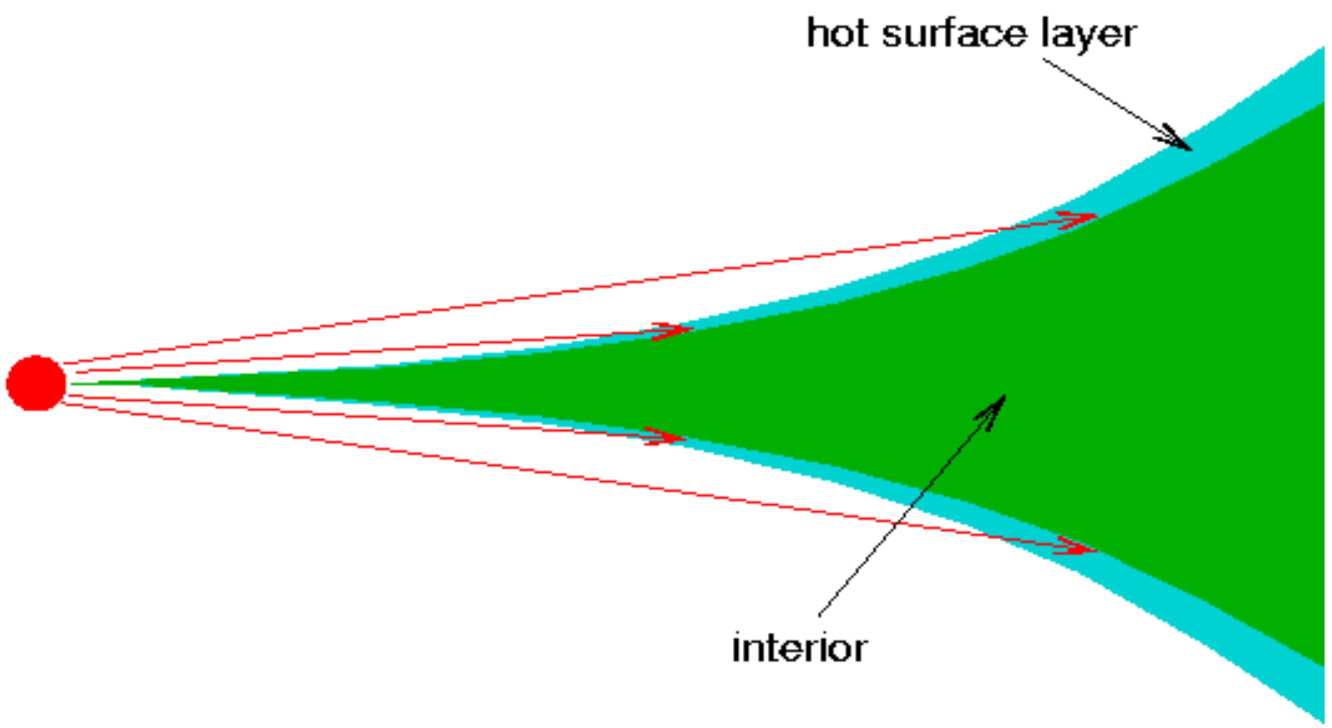
- stimulated emission
- spontaneous emission
- thermal emission
- energy scattered into the beam

$$\frac{dI_\lambda(s, \overrightarrow{n})}{ds} = j_\lambda(s) - \alpha_\lambda(s)\, I_\lambda(s, \overrightarrow{n})$$

$$I_\lambda(s, \overrightarrow{n}) = I_\lambda(s_0, \overrightarrow{n})\, e^{-\tau_\lambda(s_0, s)} + \int_{s_0}^{s} j_\lambda(s')\, e^{-\tau(s_0, s')}\, ds$$

Spectroscopic features

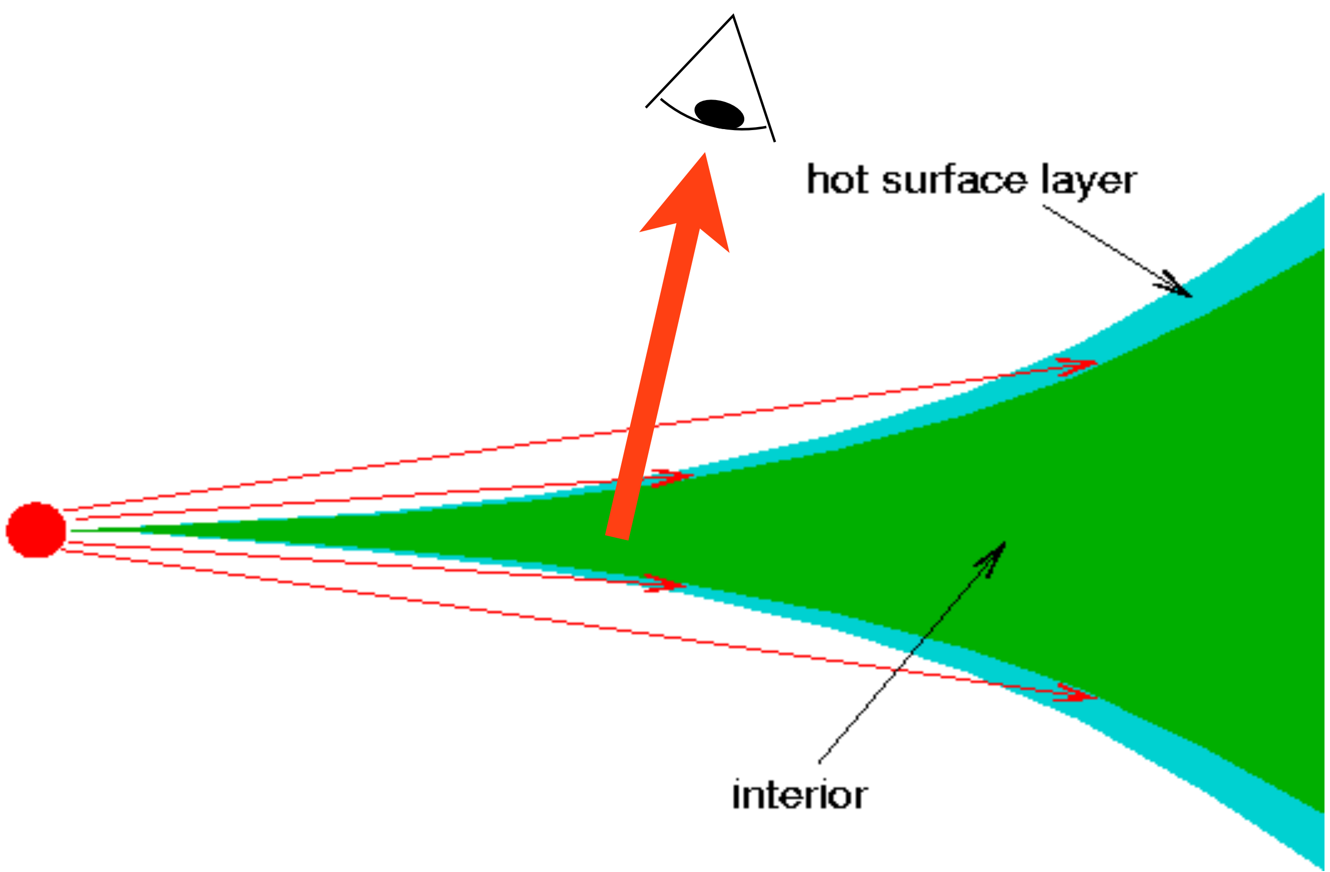
Optically thick disk



$$I_\lambda^{\text{observed}} = I_\lambda^{\text{bg}}\, e^{-\tau_\lambda} + I_\lambda^{\text{fg}}\, (1 - e^{-\tau_\lambda})$$

Spectroscopic features

Optically thick disk

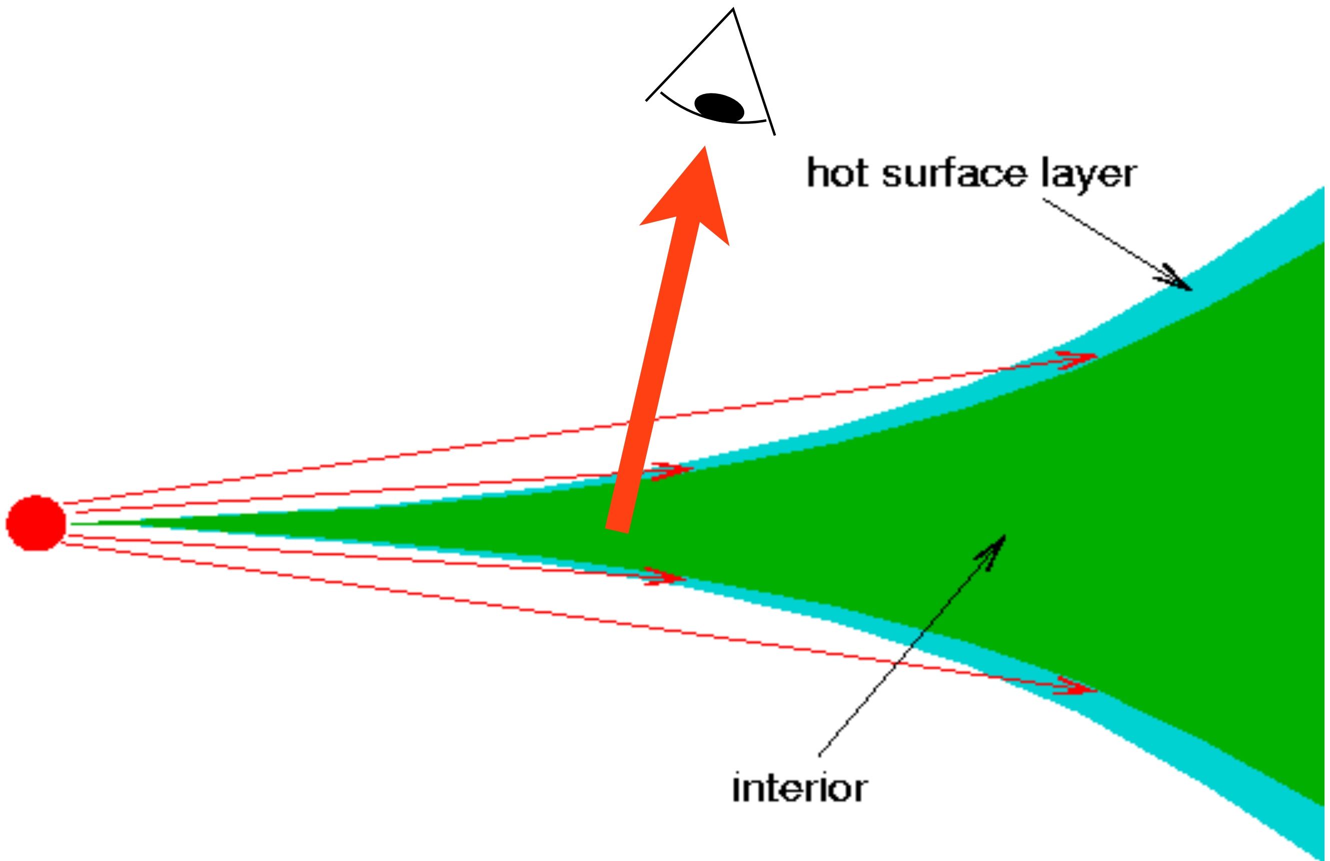


- $T^{\rm fg} > T^{\rm bg}$ emission features

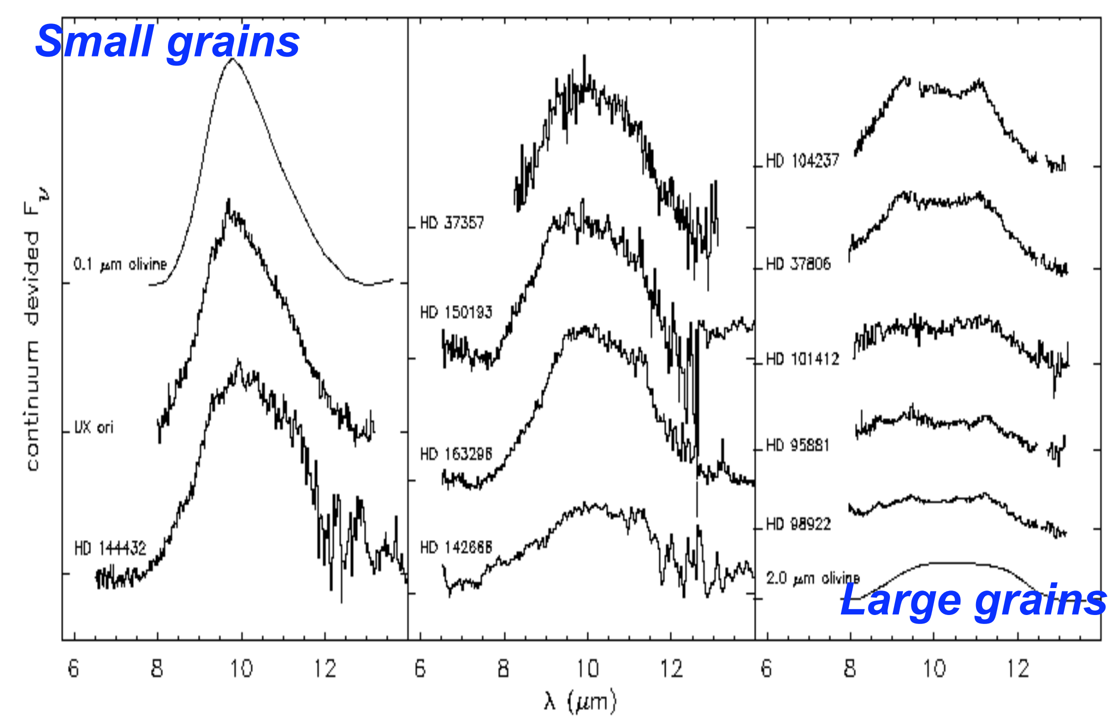
$$I_\lambda^{\rm observed} = I_\lambda^{\rm bg}\, e^{-\tau_\lambda} + I_\lambda^{\rm fg}\,(1 - e^{-\tau_\lambda})$$

Spectroscopic features

Optically thick disk



- $T^{\rm fg} > T^{\rm bg}$ emission features

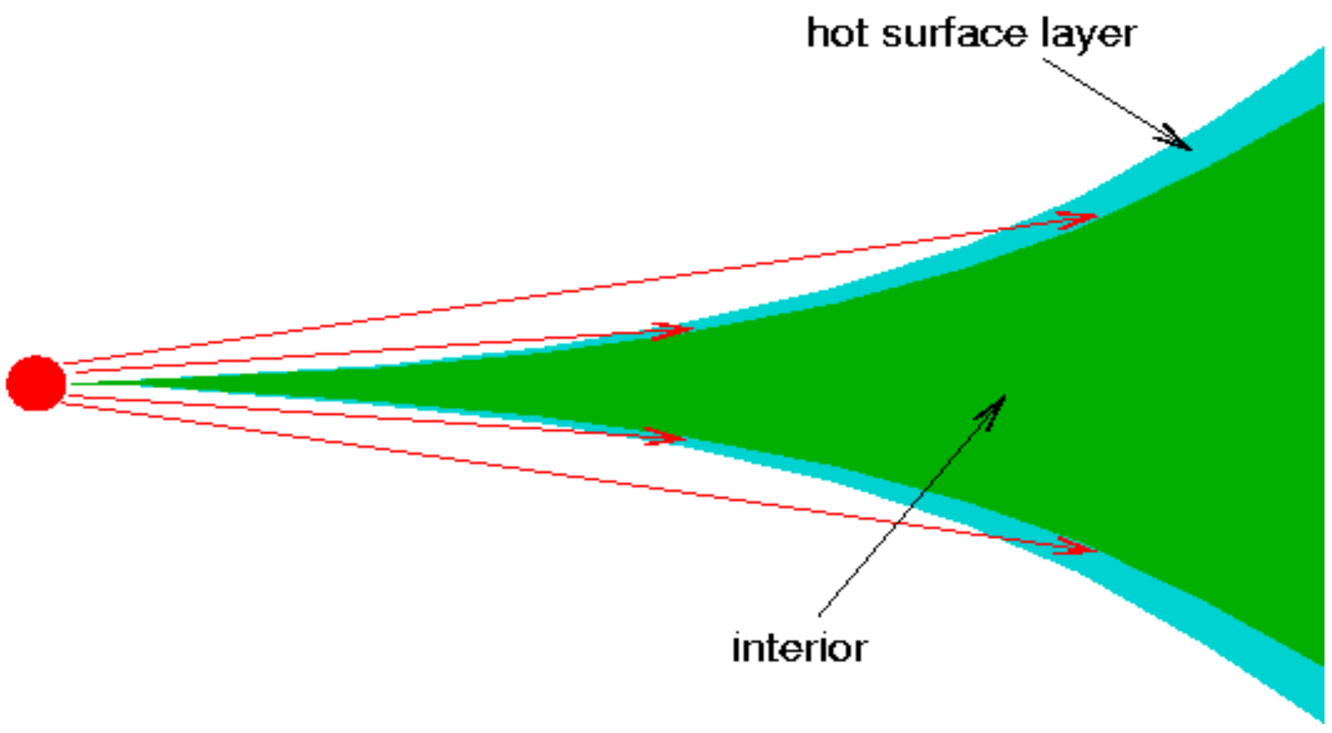


v. Boekel et al 2003

$$I_\lambda^{\rm observed} = I_\lambda^{\rm bg}\, e^{-\tau_\lambda} + I_\lambda^{\rm fg}\,(1 - e^{-\tau_\lambda})$$

Spectrosocopic features

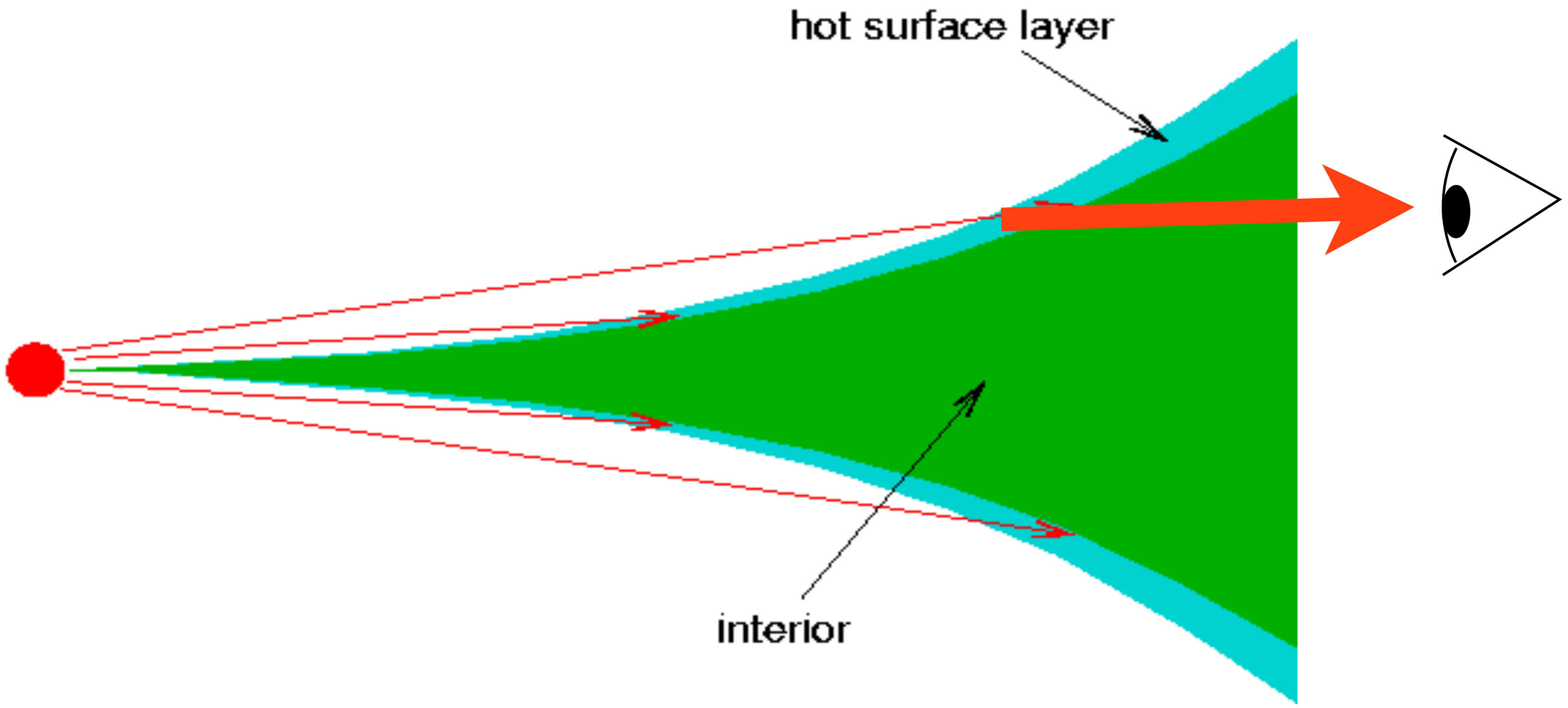
Optically thick disk



$$I_\lambda^{\rm observed} = I_\lambda^{\rm bg}\, e^{-\tau_\lambda} + I_\lambda^{\rm fg}\,(1 - e^{-\tau_\lambda})$$

Spectrosocopic features

Optically thick disk



- $T^{\rm fg} < T^{\rm bg}$
  absorption features

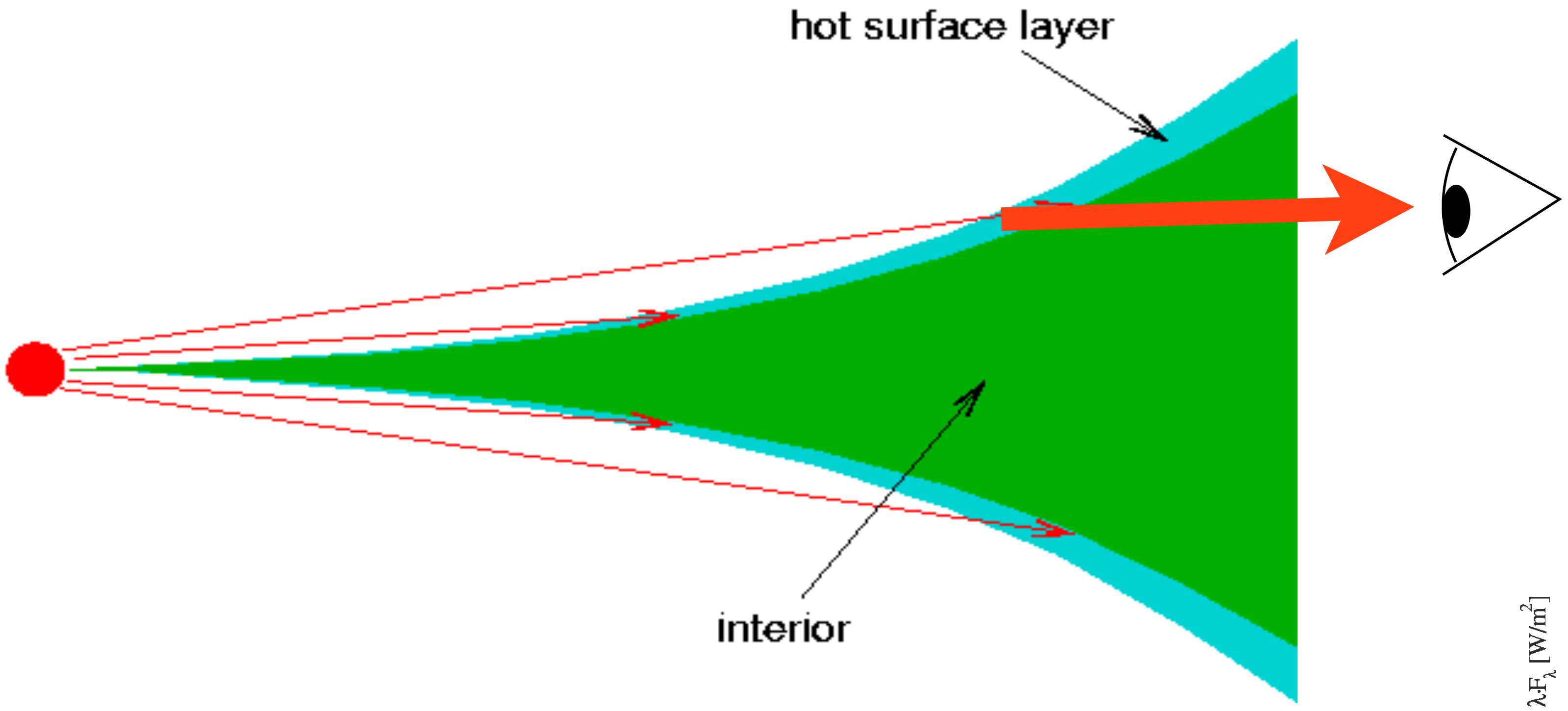
$$I_\lambda^{\rm observed} = I_\lambda^{\rm bg}\, e^{-\tau_\lambda} + I_\lambda^{\rm fg}\,(1 - e^{-\tau_\lambda})$$

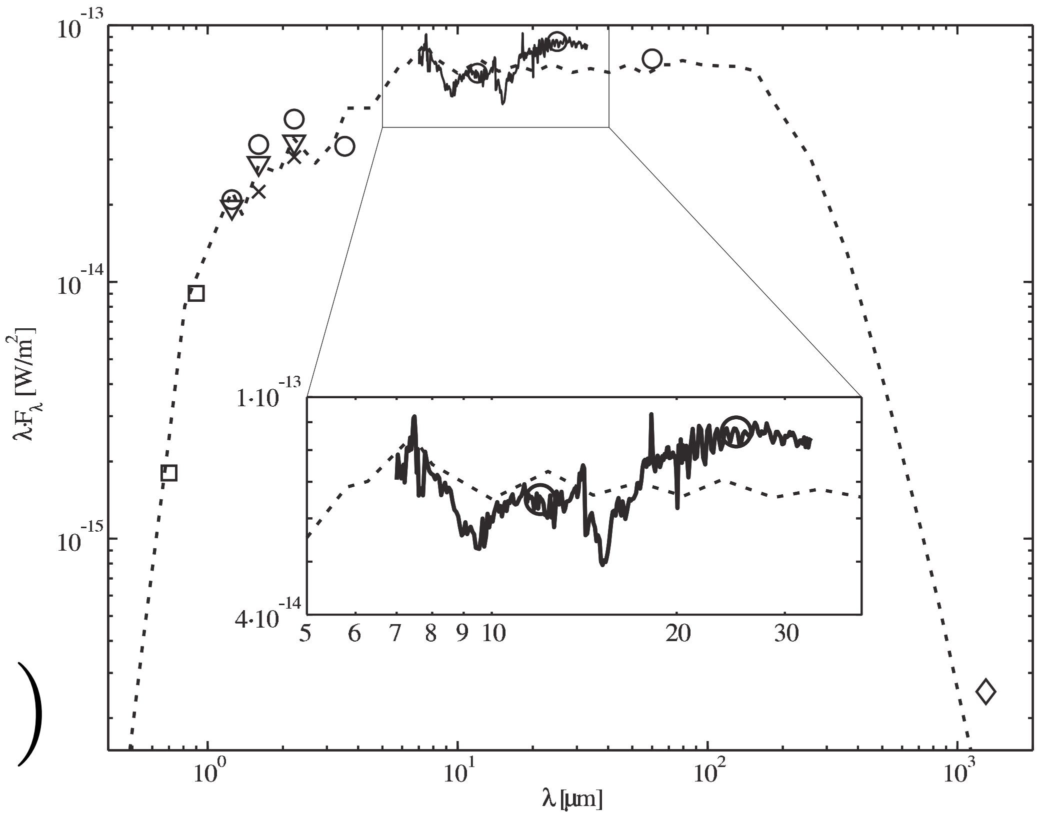
Spectrosocopic features

Optically thick disk

- $T^{\mathrm{fg}} < T^{\mathrm{bg}}$

absorption features





$$I_\lambda^{\mathrm{observed}} = I_\lambda^{\mathrm{bg}}\, e^{-\tau_\lambda} + I_\lambda^{\mathrm{fg}}\,(1 - e^{-\tau_\lambda})$$

Kirchoff's law

Suppose we have a medium at equilibrium at a temperature T :

$$\frac{\mathrm{d}I_\lambda(s, \overrightarrow{n})}{\mathrm{d}s} = 0 \quad \text{and} \quad I_\lambda = B_\lambda(T)$$

$$j_\lambda(s) - \alpha_\lambda(s)\, I_\lambda(s, \overrightarrow{n}) = j_\lambda(s) - \alpha_\lambda(s)\, B_\lambda(T) = 0$$

Kirchoff's law

Suppose we have a medium at equilibrium at a temperature T :

$$\frac{\mathrm{d}I_\lambda(s,\overrightarrow{n})}{\mathrm{d}s} = 0 \quad \text{and} \quad I_\lambda = B_\lambda(T)$$

$$j_\lambda(s) - \alpha_\lambda(s)\, I_\lambda(s,\overrightarrow{n}) = j_\lambda(s) - \alpha_\lambda(s)\, B_\lambda(T) = 0$$

At radiative equilibrium, a good absorber is a good emitter, and a poor absorber is a poor emitter

$$j_\lambda = \alpha_\lambda\, B_\lambda(T)$$

Source function

In the general case, we define

$$S_\lambda \equiv \frac{j_\lambda}{\alpha_\lambda}$$

Source function

In the general case, we define

$$S_\lambda \equiv \frac{j_\lambda}{\alpha_\lambda}$$

The RT equation can be written:

$$\frac{dI_\lambda(s,\overrightarrow{n})}{ds} = \alpha_\lambda(s)\,S_\lambda(s) - \alpha_\lambda(s)\,I_\lambda(s,\overrightarrow{n})$$

or :

$$\frac{dI_\lambda(s,\overrightarrow{n})}{d\tau} = S_\lambda(s) - I_\lambda(s,\overrightarrow{n})$$

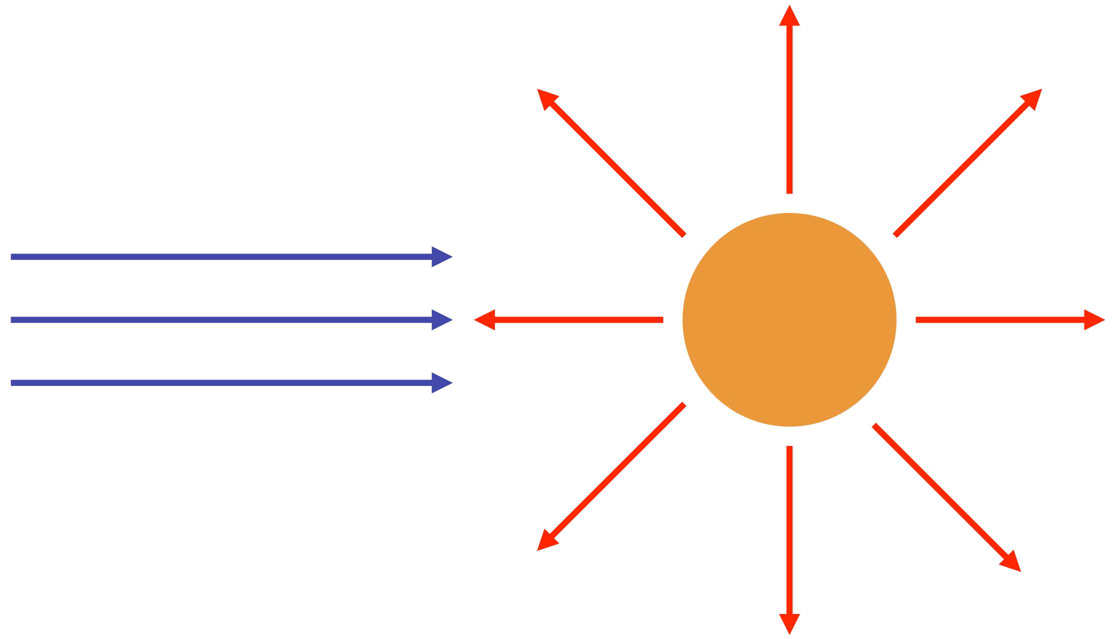
Temperature of a dust grain

Heating:

$$Q_+ = 4\pi \int_0^\infty \kappa_\lambda^{\mathrm{abs}} \, J_\lambda \, \mathrm{d}\lambda$$

Cooling:

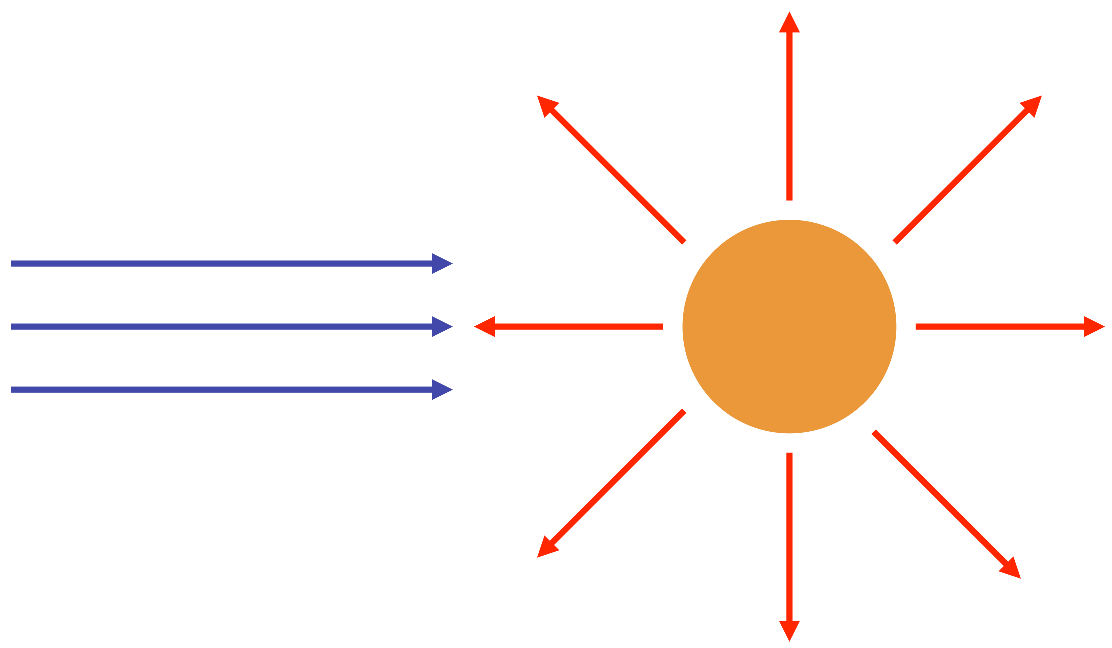
$$Q_- = 4\pi \int_0^\infty \kappa_\lambda^{\mathrm{abs}} \, B_\lambda(T) \, \mathrm{d}\lambda$$



Temperature of a dust grain

Heating:

$$Q_{+} = 4\pi \int_0^\infty \kappa_\lambda^{\rm abs} \, J_\lambda \, {\rm d}\lambda$$



Cooling:

$$Q_{-} = 4\pi \int_0^\infty \kappa_\lambda^{\rm abs} \, B_\lambda(T) \, {\rm d}\lambda$$

Thermal balance:

$$\int_0^\infty \kappa_\lambda^{\rm abs} \, B_\lambda(T) \, {\rm d}\lambda = \int_0^\infty \kappa_\lambda^{\rm abs} \, J_\lambda \, {\rm d}\lambda$$

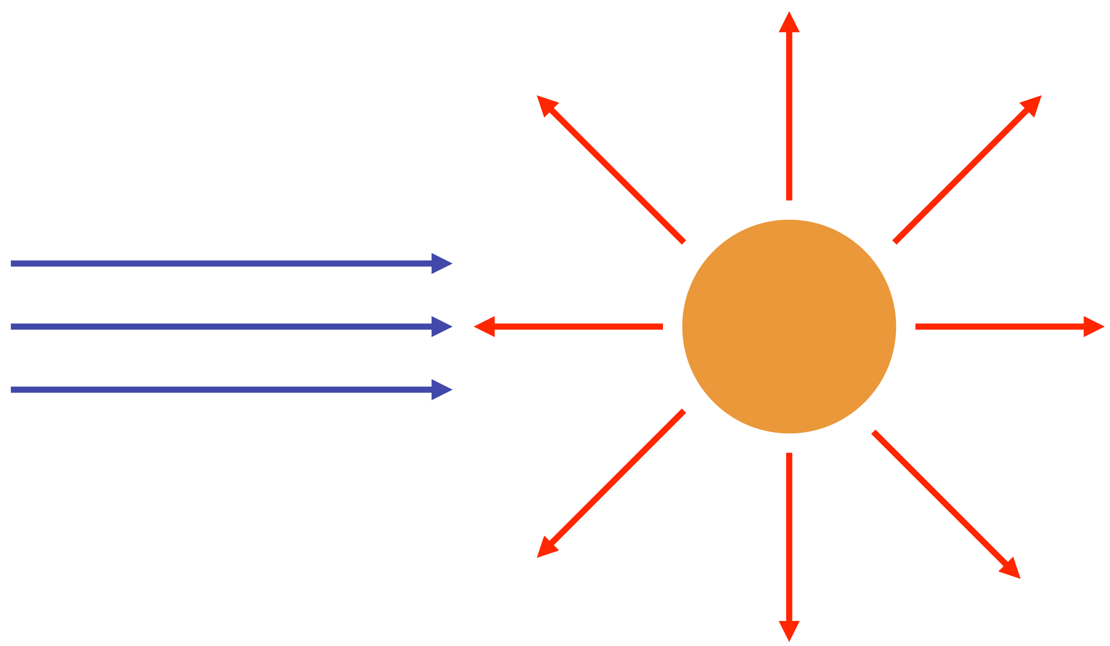
Temperature of a dust grain

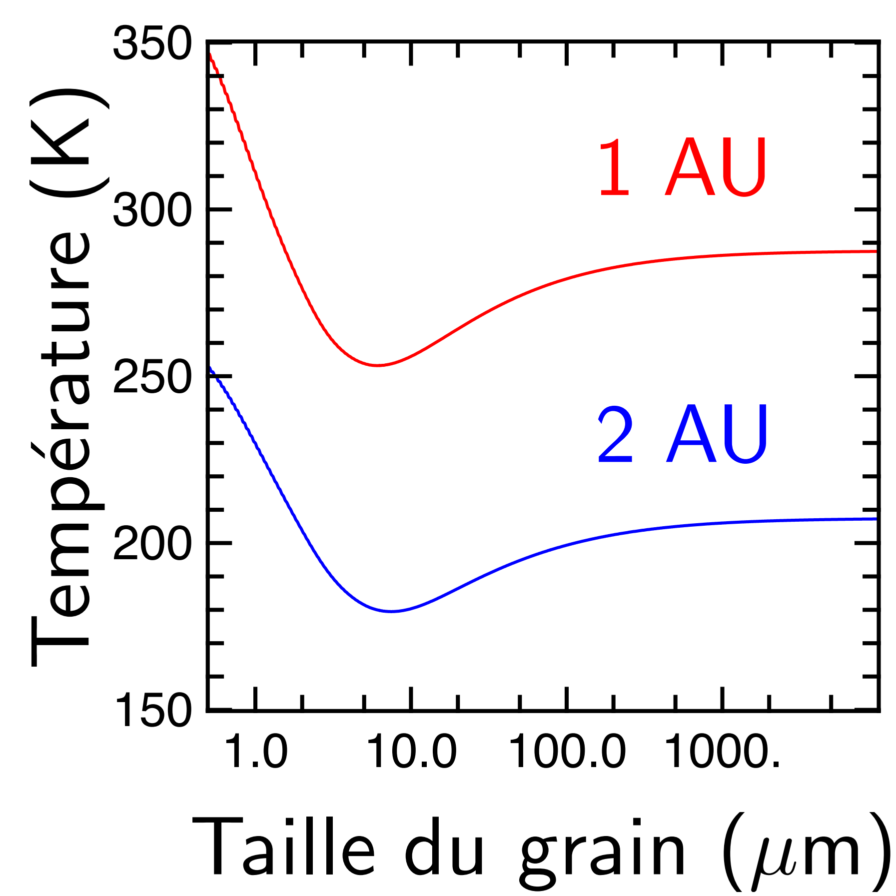
Heating:

$$Q_+ = 4\pi \int_0^\infty \kappa_\lambda^{\rm abs}\, J_\lambda \,{\rm d}\lambda$$

Cooling:

$$Q_- = 4\pi \int_0^\infty \kappa_\lambda^{\rm abs}\, B_\lambda(T) \,{\rm d}\lambda$$

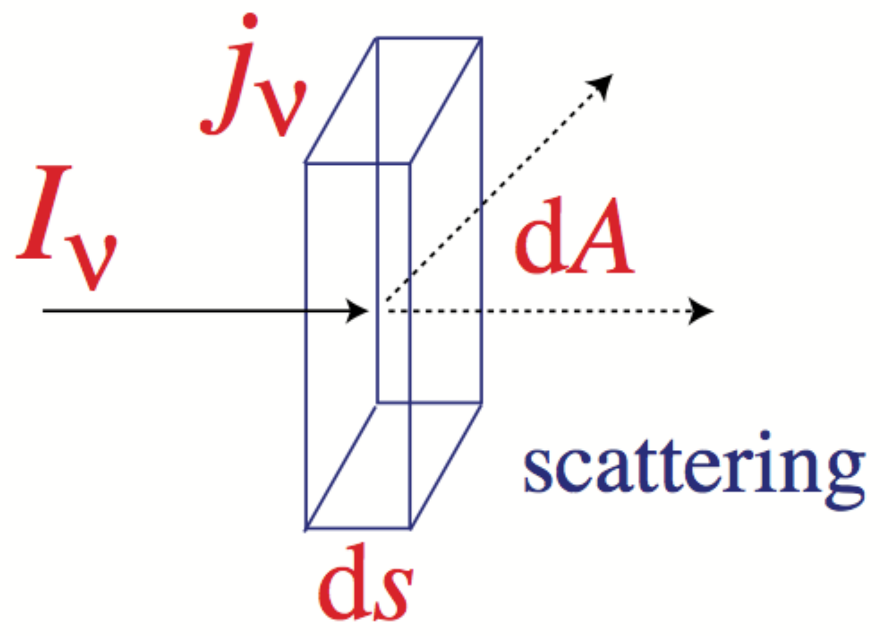




Thermal balance:

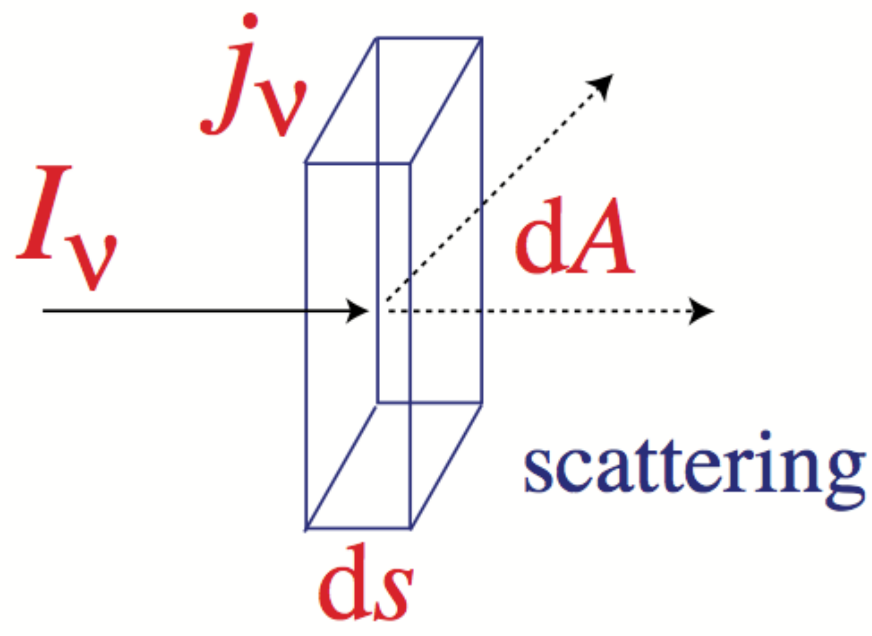
$$\int_0^\infty \kappa_\lambda^{\rm abs}\, B_\lambda(T) \,{\rm d}\lambda = \int_0^\infty \kappa_\lambda^{\rm abs}\, J_\lambda \,{\rm d}\lambda$$

Scattering



$$\frac{dI_\lambda(s,\overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\mathrm{sca}}(s)\,I_\lambda(s,\overrightarrow{n}) + \alpha_\lambda^{\mathrm{scatt}}(s)\,\frac{1}{4\pi}\int_\Omega \psi_\lambda(s,\overrightarrow{n}',\overrightarrow{n})\,I_\lambda(s,\overrightarrow{n}')\,d\Omega'$$

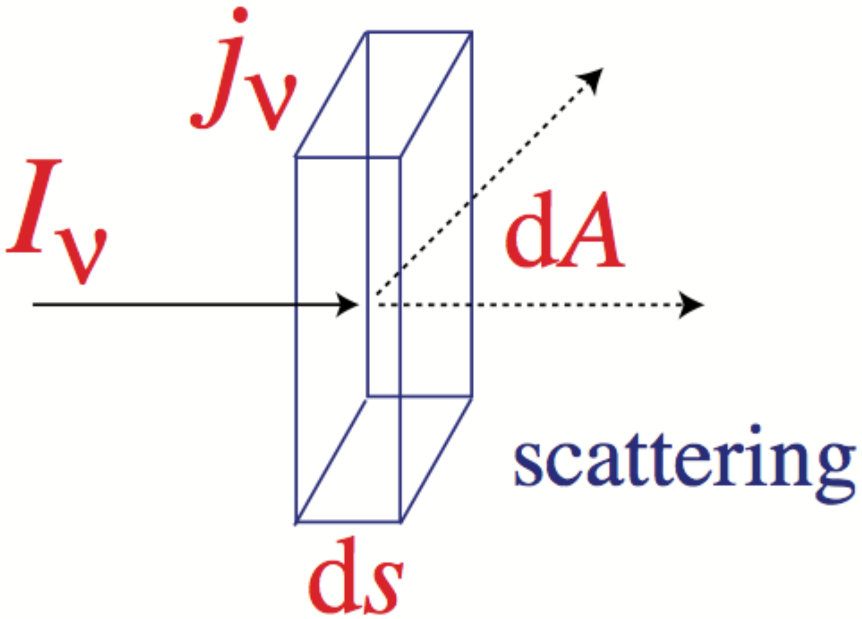
Scattering



$$\frac{dI_\lambda(s,\vec{n})}{ds} = -\,\alpha_\lambda^{\mathrm{sca}}(s)\,I_\lambda(s,\vec{n}) + \alpha_\lambda^{\mathrm{scatt}}(s)\,\frac{1}{4\pi}\int_\Omega \psi_\lambda(s,\vec{n}',\vec{n})\,I_\lambda(s,\vec{n}')\,d\Omega'$$

Phase function: anisotropic

Scattering



$$\frac{dI_\lambda(s,\vec{n})}{ds} = -\,\alpha_\lambda^{\rm sca}(s)\, I_\lambda(s,\vec{n}) + \alpha_\lambda^{\rm scatt}(s)\,\frac{1}{4\pi}\int_\Omega \psi_\lambda(s,\vec{n}',\vec{n})\, I_\lambda(s,\vec{n}')\, d\Omega'$$

Phase function: anisotropic

RT equation is now integro-differential

Dust RT equations

$$\frac{dI_\lambda(s, \overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\rm ext}(s)\, I_\lambda(s, \overrightarrow{n}) + \alpha_\lambda^{\rm abs}(s)\, B_\lambda(T(s)) + \alpha_\lambda^{\rm scatt}(s)\, \frac{1}{4\pi} \int_\Omega \psi_\lambda(s, \overrightarrow{n}', \overrightarrow{n})\, I_\lambda(s, \overrightarrow{n}')\, d\Omega'$$

$$\int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) B_\lambda(T(\overrightarrow{r}))\, d\lambda = \int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) J_\lambda(\overrightarrow{r})\, d\lambda$$

Dust RT equations

$$\frac{dI_\lambda(s,\overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\rm ext}(s)\,I_\lambda(s,\overrightarrow{n}) + \alpha_\lambda^{\rm abs}(s)\,B_\lambda(T(s)) + \alpha_\lambda^{\rm scatt}(s)\,\frac{1}{4\pi}\int_\Omega \psi_\lambda(s,\overrightarrow{n}',\overrightarrow{n})\,I_\lambda(s,\overrightarrow{n}')\,d\Omega'$$

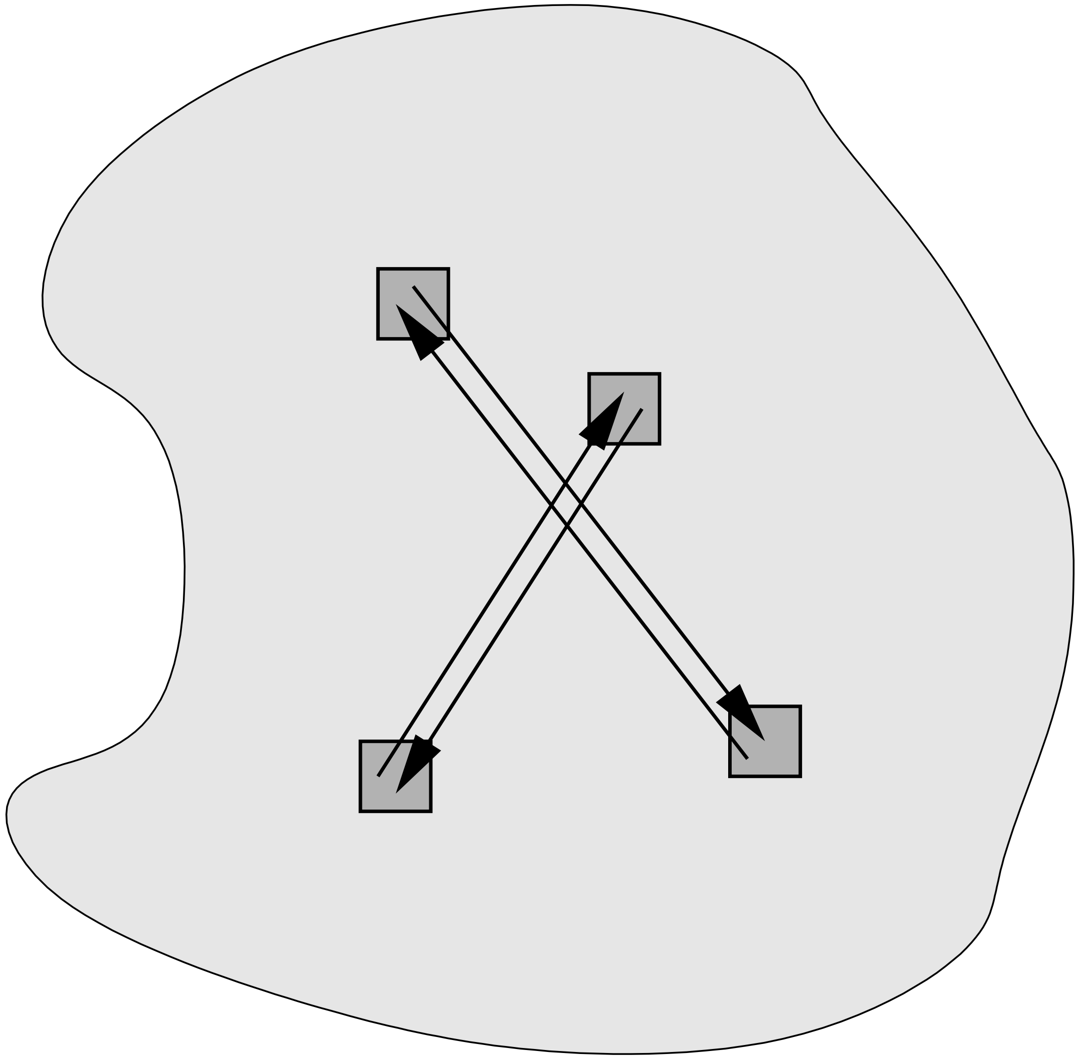
$$\int_0^\infty \kappa^{\rm abs}(\lambda,\overrightarrow{r})B_\lambda(T(\overrightarrow{r}))\,d\lambda = \int_0^\infty \kappa^{\rm abs}(\lambda,\overrightarrow{r})J_\lambda(\overrightarrow{r})\,d\lambda$$

Dust RT equations

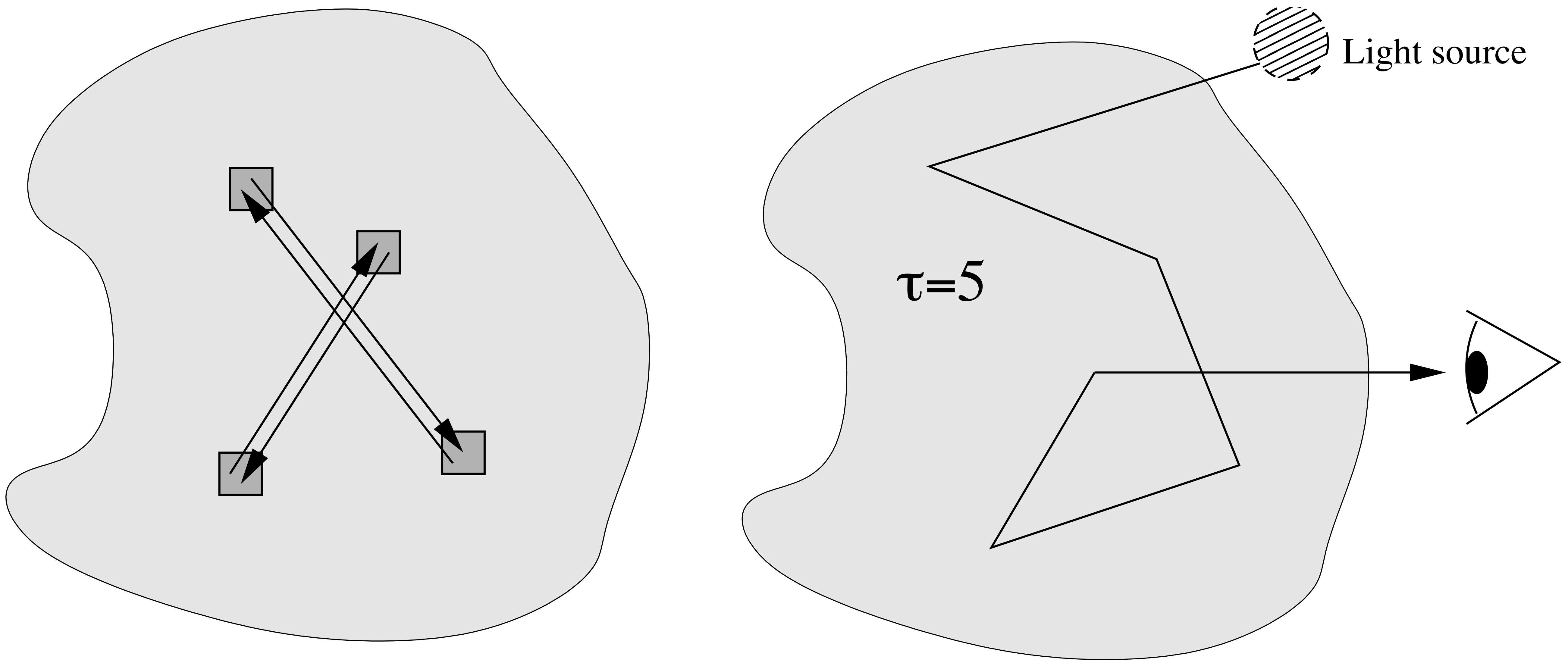
$$\frac{dI_\lambda(s,\overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\rm ext}(s)\,I_\lambda(s,\overrightarrow{n}) + \alpha_\lambda^{\rm abs}(s)\,B_\lambda(T(s)) + \alpha_\lambda^{\rm scatt}(s)\,\frac{1}{4\pi}\int_\Omega \psi_\lambda(s,\overrightarrow{n}',\overrightarrow{n})\,I_\lambda(s,\overrightarrow{n}')\,d\Omega'$$

$$\int_0^\infty \kappa^{\rm abs}(\lambda,\overrightarrow{r})B_\lambda(T(\overrightarrow{r}))\,d\lambda = \int_0^\infty \kappa^{\rm abs}(\lambda,\overrightarrow{r})J_\lambda(\overrightarrow{r})\,d\lambda$$

Dust RT equations



Dust RT equations



Remark 1: time dependance

$$\frac{1}{c}\frac{\partial I_\lambda(s,\overrightarrow{n},t)}{\partial t} + \frac{\partial I_\lambda(s,\overrightarrow{n},t)}{\partial s} = j_\lambda(s) - \alpha_\lambda(s)\, I_\lambda(s,\overrightarrow{n},t)$$

We will assume that light propagation is much faster than the timescale at which the object changes

Not always true !

See for instance Harris et al, 2011

Remark 2: Polarisation

Polarization state of the light can be described by the Stokes parameters

$$I = E$$

$$Q = (E_{\updownarrow} - E_{\leftrightarrow})$$

$$U = (E_{\nearrow\!\!\!\swarrow} - E_{\nwarrow\!\!\!\searrow})$$

$$V = (E_{\circlearrowright} - E_{\circlearrowleft})$$

Remark 2: Polarisation

$$\frac{d\mathbf{S}_\lambda(s, \overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\mathrm{ext}}(s)\,\mathbf{S}_\lambda(s, \overrightarrow{n}) + \alpha_\lambda^{\mathrm{abs}}(s)\,B_\lambda(T(s)) + \alpha_\lambda^{\mathrm{scatt}}(s)\,\frac{1}{4\pi}\int_\Omega \mathcal{M}_\lambda(s, \overrightarrow{n}', \overrightarrow{n})\,\mathbf{S}_\lambda(s, \overrightarrow{n}')\,d\Omega'$$

Remark 2: Polarisation

$$\frac{d\mathbf{S}_\lambda(s, \overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\rm ext}(s)\,\mathbf{S}_\lambda(s, \overrightarrow{n})$$

dichroic extinction

$$+\,\alpha_\lambda^{\rm abs}(s)\,B_\lambda(T(s))$$

$$+\,\alpha_\lambda^{\rm scatt}(s)\,\frac{1}{4\pi}\int_\Omega \mathcal{M}_\lambda(s, \overrightarrow{n}', \overrightarrow{n})\,\mathbf{S}_\lambda(s, \overrightarrow{n}')\,d\Omega'$$

Remark 2: Polarisation

$$\frac{d\mathbf{S}_\lambda(s, \overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\mathrm{ext}}(s)\,\mathbf{S}_\lambda(s, \overrightarrow{n})$$

dichroic extinction

$$+\,\alpha_\lambda^{\mathrm{abs}}(s)\,B_\lambda(T(s))$$

polarised emision

$$+\,\alpha_\lambda^{\mathrm{scatt}}(s)\,\frac{1}{4\pi}\int_\Omega \mathcal{M}_\lambda(s, \overrightarrow{n}', \overrightarrow{n})\,\mathbf{S}_\lambda(s, \overrightarrow{n}')\,d\Omega'$$

Remark 2: Polarisation

$$\frac{d\mathbf{S}_\lambda(s, \overrightarrow{n})}{ds} = -\,\alpha_\lambda^{\rm ext}(s)\,\mathbf{S}_\lambda(s, \overrightarrow{n})$$

dichroic extinction

$$+\,\alpha_\lambda^{\rm abs}(s)\,B_\lambda(T(s))$$

polarised emision

$$+\,\alpha_\lambda^{\rm scatt}(s)\,\frac{1}{4\pi}\int_\Omega \mathcal{M}_\lambda(s, \overrightarrow{n}', \overrightarrow{n})\,\mathbf{S}_\lambda(s, \overrightarrow{n}')\,d\Omega'$$

Mueller matrix

Remark 2: Polarisation

Polarisation by scattering very sensitive to dust properties

Mueller matrix (randomly oriented particles)

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_d = \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{33} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_i$$

Circular polarisation in case of multiple scattering

$$\begin{pmatrix} I = 1 \\ Q = 0 \\ U = 0 \\ V = 0 \end{pmatrix} \overset{1^{\text{ère}}\text{diff}}{\overbrace{\longrightarrow}} \begin{pmatrix} I = 1 \\ Q \neq 0 \\ U = 0 \\ V = 0 \end{pmatrix} \overset{2^{\text{ème}}\text{diff}}{\overbrace{\longrightarrow}} \begin{pmatrix} I = 1 \\ Q \neq 0 \\ U \neq 0 \\ V \neq 0 \end{pmatrix}$$

Remark 3: line transfer

$$\frac{dI_\nu}{ds} = j_\nu(s) - \alpha_\nu(s) I_\nu(s)$$

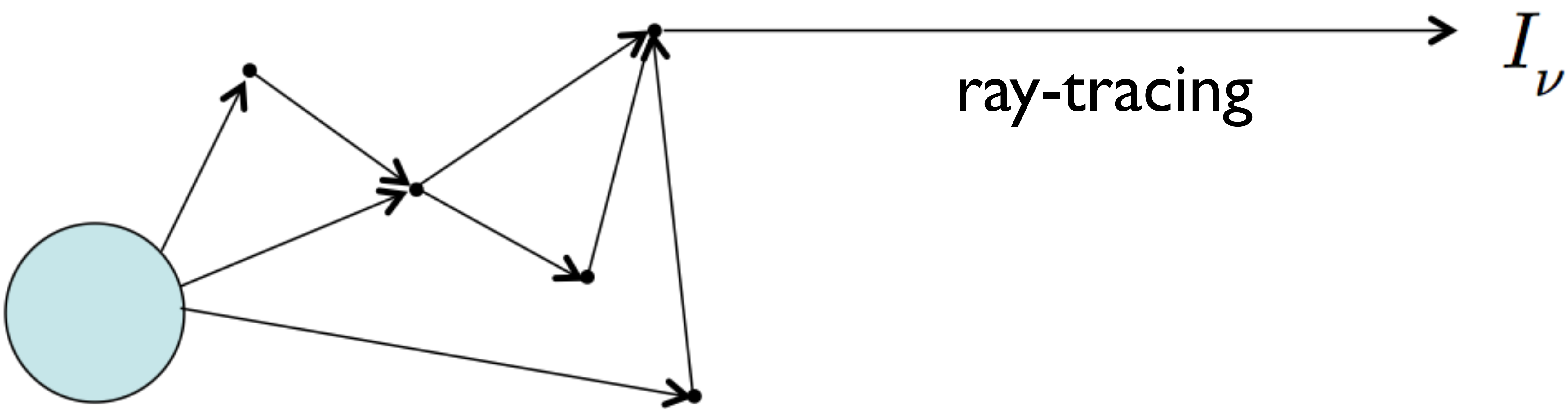
$$j_{ij,\nu} = \frac{h\nu_{ij}}{4\pi} N_i A_{ij} \phi_{ij}(\nu)$$

$$\alpha_{ij,\nu} = \frac{h\nu_{ij}}{4\pi} (N_j B_{ji} - N_i B_{ij}) \phi_{ij}(\nu)$$

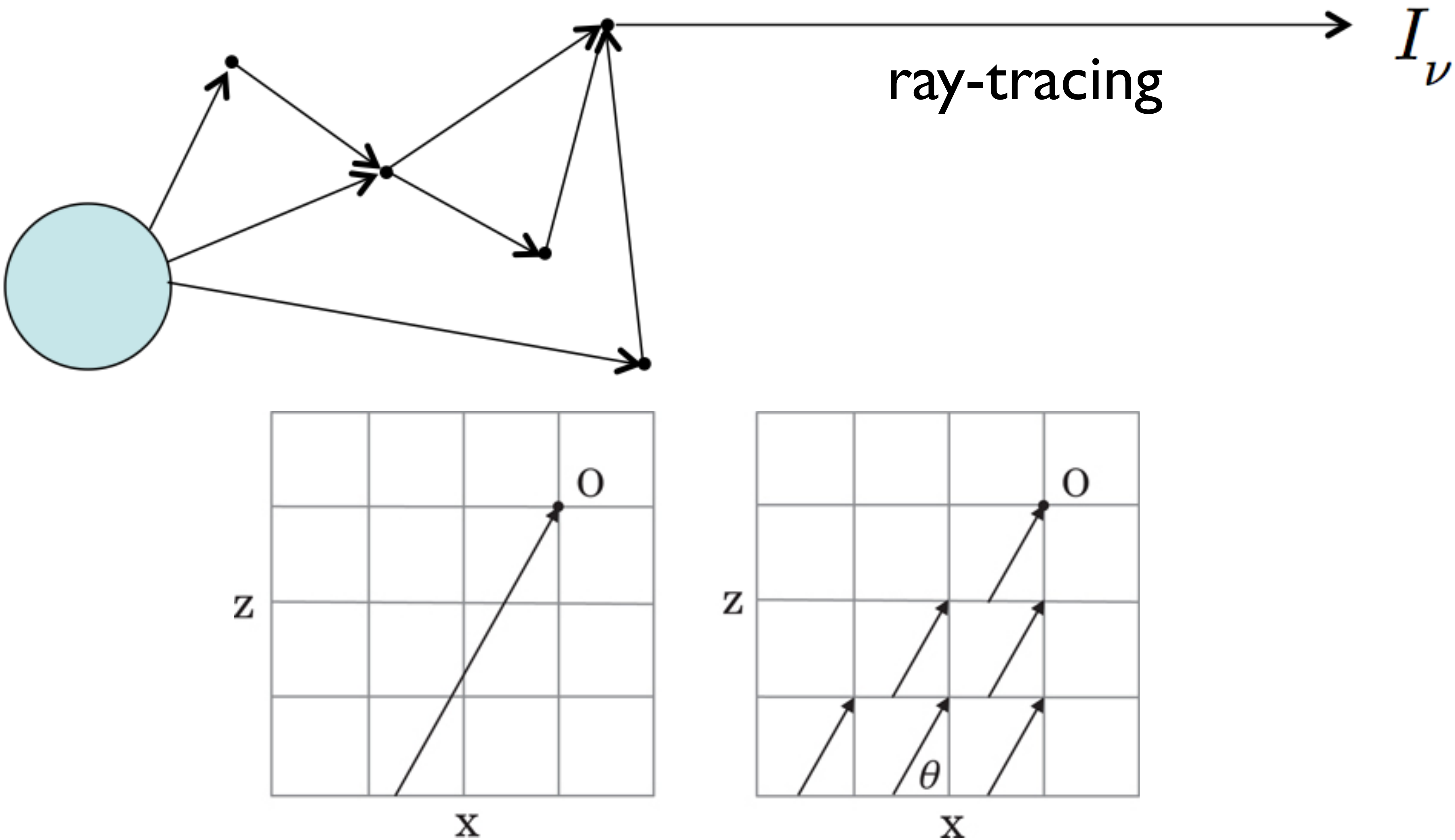
Level populations Ni, and then opacities depend on temperature

How do we solve the RT equation ?

Discrete-ordinate methods

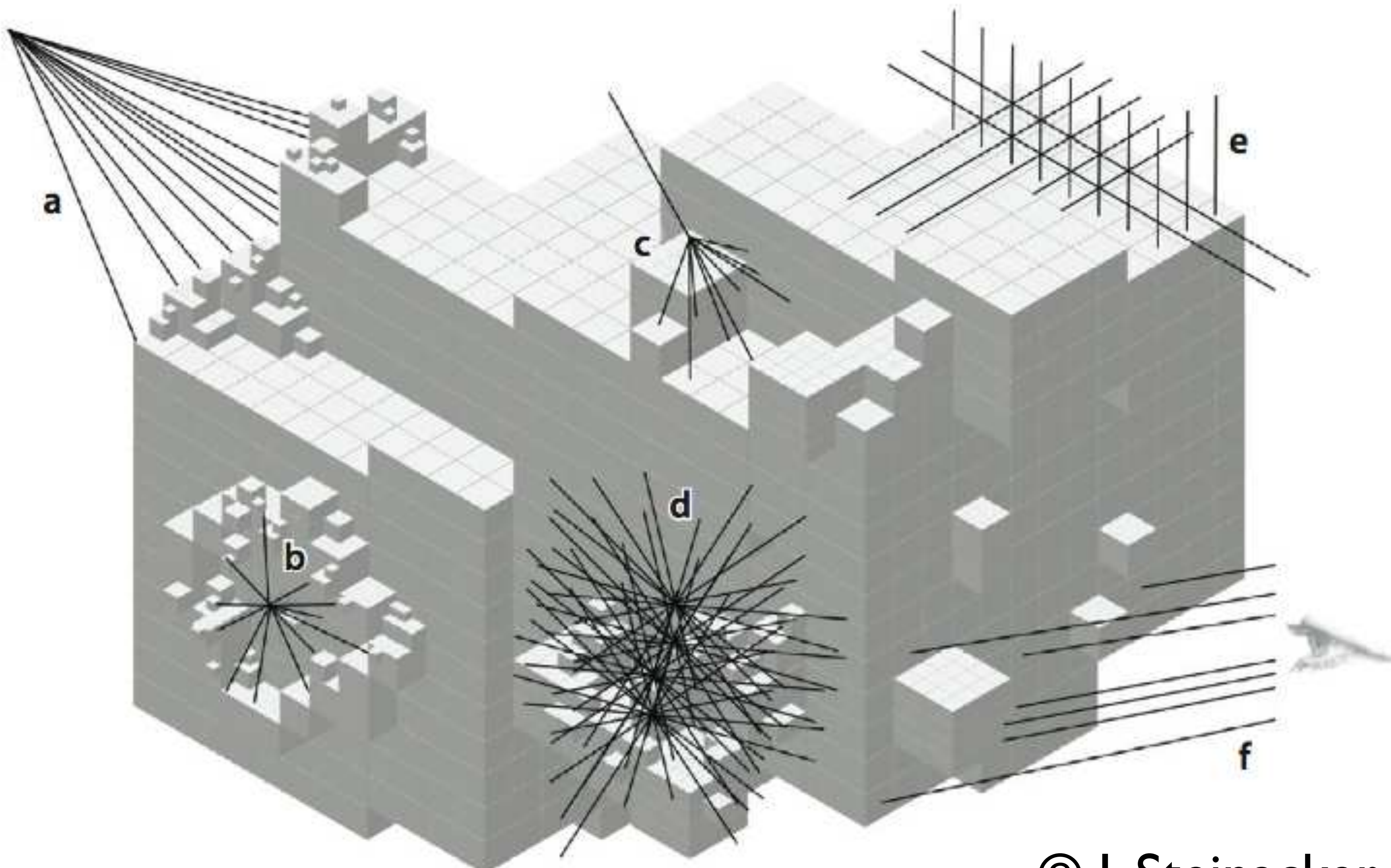


Discrete-ordinate methods



long or short characteristics

Choice of rays can become VERY complex



To my knowledge: only 1 ray-tracing code in 3D

Lambda-iteration

- Make an inital guess for J_λ, compute S_λ
- Integrate the formal RT equation along a large number of rays
- Recompute J_λ
- Loop until converged

Lambda-iteration

- Make an inital guess for J_λ, compute S_λ
- Integrate the formal RT equation along a large number of rays
- Recompute J_λ
- Loop until converged

$$J_\lambda = \Lambda[S_\lambda]$$

Lambda-iteration

- Make an inital guess for J_λ, compute S_λ
- Integrate the formal RT equation along a large number of rays
- Recompute J_λ
- Loop until converged

$$J_\lambda = \Lambda[S_\lambda]$$

Pb : need $N_{\rm iter} \gg \tau^2$

Lambda-iteration

- Make an inital guess for J_λ, compute S_λ
- Integrate the formal RT equation along a large number of rays
- Recompute J_λ
- Loop until converged

$$J_\lambda = \Lambda[S_\lambda]$$

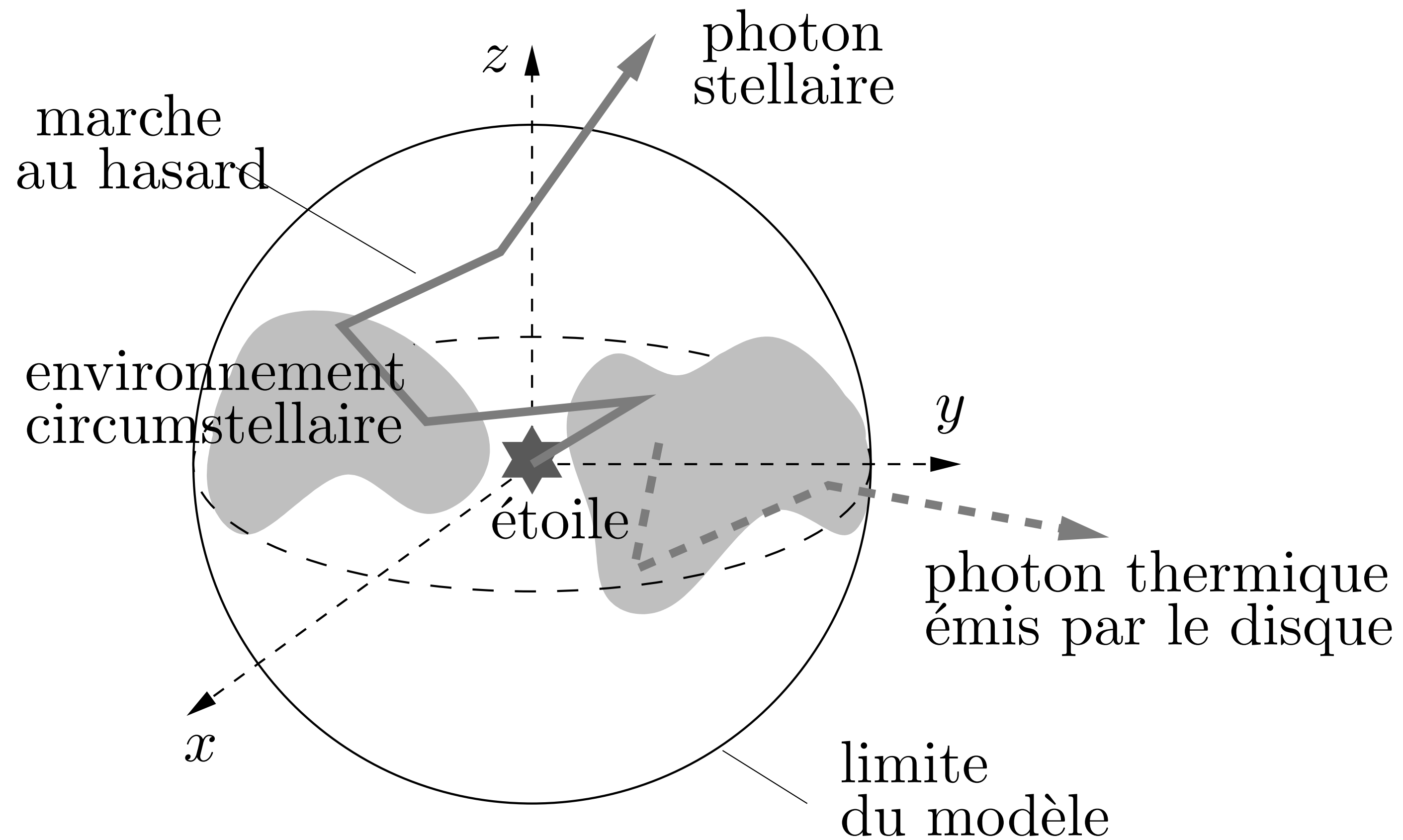
Pb : need $N_{\rm iter} \gg \tau^2$ ⇒ Accelerated - LI

$$\Lambda = \Lambda^* + (\Lambda - \Lambda^*)$$

Monte Carlo Basics

- Idea :
  Propagate many photon packets by randomly sampling from probability distribution functions for directions, wavelengths, path lengths, interactions with dust.

- mimics the motion of photons



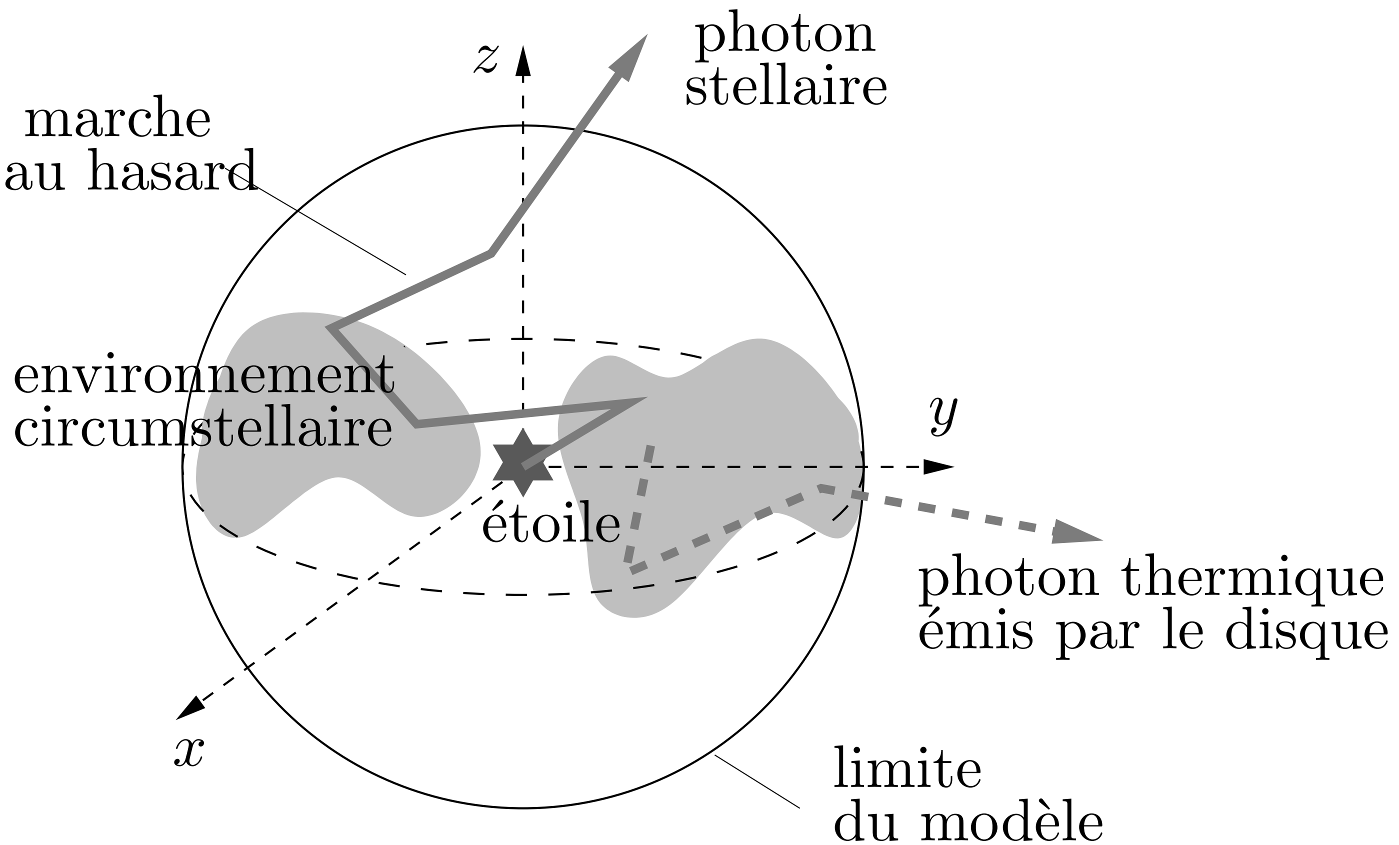
Monte Carlo Basics

- Idea :
  Propagate many photon packets by randomly sampling from probability distribution functions for directions, wavelengths, path lengths, interactions with dust.

- mimics the motion of photons

Advantages:

- easy to include physics
- intrinsically 3D
- Fast: variance reduction techniques, diffusion approx., ray-tracing



Monte Carlo Basics

- Emit a photon packet = luminosity packet
- packet travels some distance
- packet interacts with dust :
  - scattering : change direction / polarization
  - absorption : kill the packet

Loop until packet exits
10^6 to 10^9 times

Monte Carlo Basics

- Emit a photon packet = luminosity packet
- packet travels some distance
- packet interacts with dust :
    - scattering : change direction / polarization
    - absorption : kill the packet

Loop until packet exits
10^6 to 10^9 times

- Compute Temperature
- Re-emit absorbed packets according to

  $\kappa_\lambda^{\mathrm{abs}} B_\lambda(T)$

- Collect packets when they exit to make observables

Probability of interation

Intensity differential over d*l* *is* $\mathrm{d}I_\lambda = -\alpha \, I_\lambda \, \mathrm{d}l$

Probability of interaction over d*l* $\alpha \, \mathrm{d}l$

Probability of interation

Intensity differential over d*l* *is* $$\mathrm{d}I_\lambda = -\alpha\, I_\lambda\, \mathrm{d}l$$

Probability of interaction over d*l* $$\alpha\, \mathrm{d}l$$

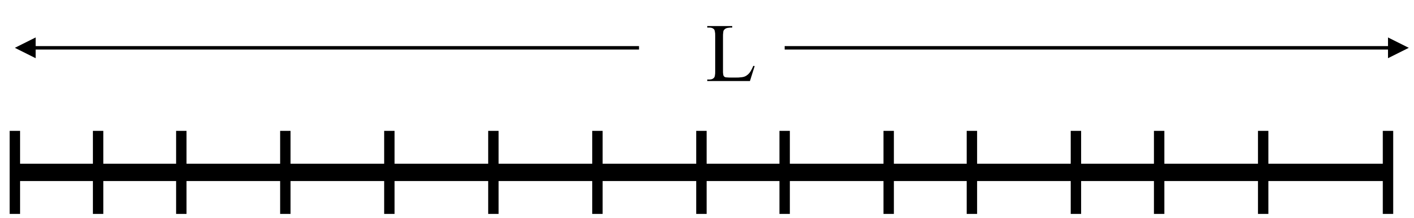
Probability of travelling d*l* without interaction $$1 - \alpha\, \mathrm{d}l$$

Probability of interation

Intensity differential over d*l* *is* $$\mathrm{d}I_\lambda = -\alpha\, I_\lambda\, \mathrm{d}l$$

Probability of interaction over d*l* $$\alpha\, \mathrm{d}l$$

Probability of travelling d*l* without interaction $$1 - \alpha\, \mathrm{d}l$$



n segments

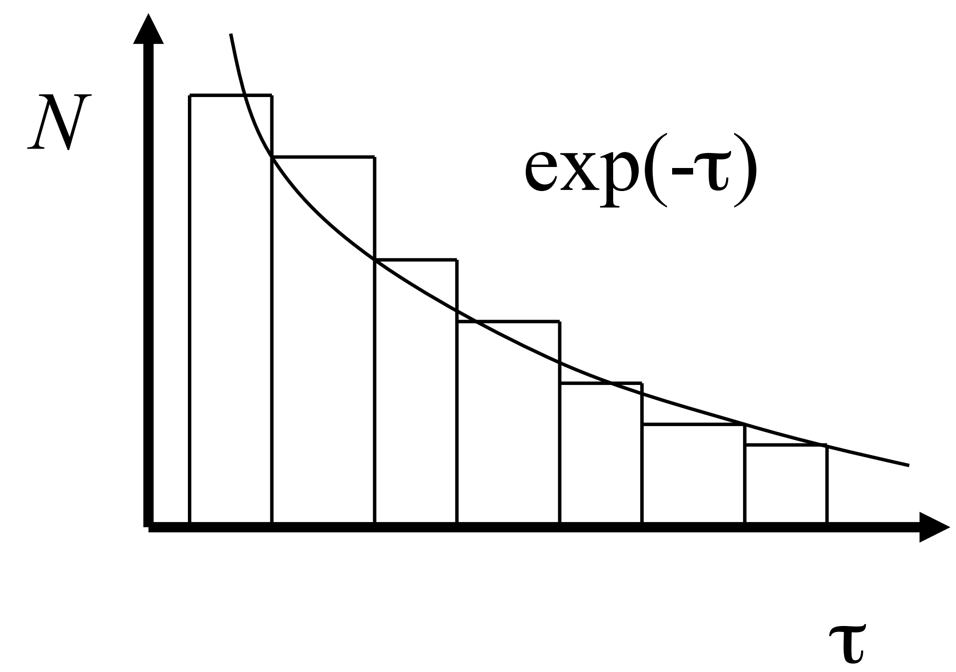
Probability of travelling L without interaction:

$$P(L) = \left(1 - \alpha \frac{L}{n}\right)^n \underset{n \to \infty}{\rightarrow} \exp(-\alpha L) = \exp(-\tau)$$

Probability distribution function

PDF for photon to travel τ is exp(-τ)

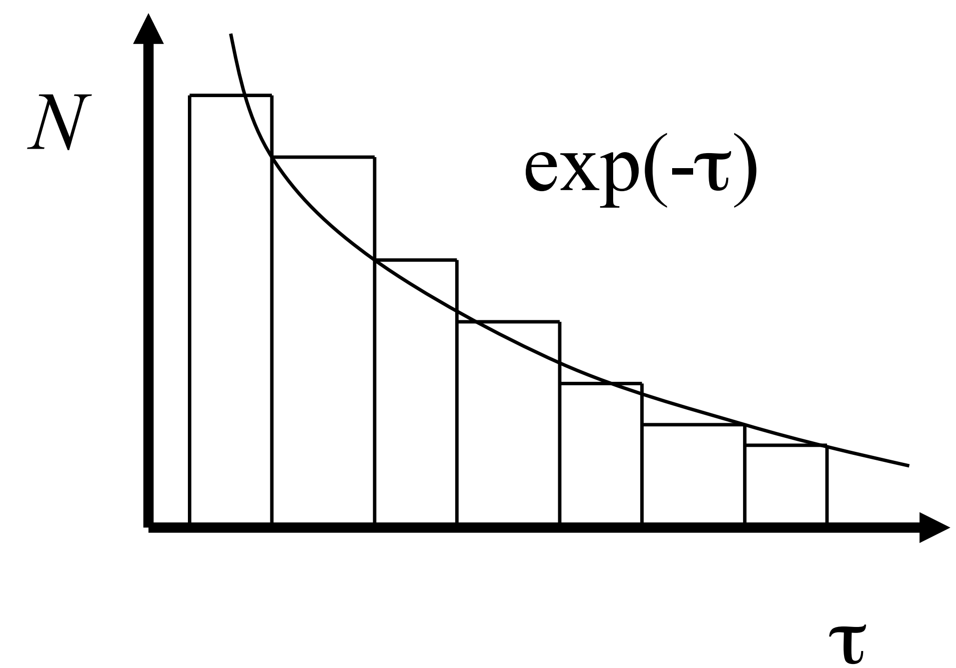
We want to pick a lot of small τ and fewer large τ



Probability distribution function

PDF for photon to travel τ is exp(-τ)

We want to pick a lot of small τ and fewer large τ



Same for all quantities: position, emission angle, scattering angle, wavelength, ...

Cumulative distribution function

We want to map any probability distribution to an uniform distribution from 0 to 1

$$\mathrm{CDF} = F(x) = \int_a^x p(x')\,\mathrm{d}x'$$

$$\int_a^b p(x')\,\mathrm{d}x' = 1$$

$$Y = F(X)$$

uniform distribution in [0,1]

$$F(a) = 0$$

$$F(b) = 1$$

Cumulative distribution function

We want to map any probability distribution to an uniform distribution from 0 to 1

$$\mathrm{CDF} = F(x) = \int_a^x p(x')\,\mathrm{d}x' \qquad \int_a^b p(x')\,\mathrm{d}x' = 1$$

$Y = F(X)$ uniform distribution in [0,1]

$$F(a) = 0$$
$$F(b) = 1$$

Pick a random number $\mathcal{A}_i = F(x) \text{ in } [0,1]$

$x = F^{-1}(\mathcal{A}_i)$ is following $p(x)$

Cumulative distribution function

$$\mathcal{A} = \int_0^{\tau} e^{-\tau'}\,\mathrm{d}\tau' = 1 - e^{-\tau} \quad \Rightarrow \quad \tau = -\log(1-\mathcal{A})$$

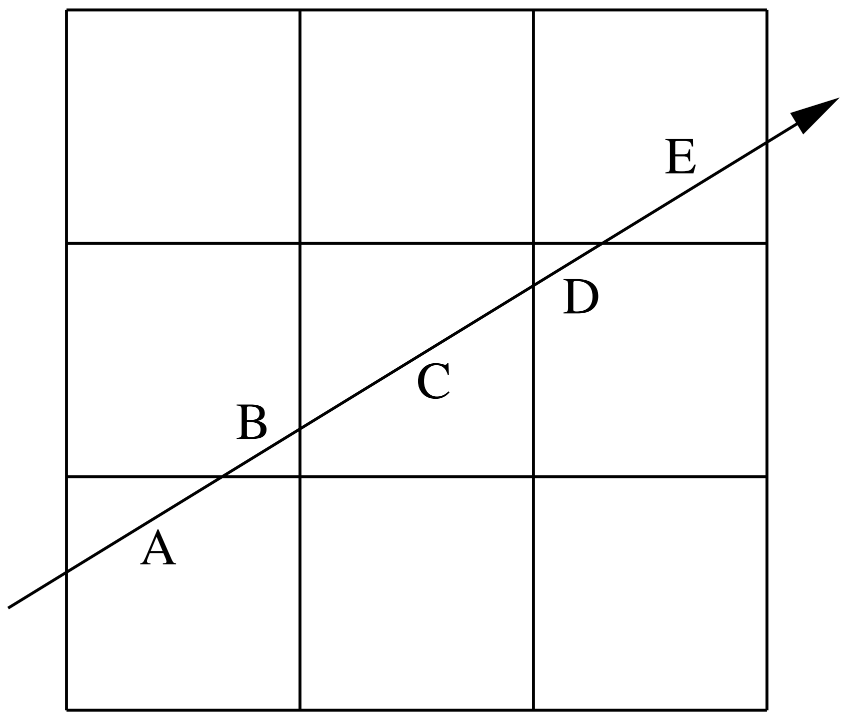
$$\text{or } \tau = -\log \mathcal{A}$$

Cumulative distribution function

$$\mathcal{A} = \int_0^{\tau} e^{-\tau'}\,\mathrm{d}\tau' = 1 - e^{-\tau} \quad \Rightarrow \quad \tau = -\log(1 - \mathcal{A})$$

or $\tau = -\log \mathcal{A}$

$$\tau = \int_0^{l} \alpha \,\mathrm{d}s$$

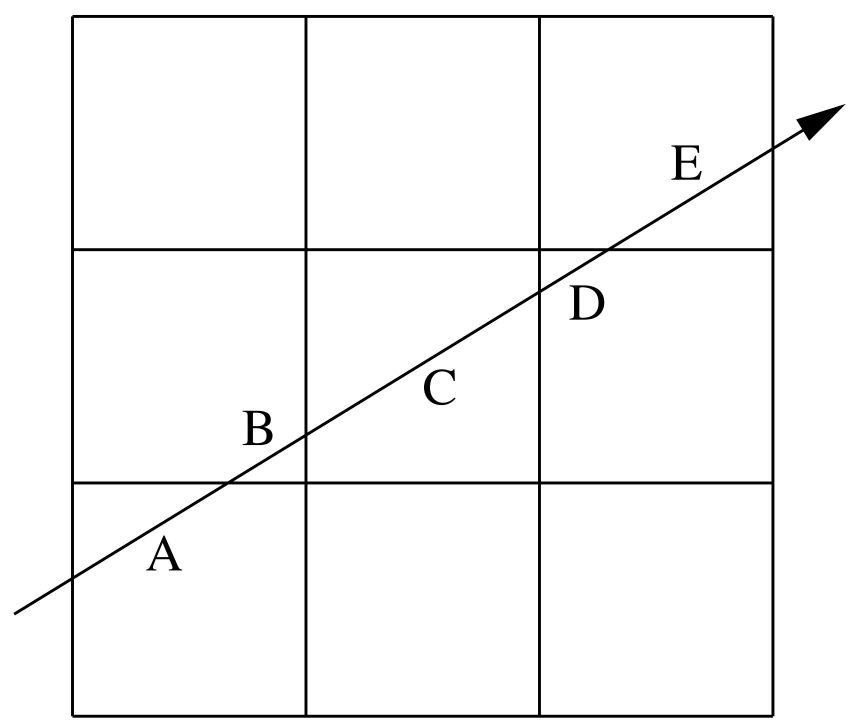


Cumulative distribution function

$$\mathcal{A} = \int_0^{\tau} e^{-\tau'}\,\mathrm{d}\tau' = 1 - e^{-\tau} \quad \Rightarrow \quad \tau = -\log(1-\mathcal{A})$$

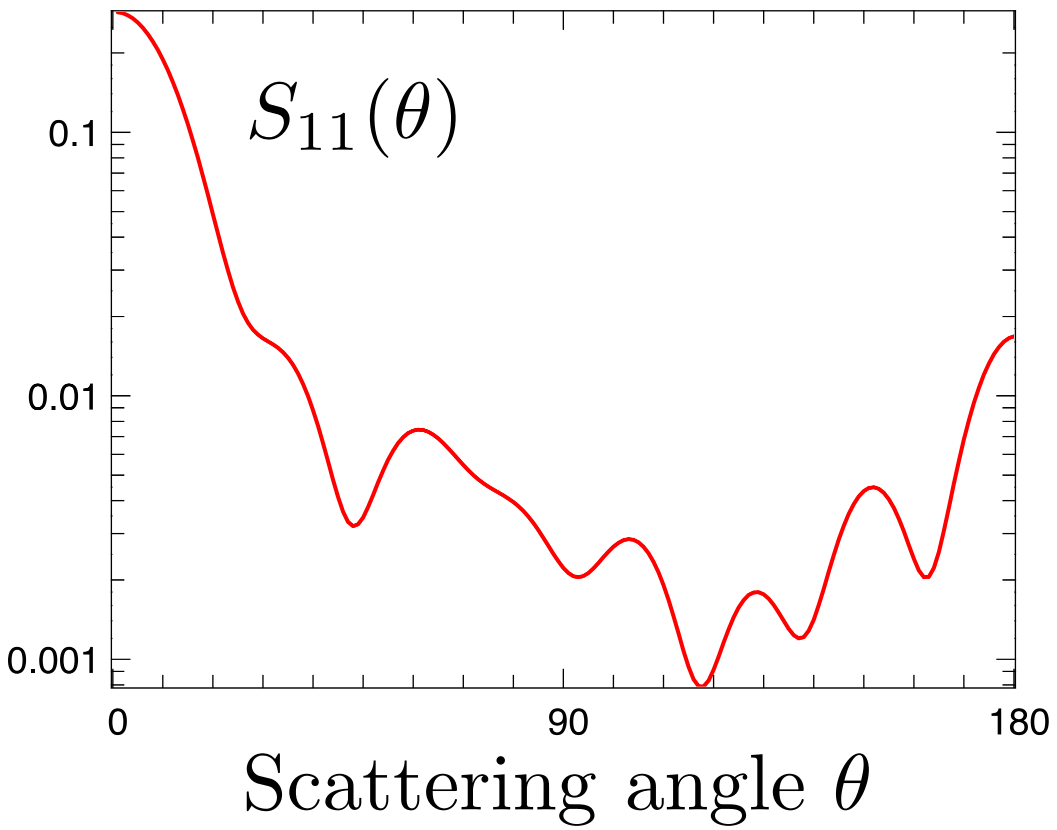
or $\tau = -\log \mathcal{A}$

$$\tau = \int_0^{l} \alpha\,\mathrm{d}s$$



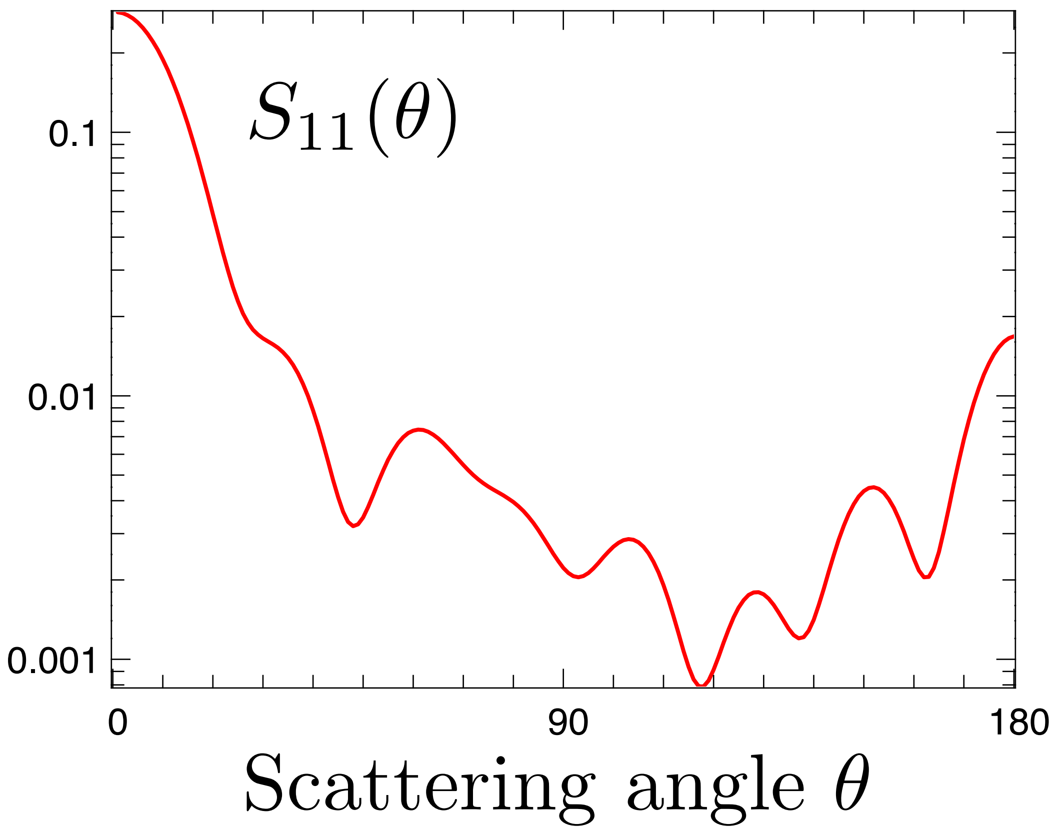
This is the most expensive part !

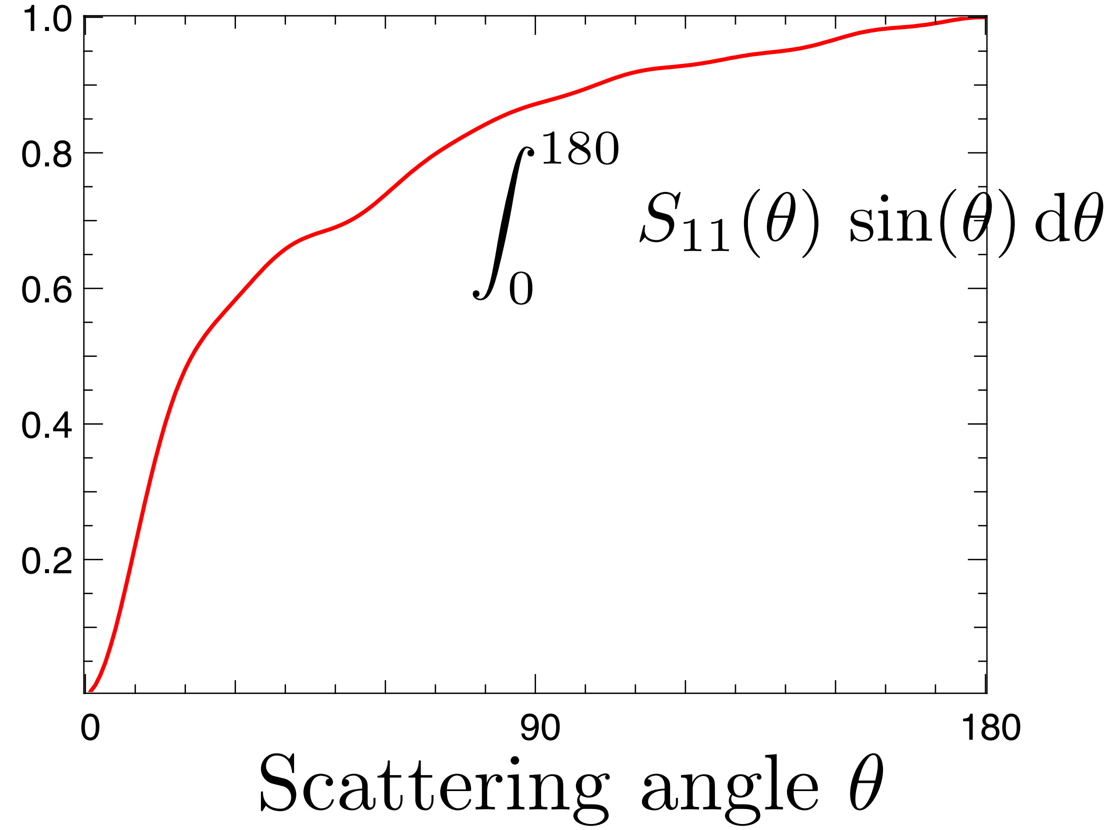
Scattering Probability distribution



This is not a realistic phase function

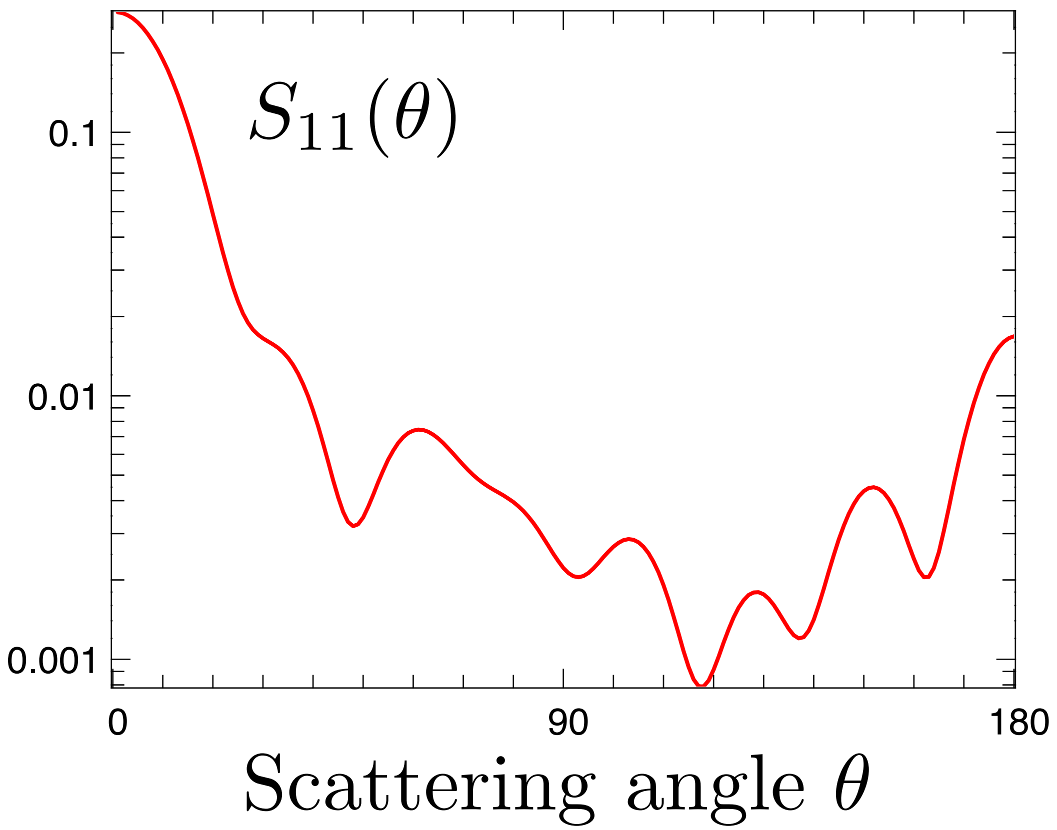
Scattering Probability distribution

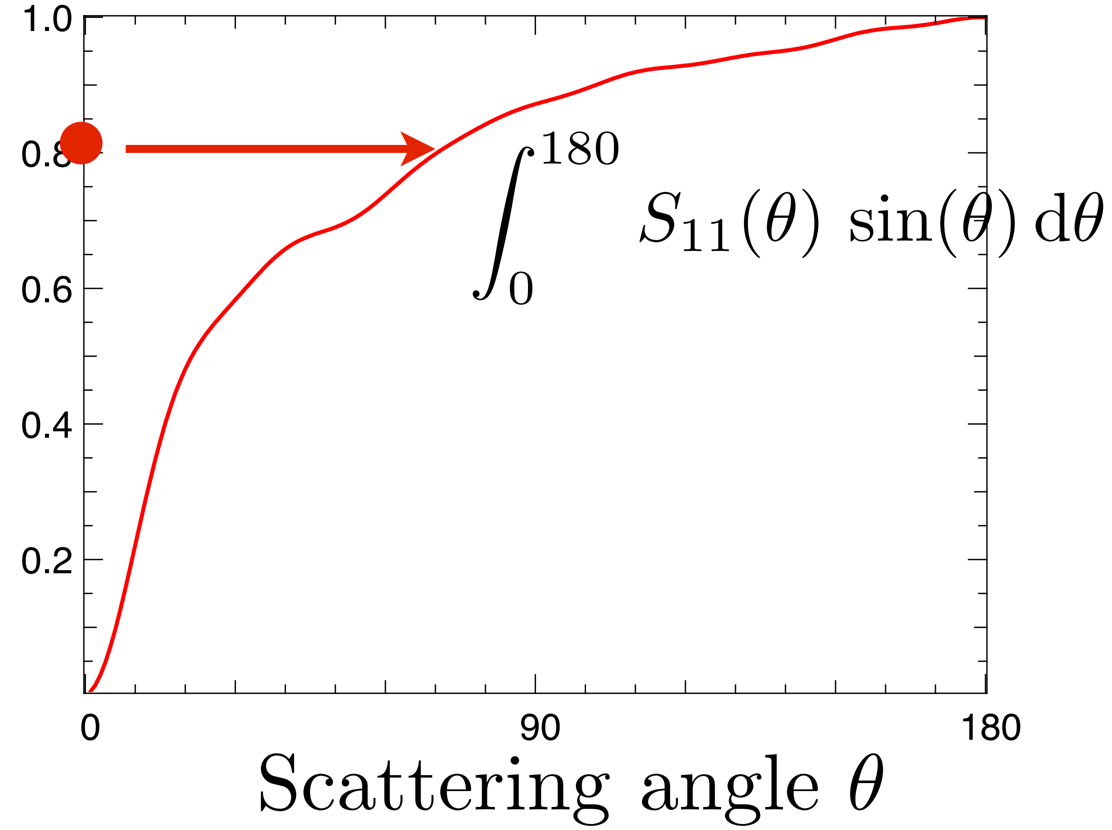




This is not a realistic phase function

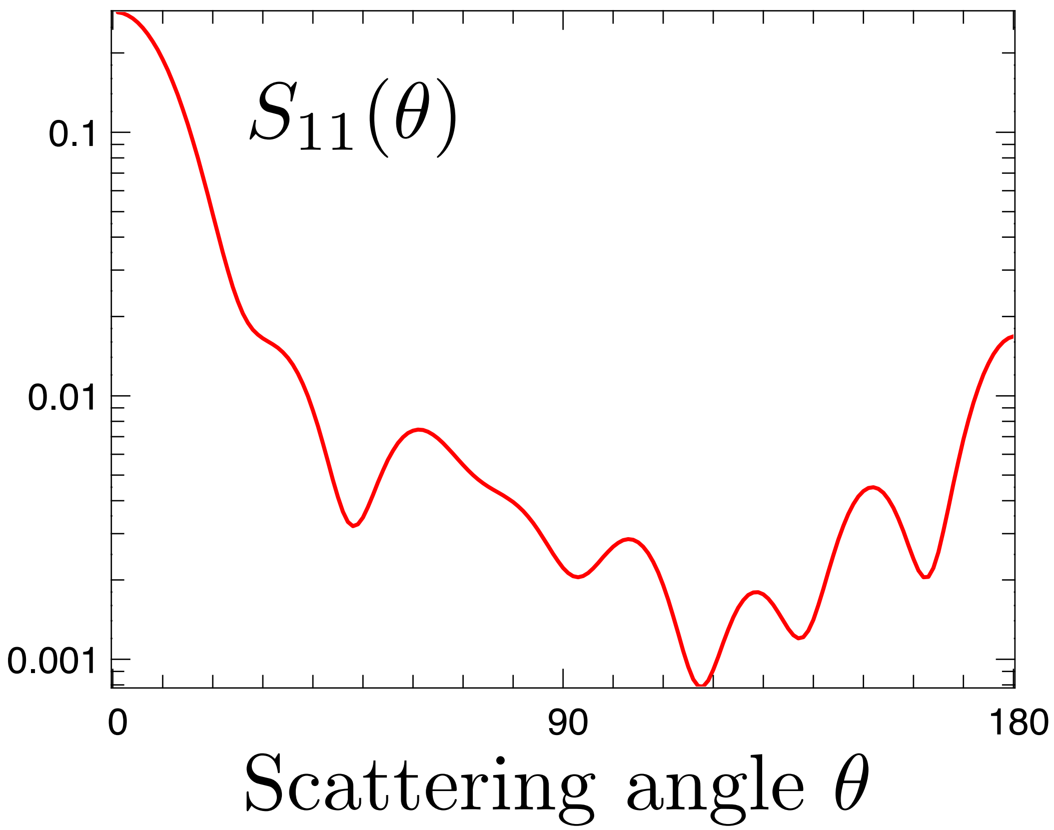
Scattering Probability distribution

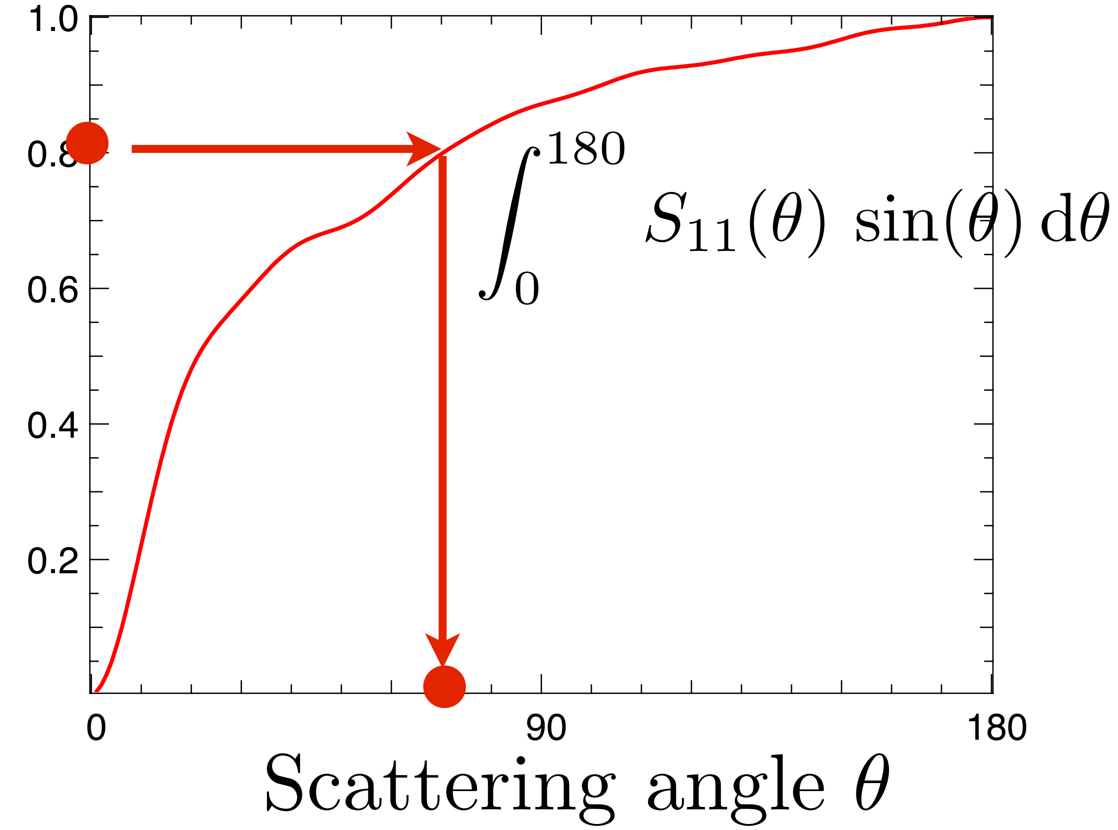




This is not a realistic phase function

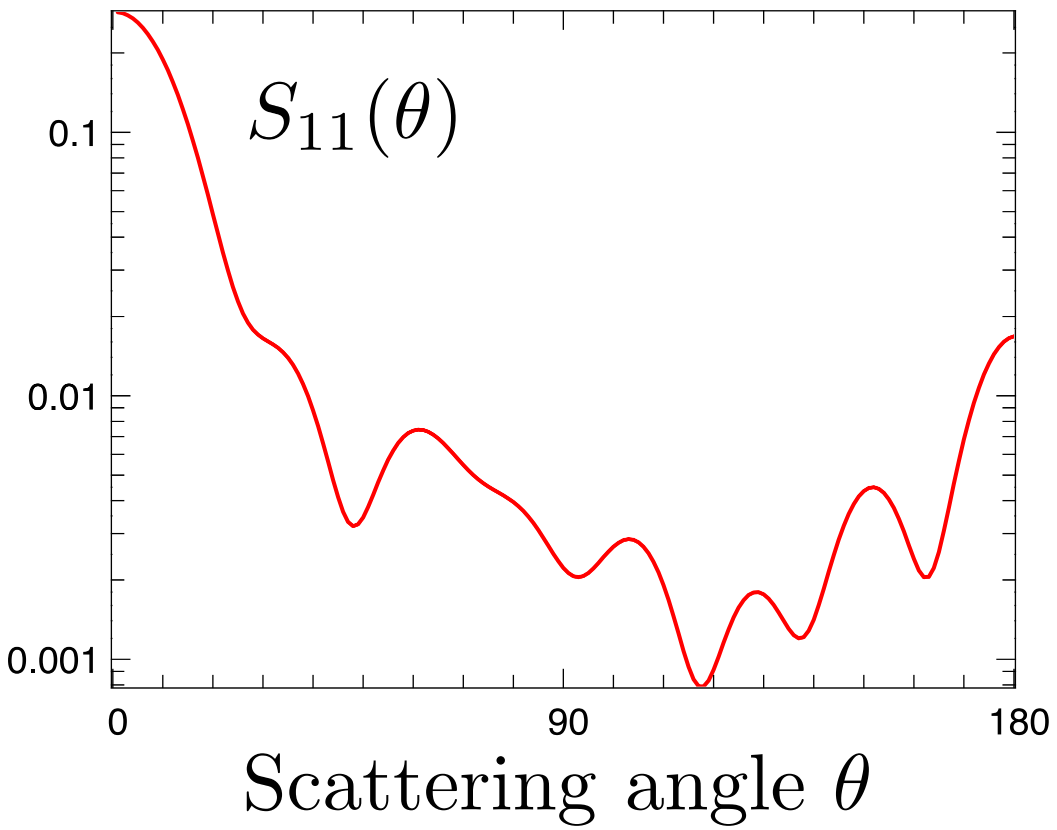
Scattering Probability distribution

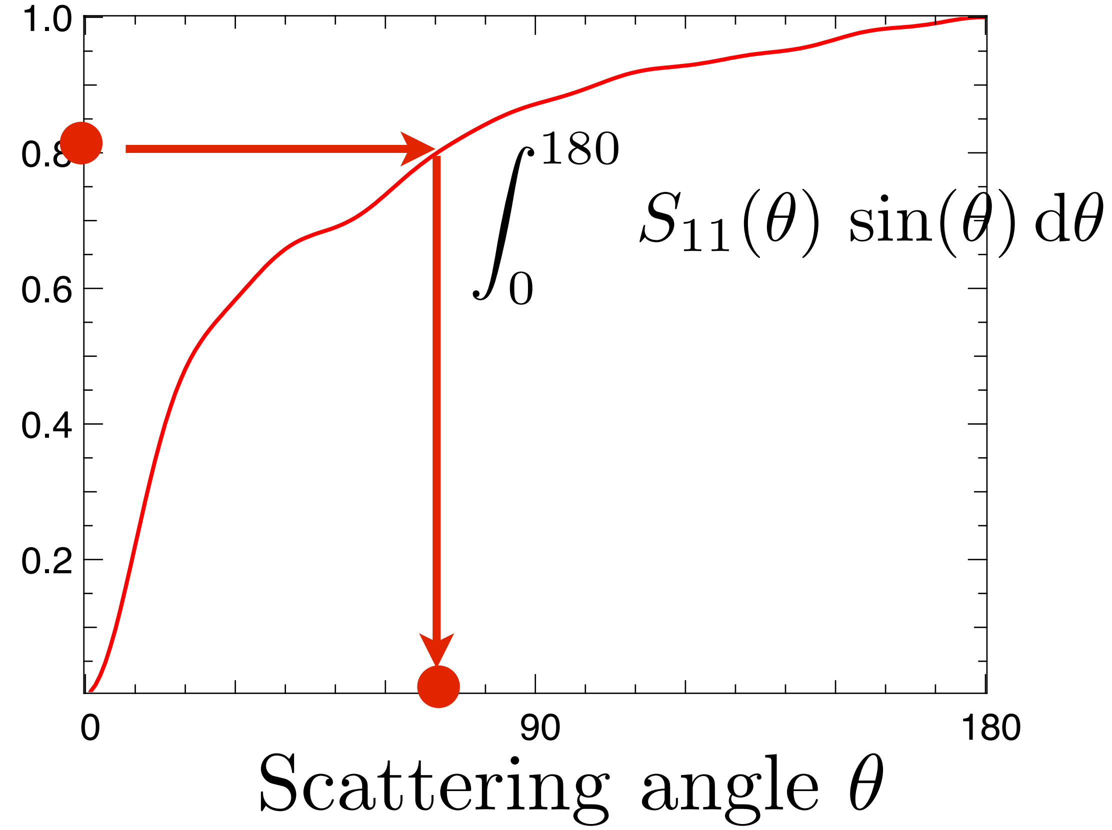




This is not a realistic phase function

Scattering Probability distribution





This is not a realistic phase function

We can sample any PDF

Problem solved ?

Yes and No

Monte-Carlo RT will eventually give the right result, but for most real-life cases it will often be inefficient in its basic implementation.

When is MC inefficient ?

- When the dust is very optically thick

When is MC inefficient ?

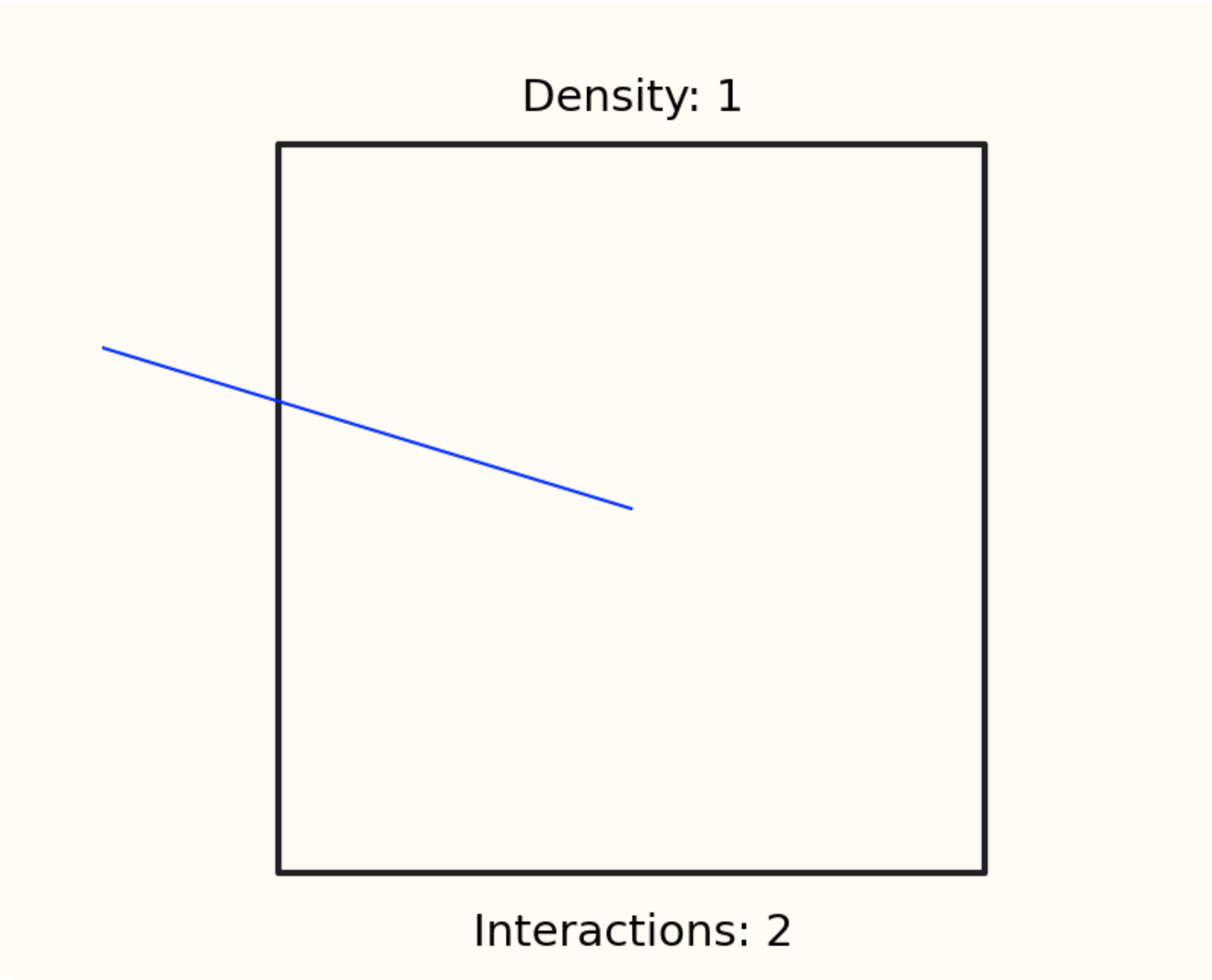
- When the dust is very optically thick
- When the dust is very optically thin

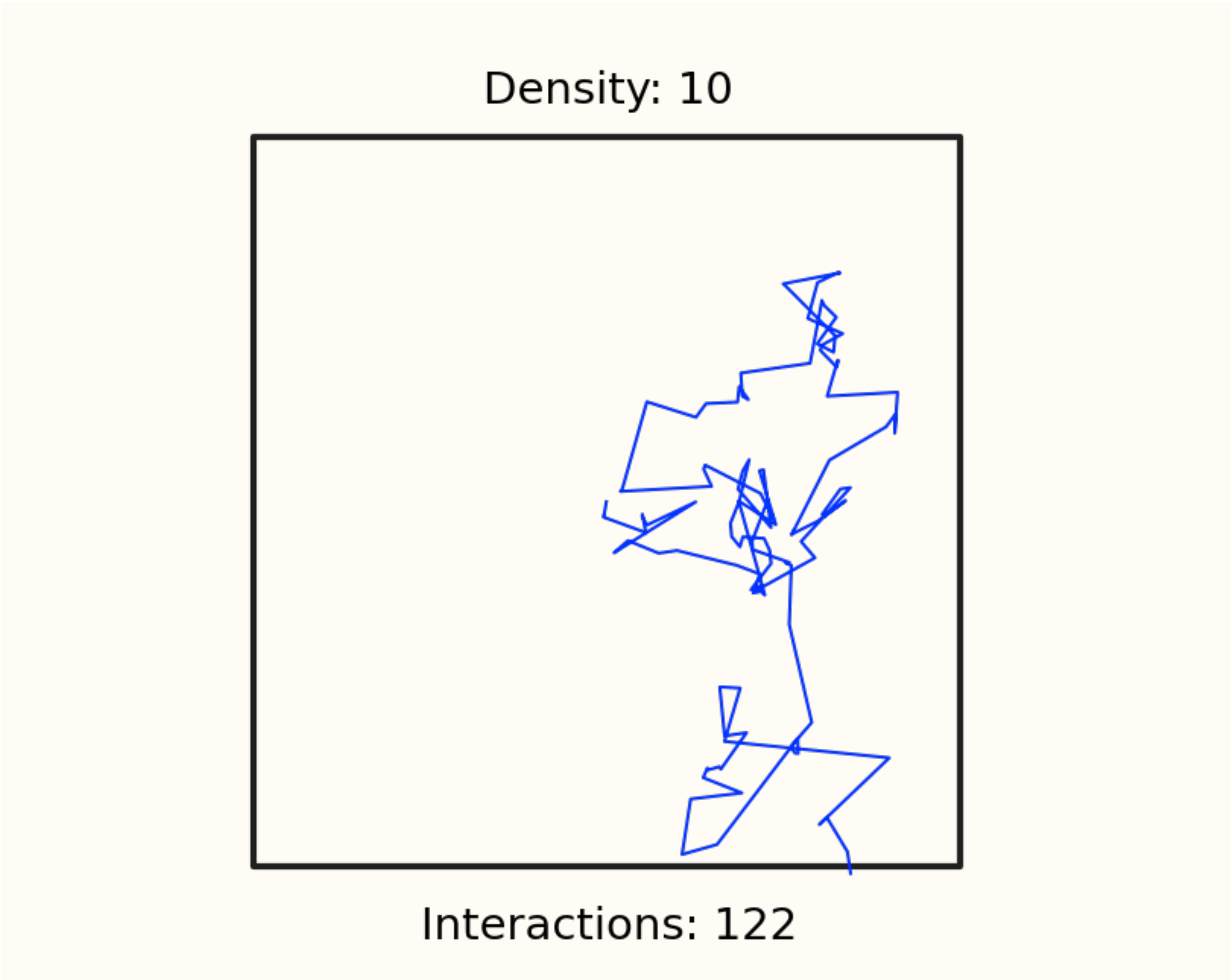
When is MC inefficient ?

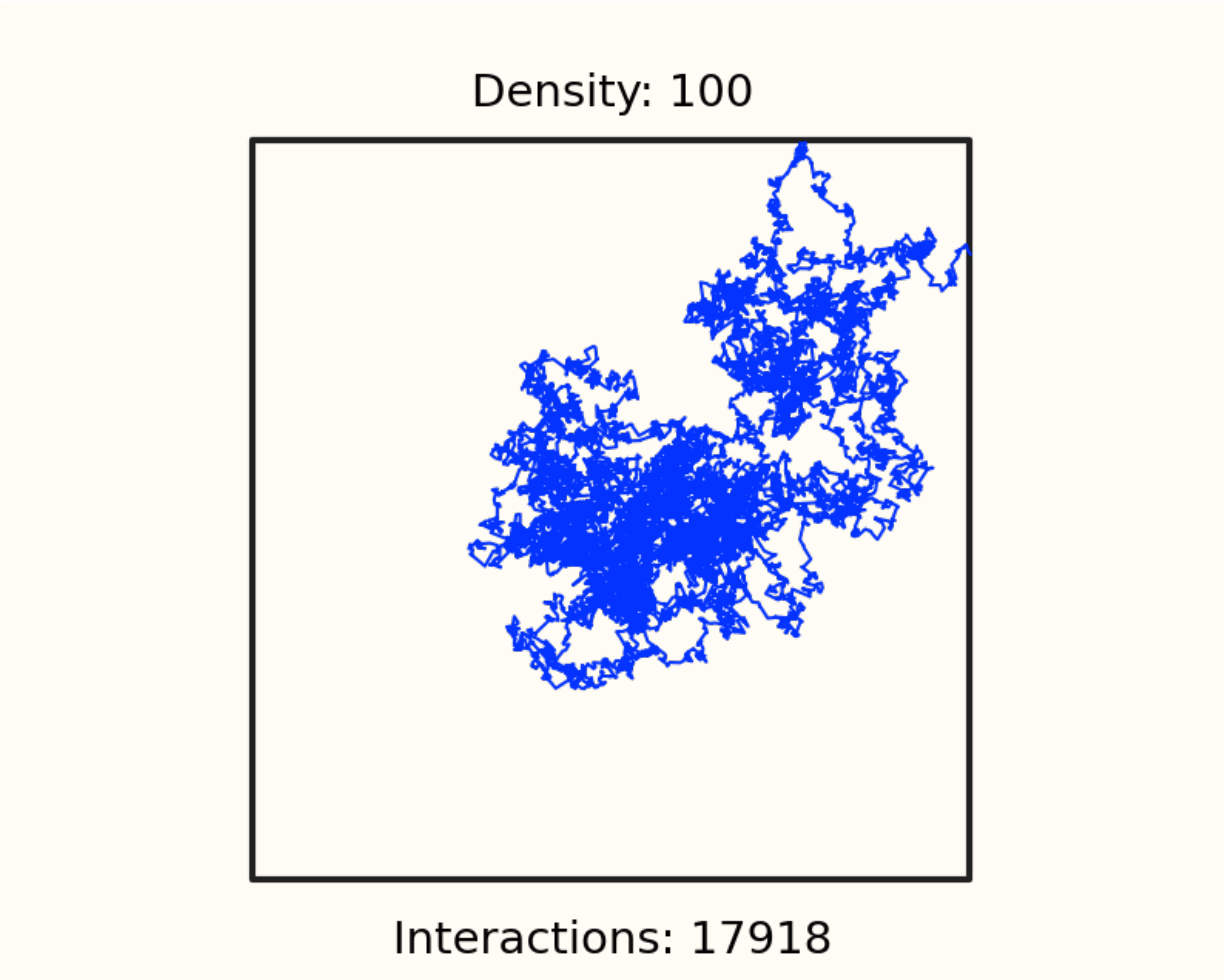
- When the dust is very optically thick
- When the dust is very optically thin
- When we only use escaping photons to produce observables

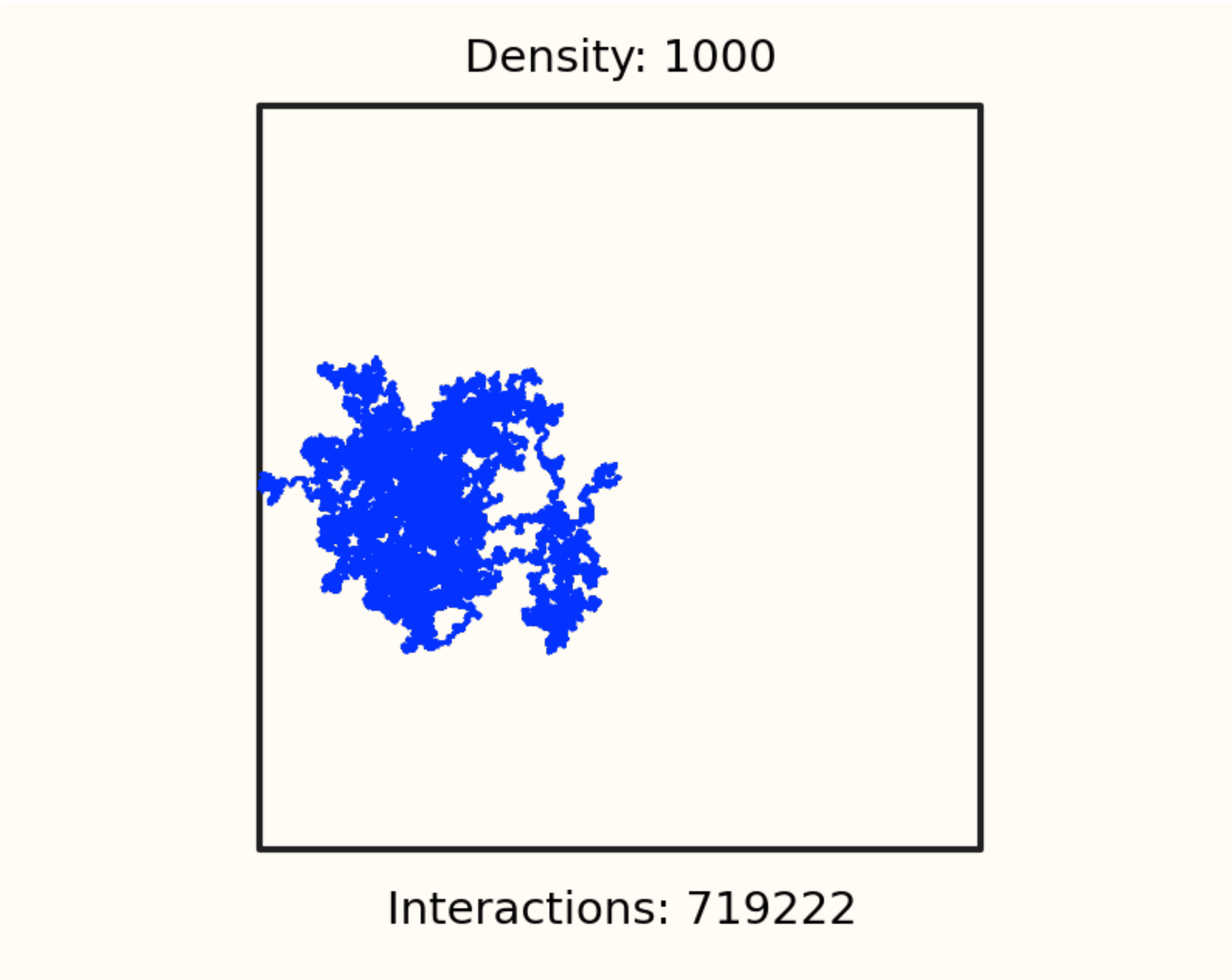
When is MC inefficient ?

- When the dust is very optically thick
- When the dust is very optically thin
- When we only use escaping photons to produce observables
- When we look away from the peaks of emissivity


Density: 1
Interactions: 2


Density: 10
Interactions: 122


Density: 100
Interactions: 17918


Density: 1000
Interactions: 719222

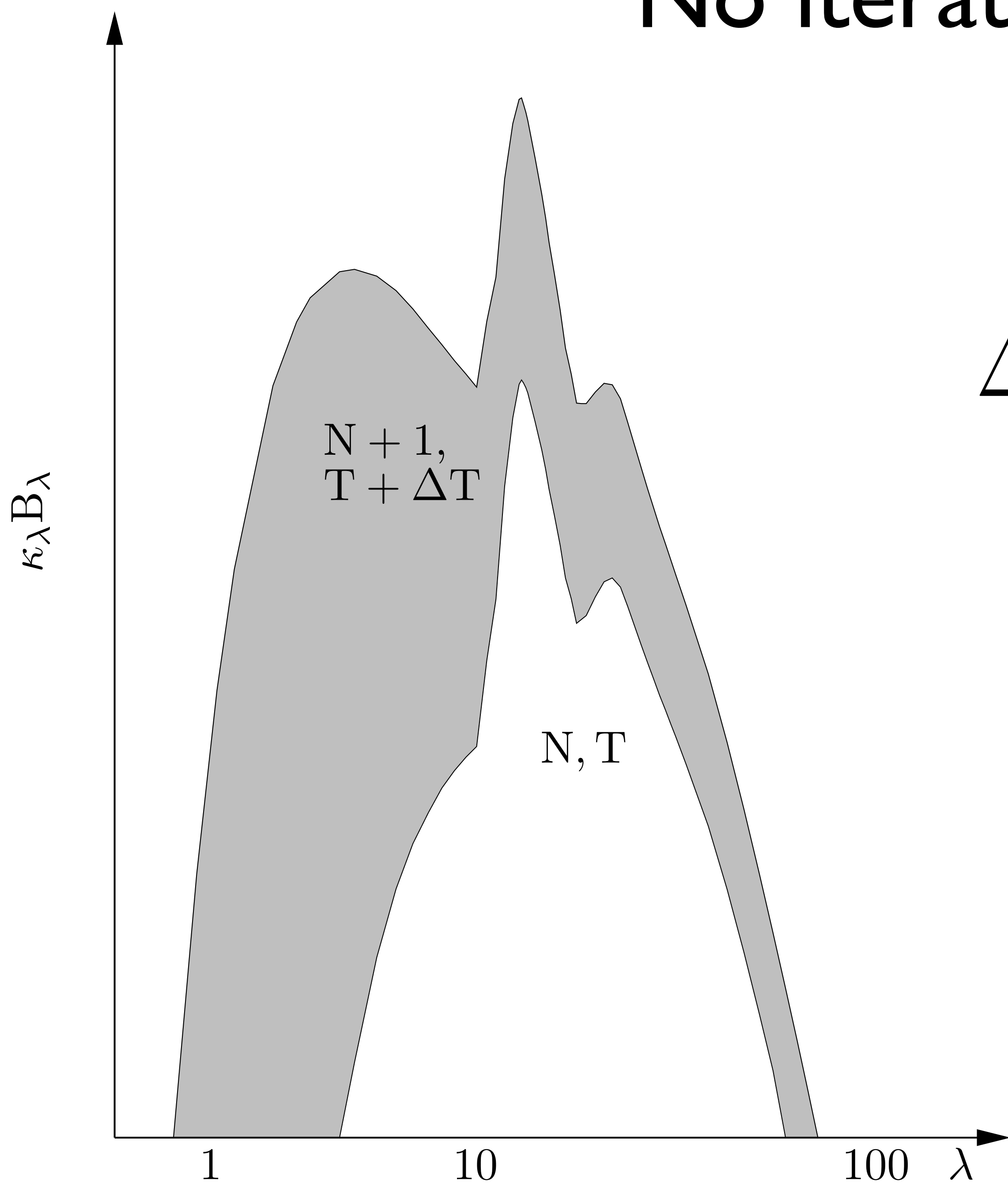
Naive implementation is VERY slow

- many interactions : many photon paths to calculate.
  Even if it happens only to a few photons, this can dominate the computation time

- many interactions : we need many iterations between J and T
  ⇒ a lots of photons are computed for nothing

Immediate re-emission

Directly re-emits photon after absorption:
No iteration any more



$$\Delta j_\lambda = \kappa_\lambda B_\lambda(T+\Delta T) - \kappa_\lambda B_\lambda(T)$$
$$\approx \kappa_\lambda \Delta T \left(\frac{\mathrm{d}B_\lambda(T)}{\mathrm{d}T}\right)$$

No equivalent in
ray-tracing methods

Bjorkman & Wood 2001

Diffusion approximation

- High optical depth

$$S_\lambda = B_\lambda$$

- Moment equations

$$\overrightarrow{F_\lambda} = -\frac{4\pi}{3\alpha}\nabla J_\lambda \qquad \nabla . K_\lambda = -\alpha\,\overrightarrow{H_\lambda}$$

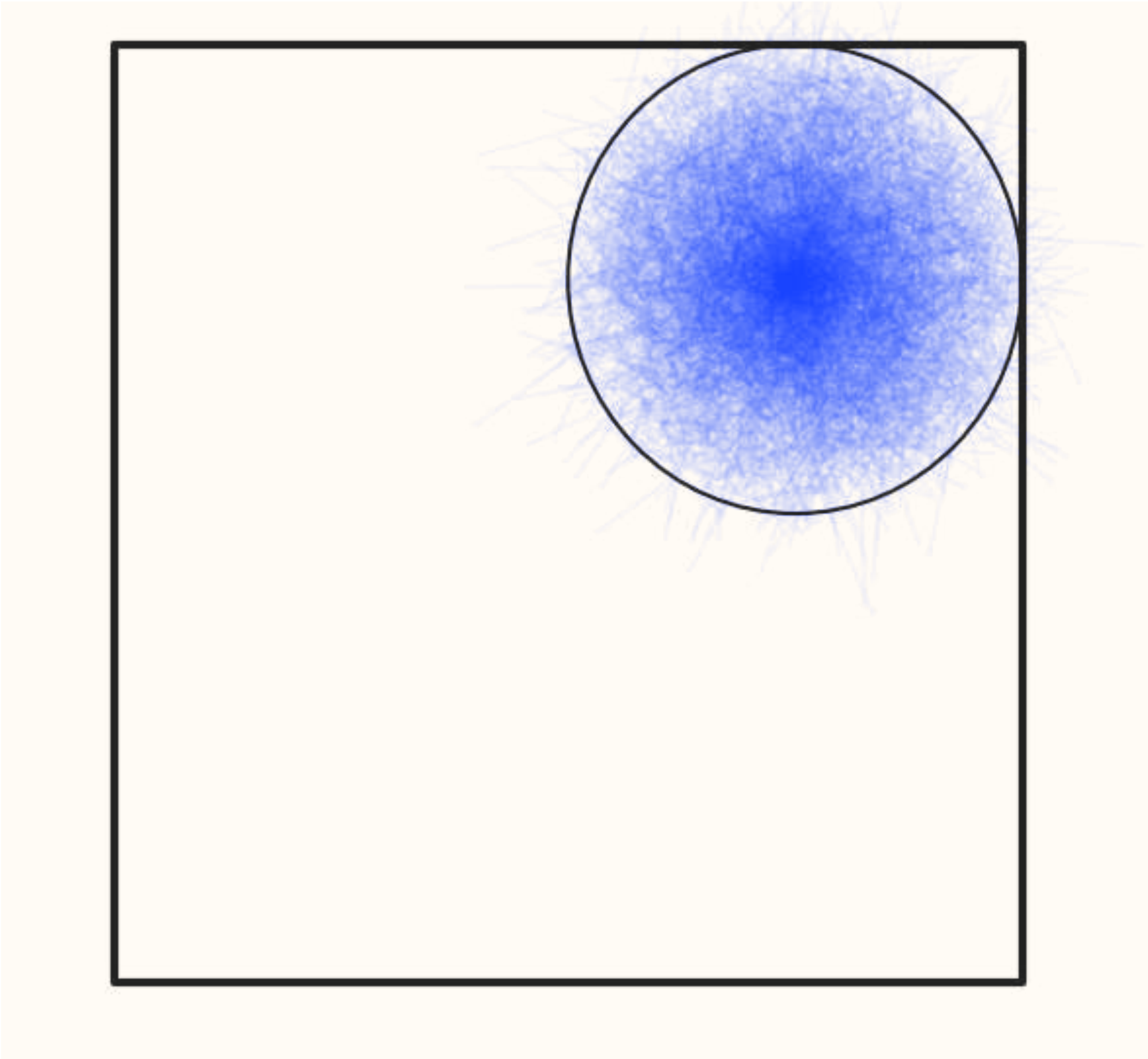
- Eddington approximation

$$K_\lambda = \frac{1}{3}J_\lambda$$

$$\nabla . \left(D\nabla T^4\right) = 0$$

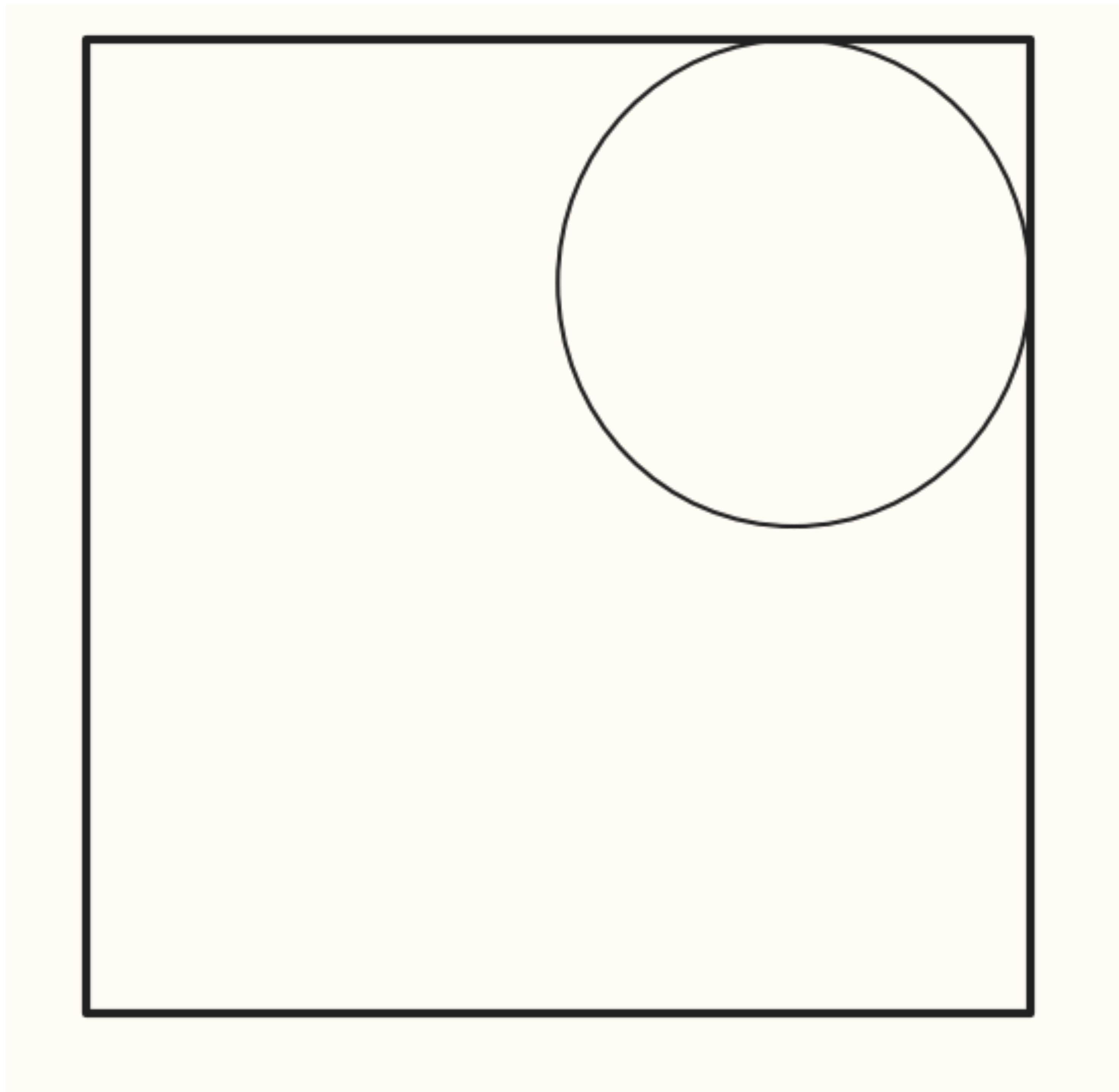
Extremely fast

Modified random-walk



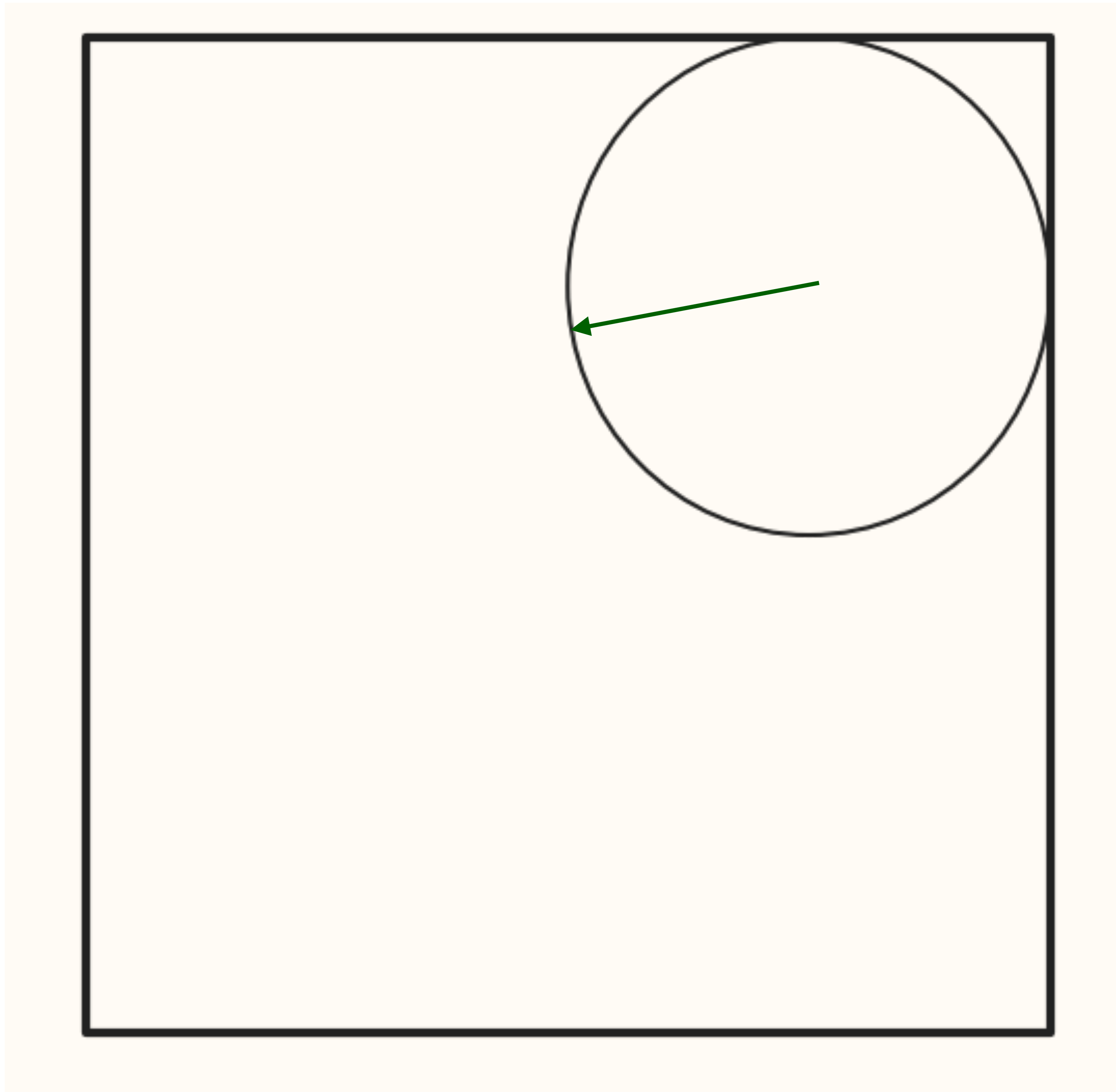
Min et al 2009

Modified random-walk



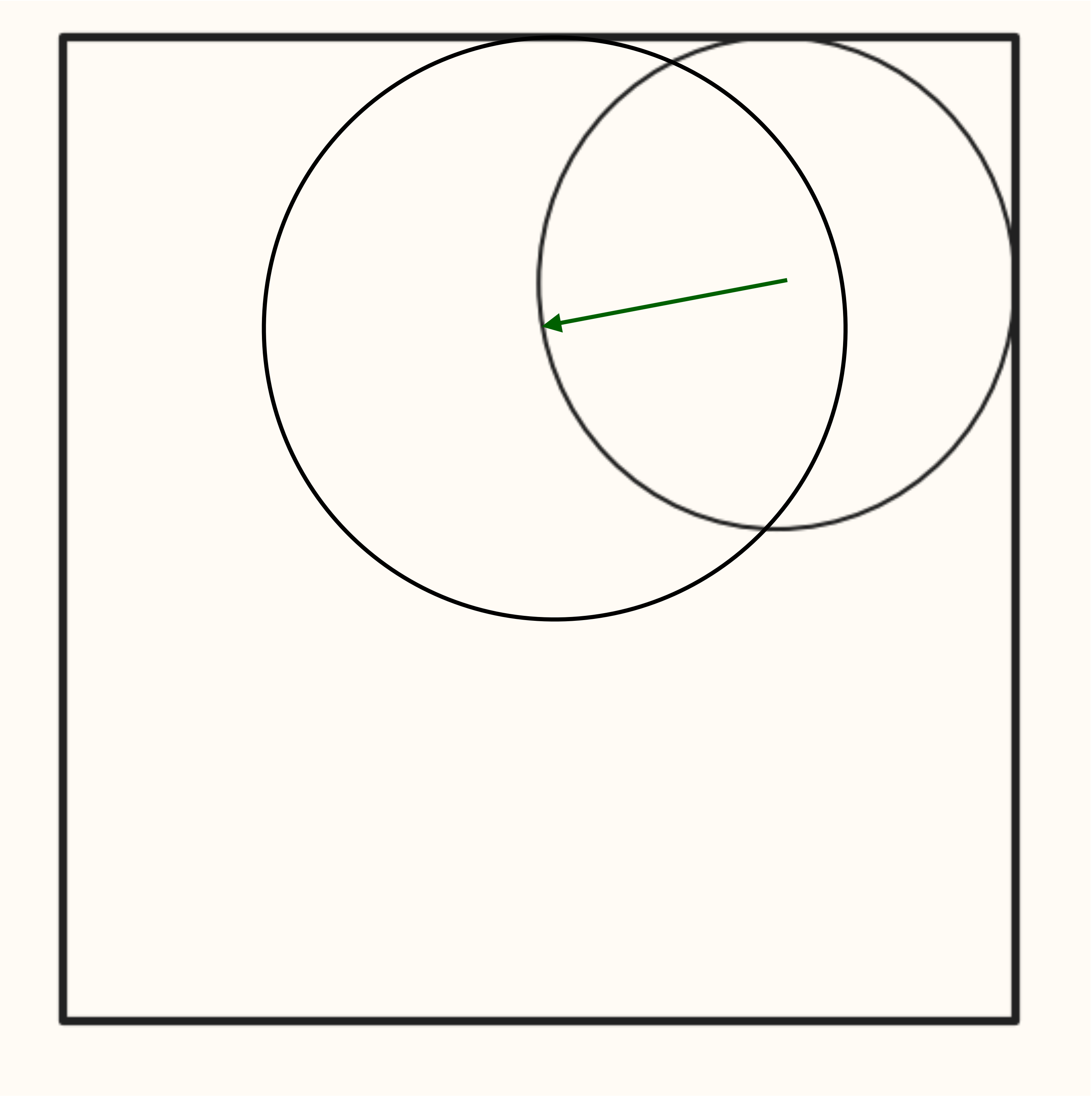
Min et al 2009

Modified random-walk



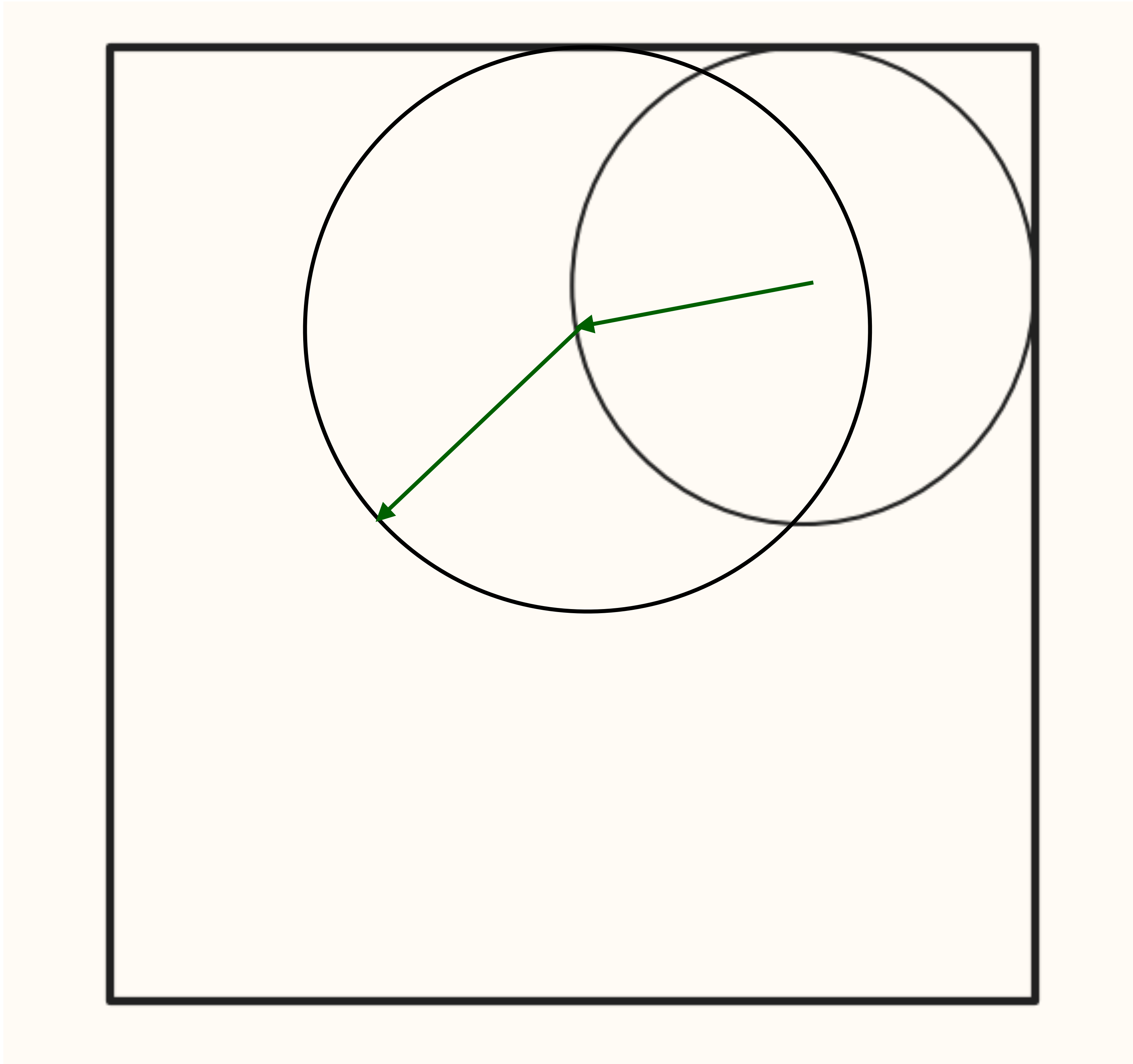
Min et al 2009

Modified random-walk



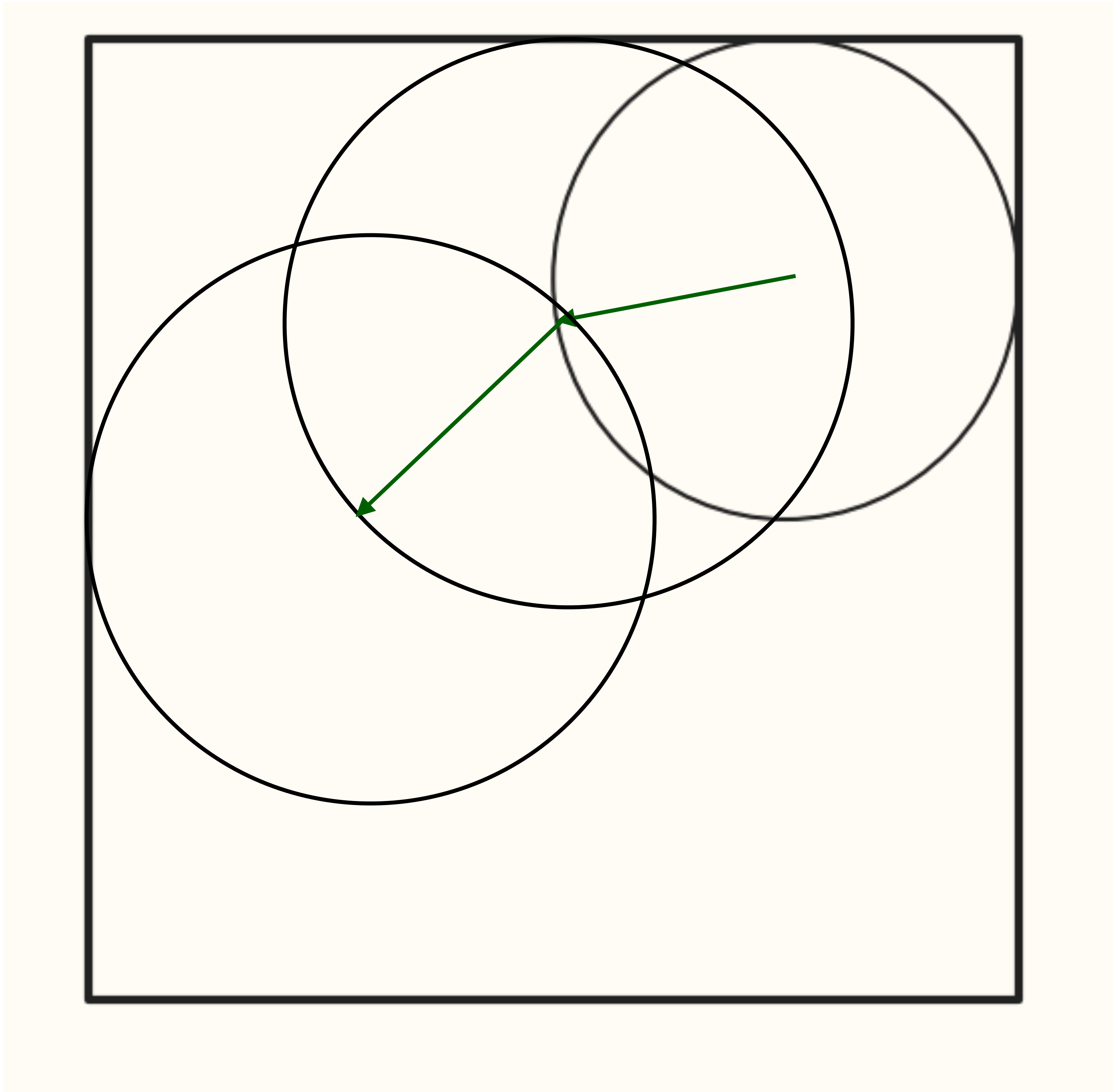
Min et al 2009

Modified random-walk



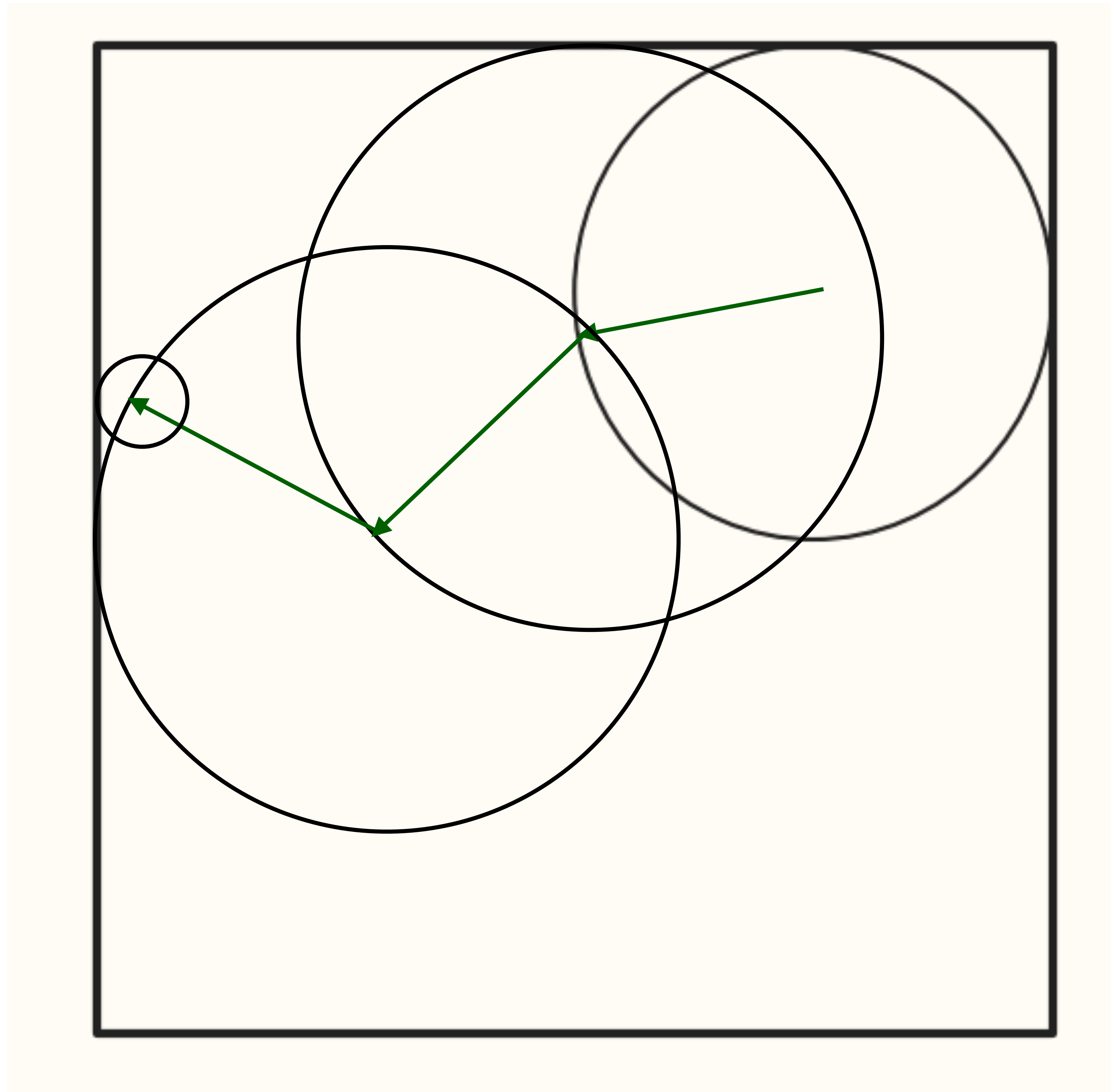
Min et al 2009

Modified random-walk



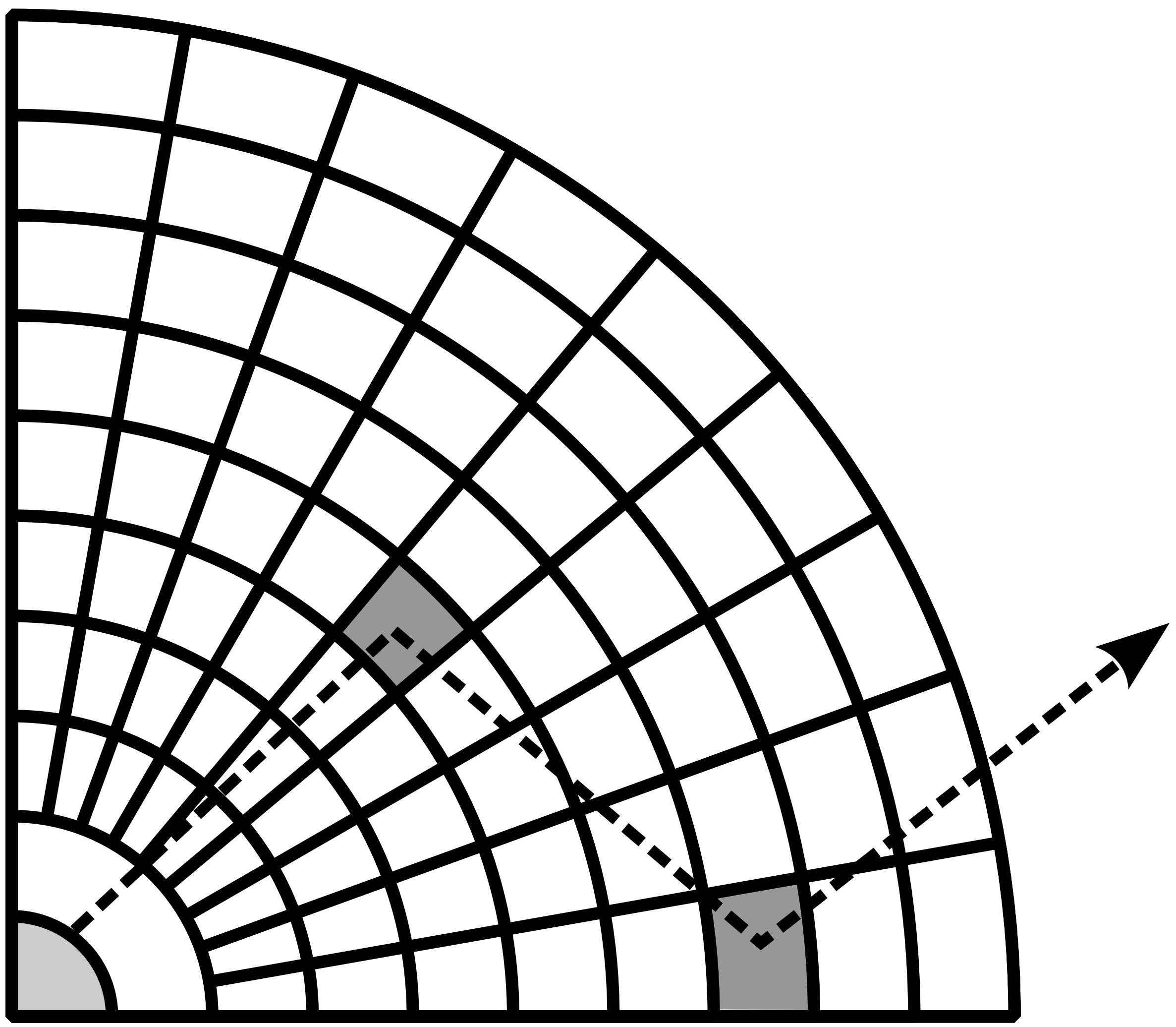
Min et al 2009

Modified random-walk



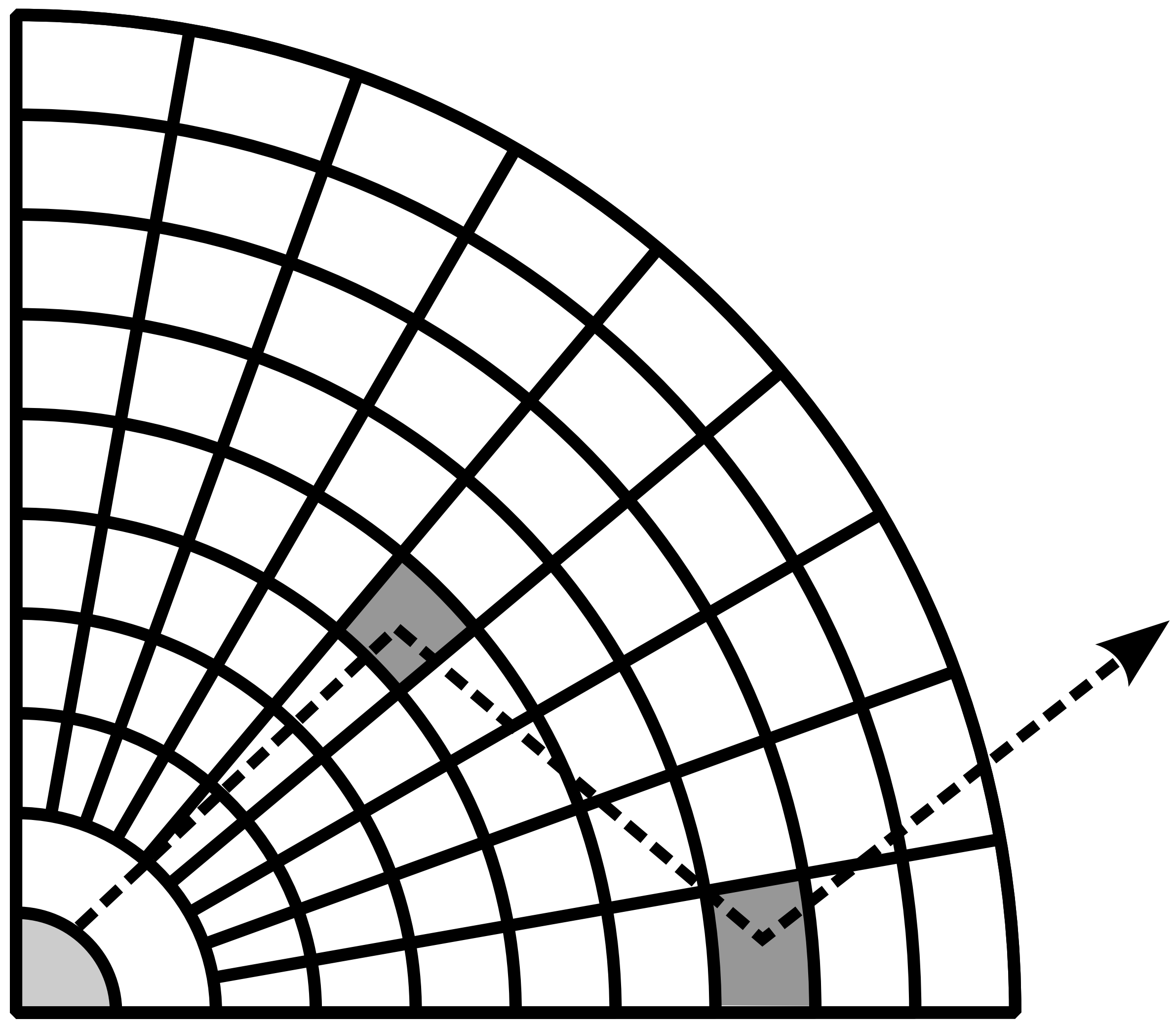
Min et al 2009

Photons only create information where they interact



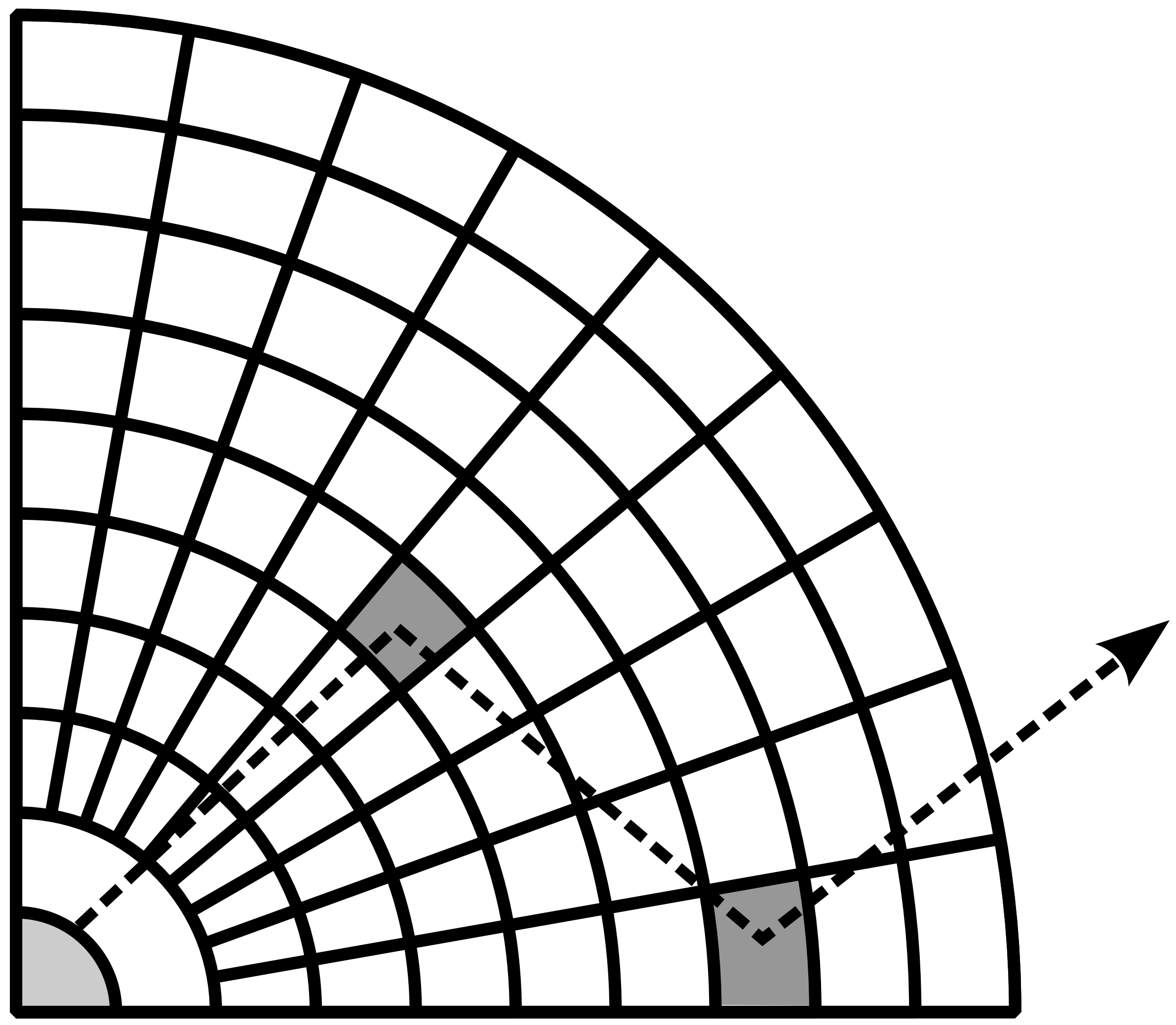
Photons only create information where they interact

$$\int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) B_\lambda(T(\overrightarrow{r}))\, d\lambda = \int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) J_\lambda(\overrightarrow{r})\, d\lambda$$



Photons only create information where they interact

$$\int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) B_\lambda(T(\overrightarrow{r}))\, d\lambda = \int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) J_\lambda(\overrightarrow{r})\, d\lambda$$

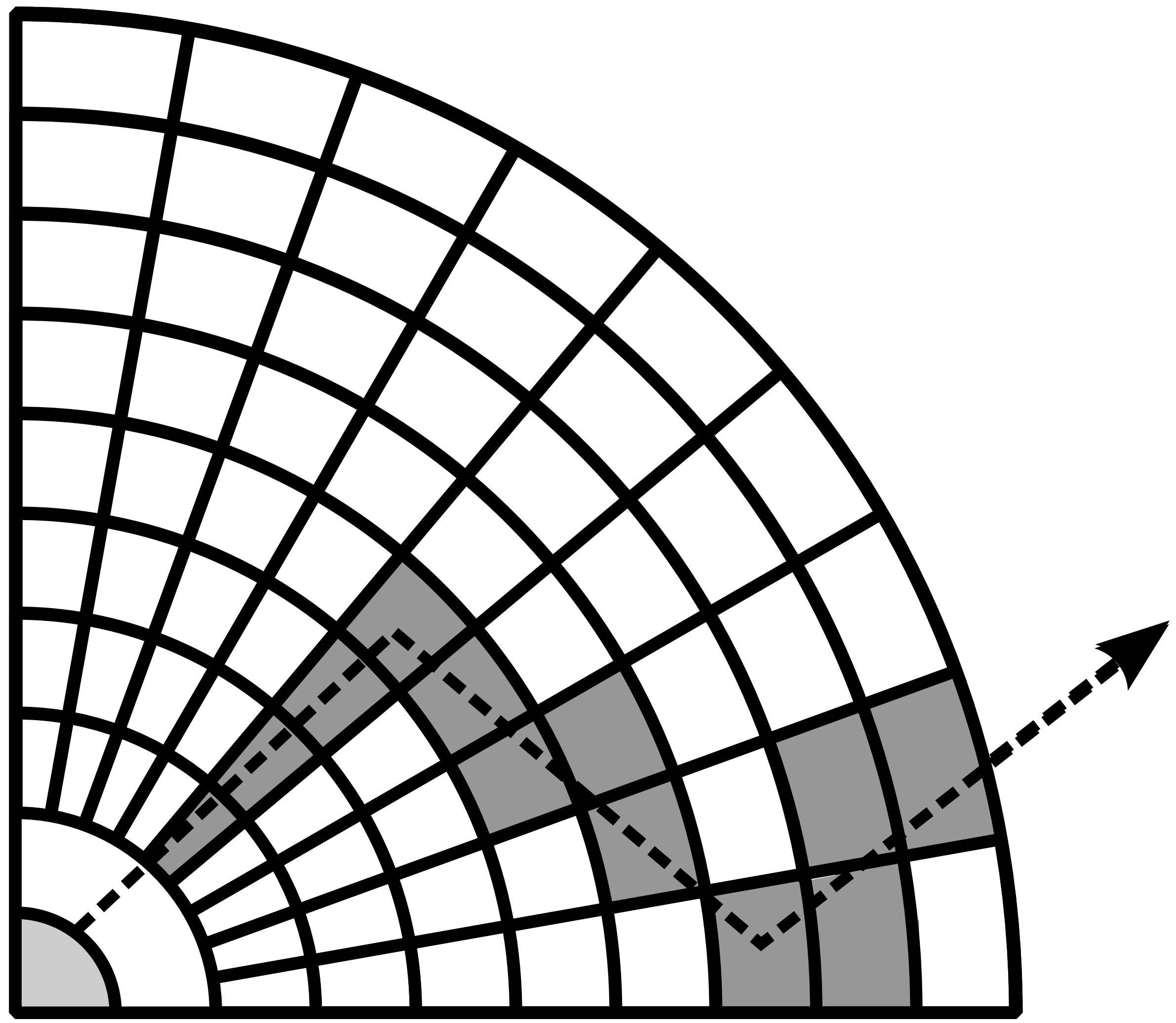


$$\mathrm{J}_\lambda = \frac{1}{4\pi \mathrm{V}_i} \sum_\gamma \varepsilon_\gamma \, \Delta l_\gamma$$

Lucy (1999)

Photons only create information where they interact

$$\int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) B_\lambda(T(\overrightarrow{r}))\, d\lambda = \int_0^\infty \kappa^{\rm abs}(\lambda, \overrightarrow{r}) J_\lambda(\overrightarrow{r})\, d\lambda$$

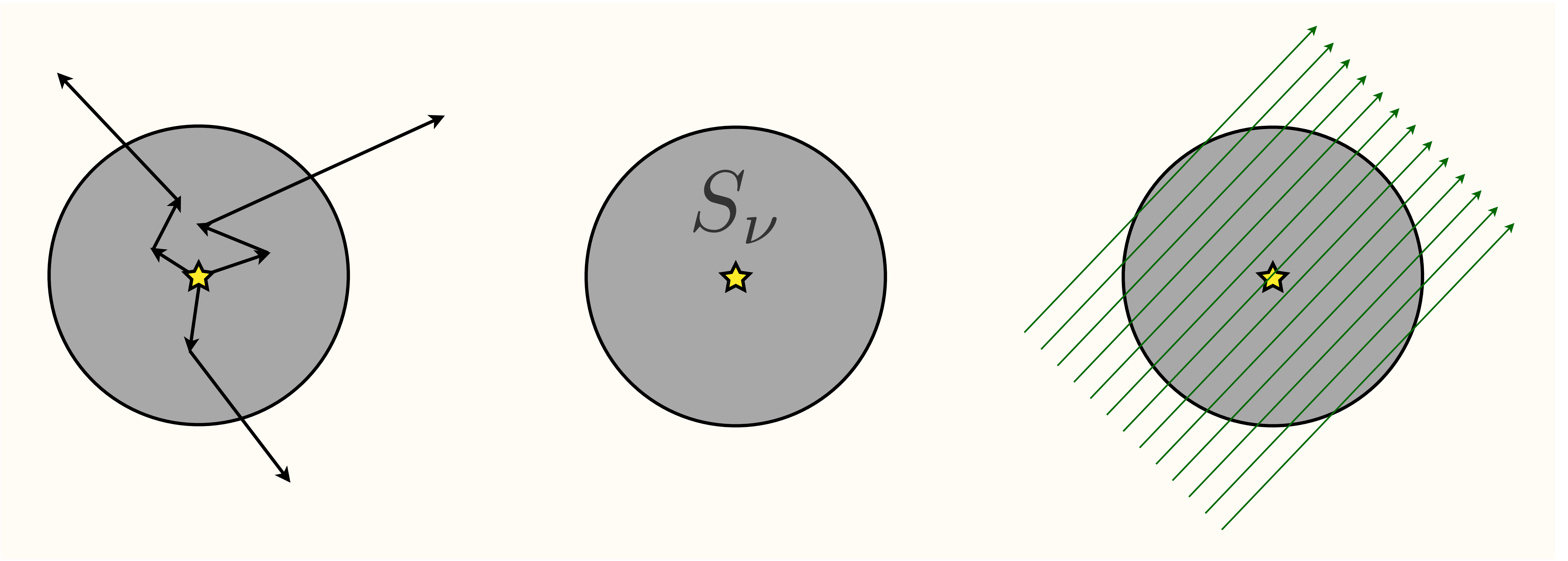


$$\mathrm{J}_\lambda = \frac{1}{4\pi \mathrm{V}_i} \sum_\gamma \varepsilon_\gamma \, \Delta l_\gamma$$

Lucy (1999)

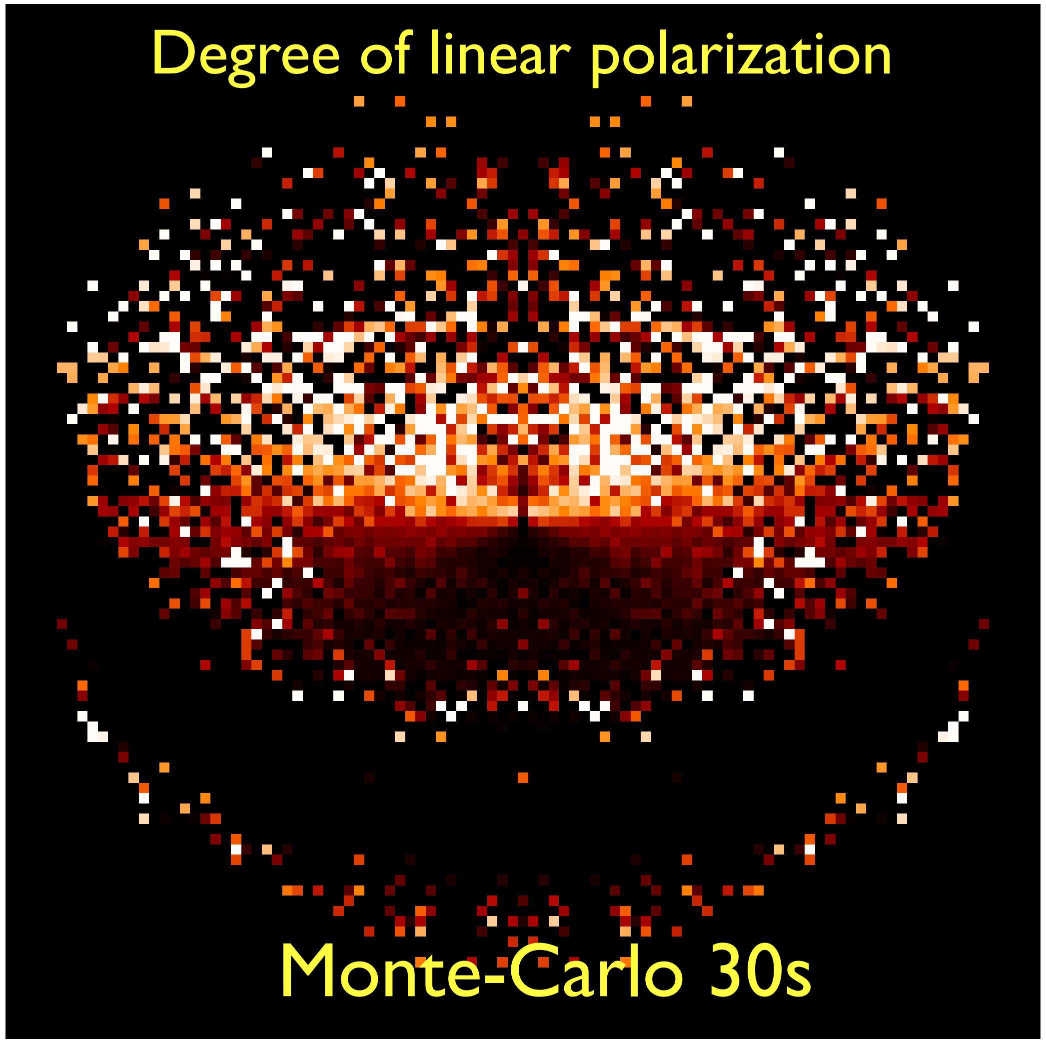
Ray tracing

We can do the same for I_λ if we also save the directions of the packets



- Pb : takes a lot of memory : $I_\lambda(x, y, z, \theta, \phi)$
- Alternative : saving scattered intensity for a few directions $\Sigma_\gamma\, \psi_\lambda(s, \vec{n}', \vec{n})\, I_\lambda(s, \vec{n}')$

MC + ray-tracing is VERY efficient



MC + ray-tracing is VERY efficient