General Relativistic Smoothed Particle Hydrodynamics

(GR-SPH)

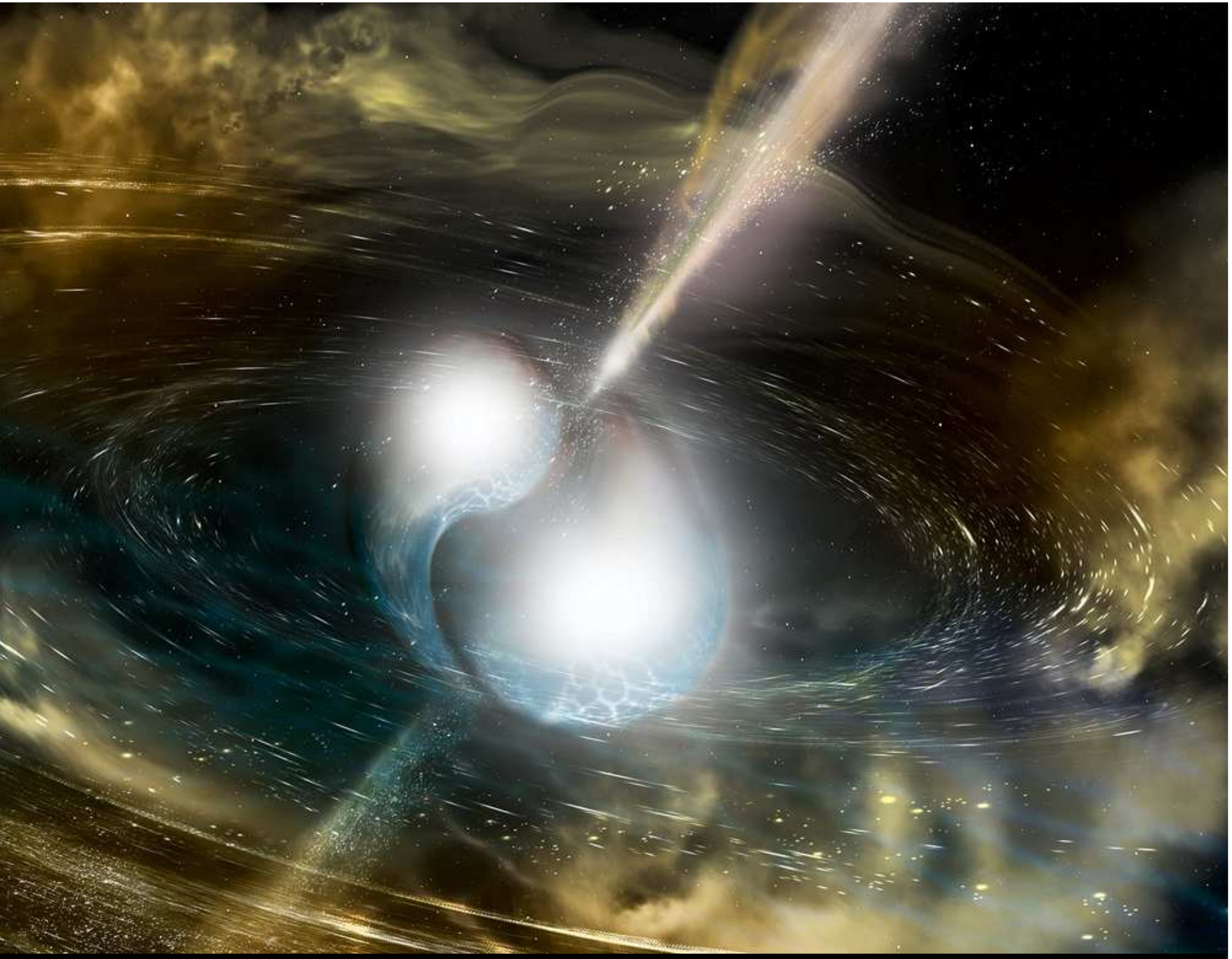
David Liptai

Supervisors: Daniel Price and Paul Lasky

MoCA Monash Centre for Astrophysics

MONASH University Science

Motivations



National Science Foundation/LIGO/Sonoma State University/A. Simonnet

Neutron star mergers!

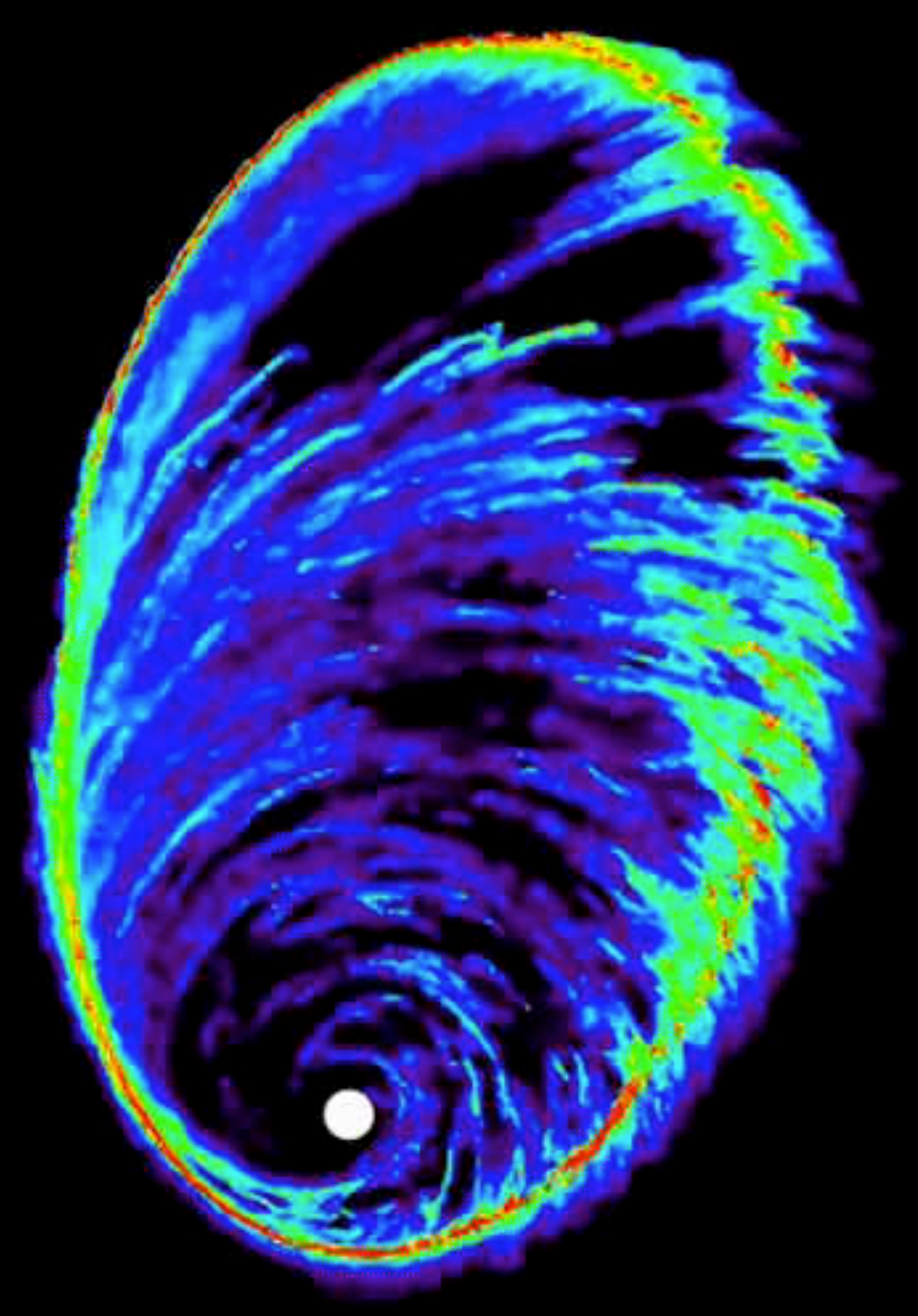
SPH perfect for NS-merger simulations

- No preferred geometry
- Resolution follows mass
- No need for background density floor

- Except….
  - No GR ☹️

Motivations

Bonnerot et al. (2016)



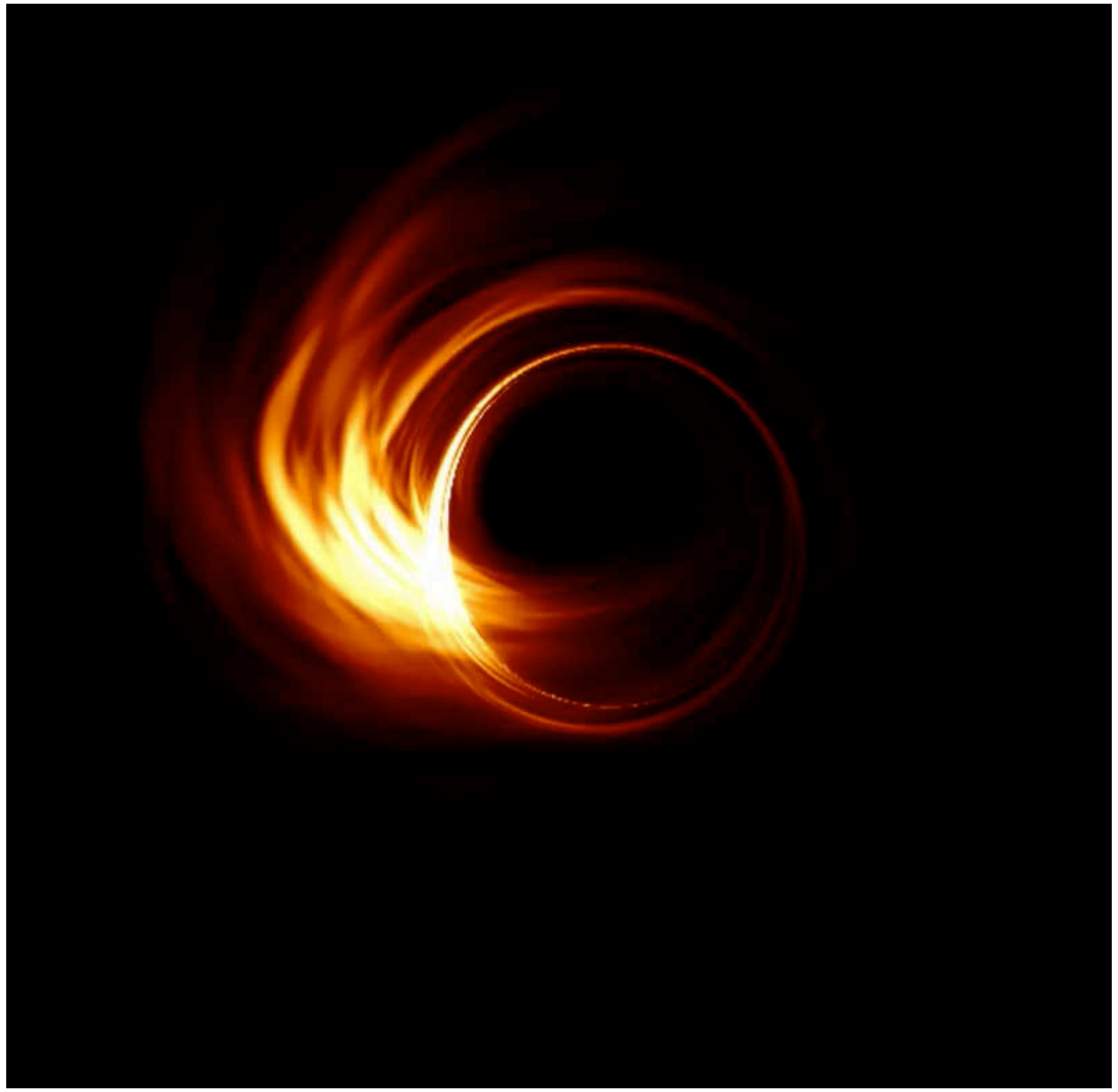
Fake GR

Nealon, Price and Nixon (2015)

Tidal Disruption Events

Tearing Discs and QPOs

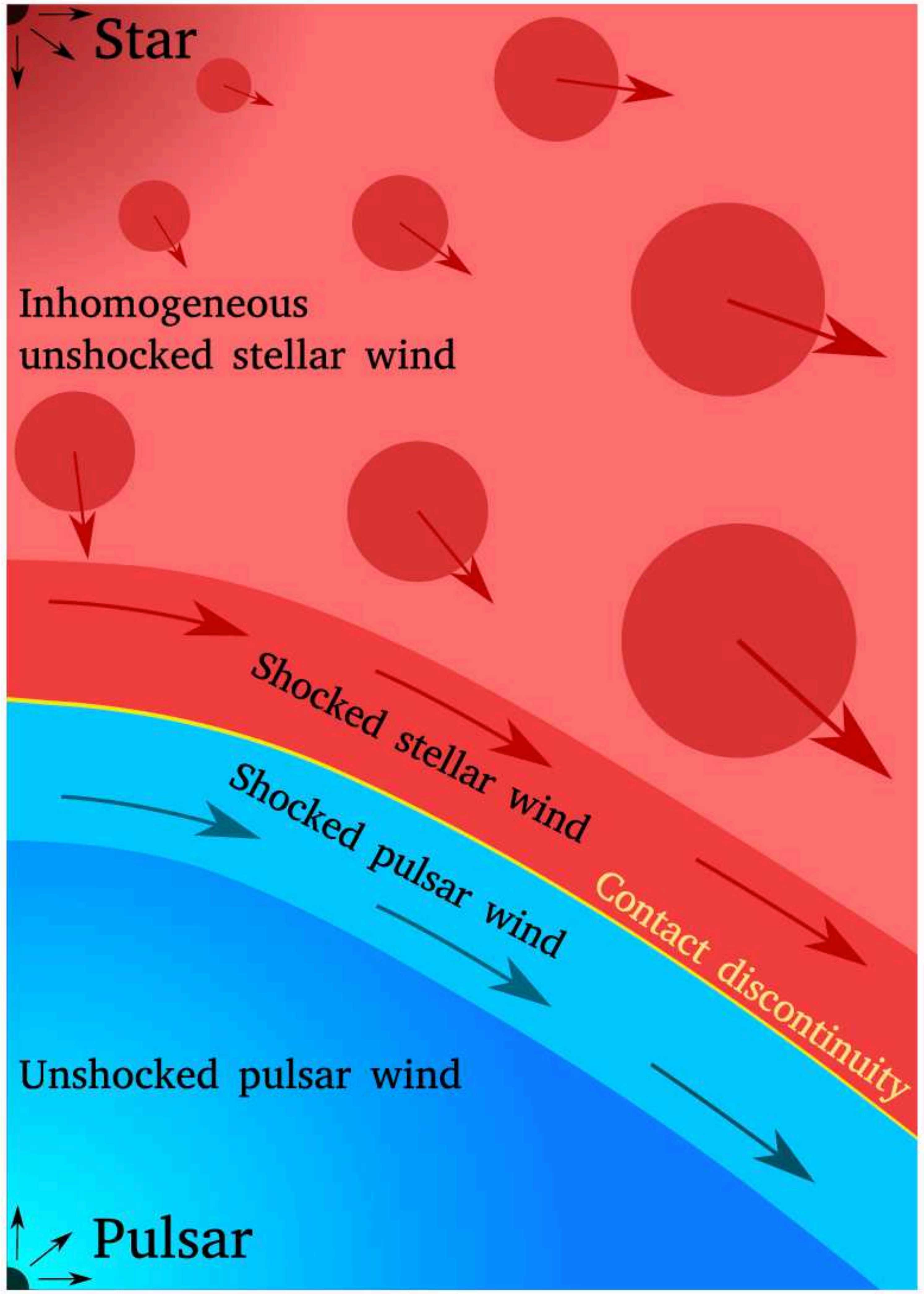
Motivations



Credit: Hotaka Shiokawa

Event Horizon Telescope

Paredes-Fortuny et al. (2015)



Relativistic Pulsar Winds

Equations of relativistic hydrodynamics

Continuity:

$$\frac{\mathrm{d}\rho^*}{\mathrm{d}t} = \underbrace{-\rho^* \frac{\partial v^i}{\partial x^i}}$$

Momentum:

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = \underbrace{-\frac{1}{\rho^*}\frac{\partial(\sqrt{-g}P)}{\partial x^i}} + \underbrace{\frac{\sqrt{-g}}{2\rho^*}\left(T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^i}\right)}_{\text{“GR”}}$$

Energy:

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \underbrace{-\frac{1}{\rho^*}\frac{\partial(\sqrt{-g}Pv^i)}{\partial x^i}}_{\text{“Hydro”}} + \xcancel{\frac{-\sqrt{-g}}{2\rho^*}\left(T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial t}\right)}$$

Equations of relativistic hydrodynamics

Continuity:

$$\cancel{\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = \frac{1}{\Omega_a} \underbrace{\sum_b m_b (v_a^i - v_b^i) \frac{\partial W_{ab}(h_a)}{\partial x^i}}},\qquad \rho_a^* = \sum_b m_b W_{ab}(h_a)$$

Momentum:

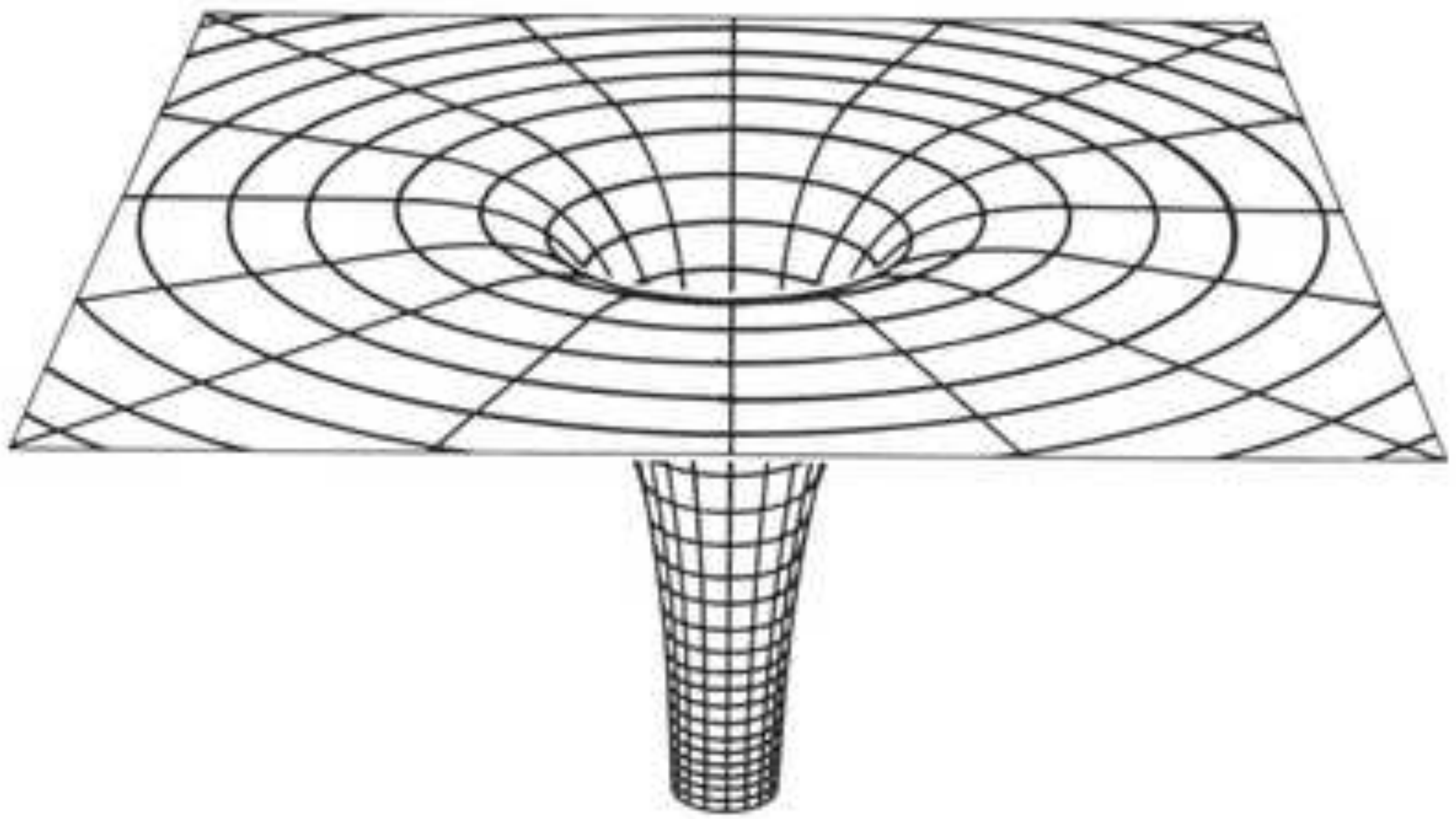
$$\frac{\mathrm{d}p_i^a}{\mathrm{d}t} = \underbrace{-\sum_b m_b \left[\frac{\sqrt{-g_a}P_a}{\Omega_a \rho_a^{*2}} \frac{\partial W_{ab}(h_a)}{\partial x^i} + \frac{\sqrt{-g_b}P_b}{\Omega_b \rho_b^{*2}} \frac{\partial W_{ab}(h_b)}{\partial x^i} \right]} + \underbrace{f_i^a}_{\text{“GR”}},$$

Energy:

$$\frac{\mathrm{d}e_a}{\mathrm{d}t} = -\sum_b \underbrace{m_b \left[\frac{\sqrt{-g_a}P_a v_b^i}{\Omega_a \rho_a^{*2}} \frac{\partial W_{ab}(h_a)}{\partial x^i} + \frac{\sqrt{-g_b}P_b v_a^i}{\Omega_b \rho_b^{*2}} \frac{\partial W_{ab}(h_b)}{\partial x^i} \right]}_{\text{“Hydro”}} + \cancel{\Lambda_a},$$

Checklist: Metrics and Coordinates

- Minkowski, Schwarzschild and Kerr
- Need in Cartesian-like coordinates
- A way to compute derivatives
- Choice of frame? (which observer?)



$$ds^2 = -\left(1 - \frac{2M}{r}\right) \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - \frac{2M}{r}} + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \,\mathrm{d}\phi^2\right)$$

$$= -\left(1 - \frac{2M}{r}\right) \mathrm{d}t^2 + \;\ldots\; \mathrm{d}x^2 + \;\ldots\; \mathrm{d}x\,\mathrm{d}y + \;\ldots\; \mathrm{d}x\,\mathrm{d}z + \;\ldots \quad ?$$

Checklist: Time Integration

- Preserve the Hamiltonian properties of the system
- Operator splitting approach
- Time reversible (conserves energy)
- Cost effective for 2nd order

Modified Leapfrog algorithm

$$p_i^{n+\frac{1}{2}} = p_i^n + \frac{\Delta t}{2} f_i^{\,\mathrm{sph}}(p_i^n, x^{i,n}),$$

$$\left\{\begin{aligned}
p_i^{m+\frac{1}{2}} &= p_i^m + \frac{\Delta t_{\mathrm{ext}}}{2} f_i^{\,\mathrm{ext}}(p_i^{m+\frac{1}{2}}, x^{i,m}), \\
x^{i,m+1} &= x^{i,m} + \frac{\Delta t_{\mathrm{ext}}}{2}\left[\frac{\mathrm{d}x^i}{\mathrm{d}t}(p_i^{m+\frac{1}{2}}, x^{i,m}) \right. \\
&\qquad \left. + \frac{\mathrm{d}x^i}{\mathrm{d}t}(p_i^{m+\frac{1}{2}}, x^{i,m+1})\right], \\
p_i^{m+1} &= p_i^{m+\frac{1}{2}} + \frac{\Delta t_{\mathrm{ext}}}{2} f_i^{\,\mathrm{ext}}(p_i^{m+\frac{1}{2}}, x^{i,m+1}),
\end{aligned}\right\}$$

$$p_i^{n+1} = p_i^{n+\frac{1}{2}} + \frac{\Delta t}{2} f_i^{\,\mathrm{sph}}(p_i^{n+1}, x^{i,n+1})$$

Checklist: Recovery of Primitive Variables

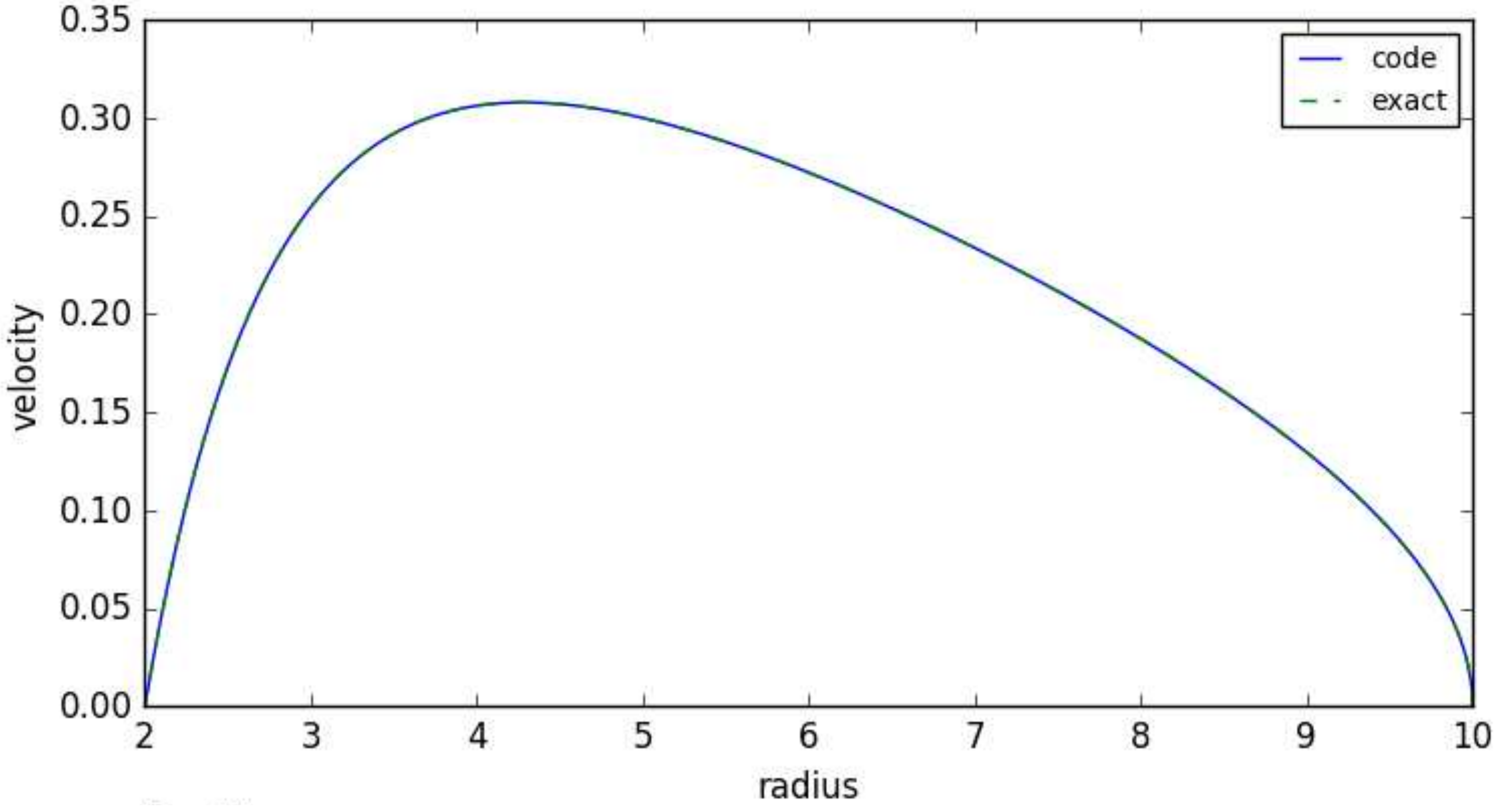
- Needs to be done after every time-step
- Needs to rigorous and cheap
- Cannot solve explicitly
- Solve numerically with a Newton-Raphson scheme
- Follow Tejeda (2012)

$$
\begin{aligned}
\rho^* &= \sqrt{-g}\rho U^0, \\
p_i &= U^0 w g_{i\mu} v^\mu, \\
e &= U^0 \left[w g_{i\mu} v^\mu v^i - (1+u) g_{\mu\nu} v^\mu v^\nu \right],
\end{aligned}
$$

$$\Downarrow$$

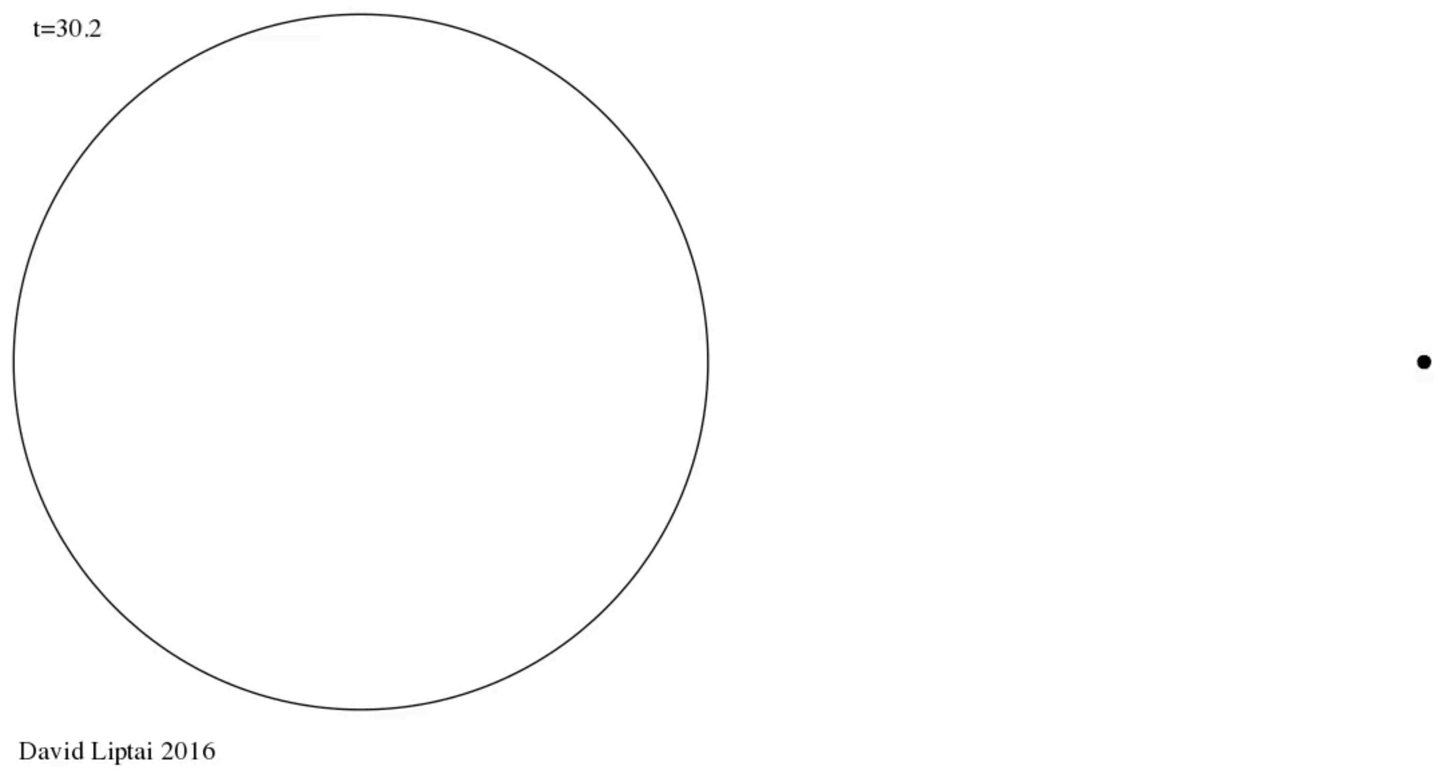
$$
\begin{aligned}
\rho &= \\
v_i &= \quad ? \\
u &=
\end{aligned}
$$

Tests: Schwarzschild metric



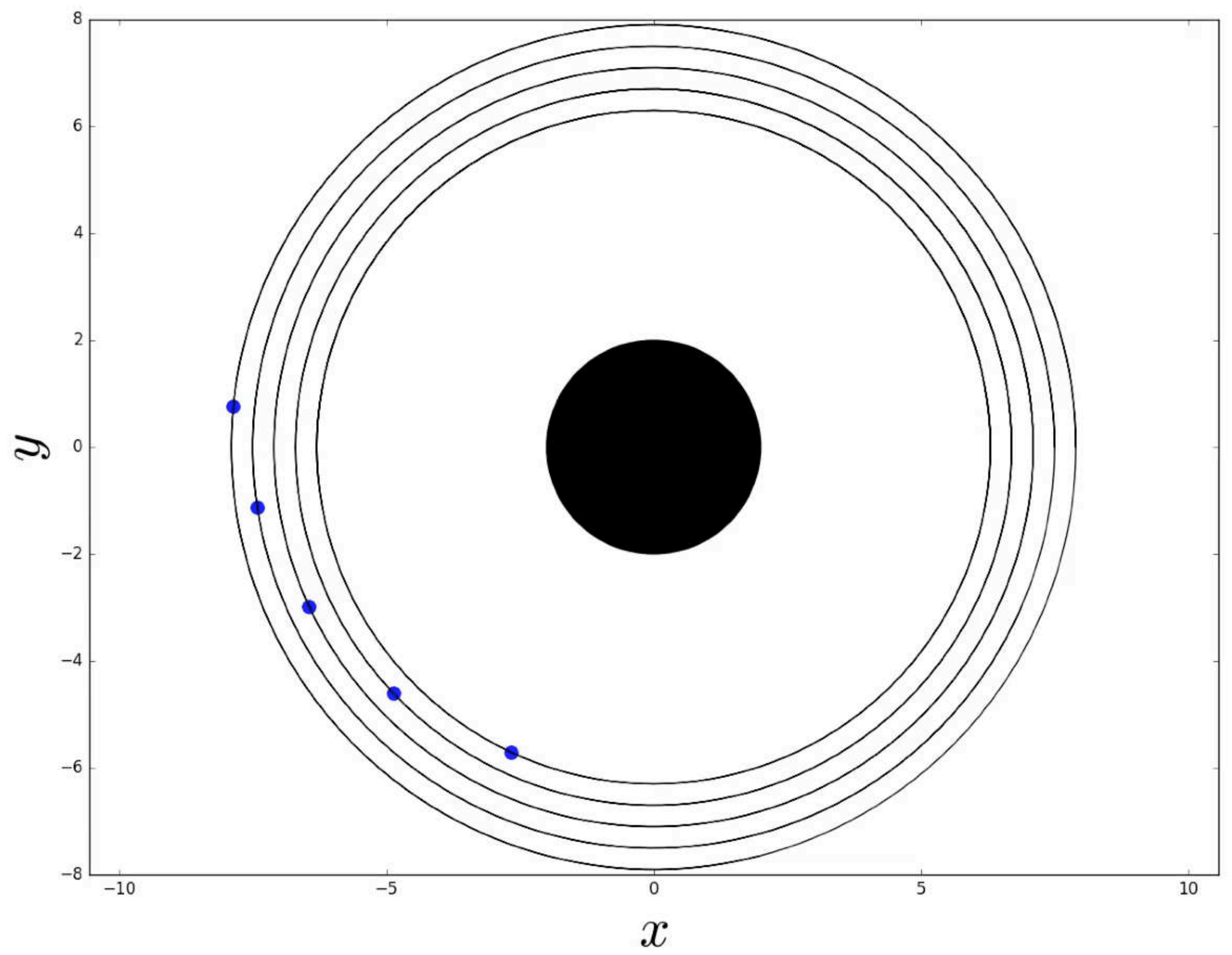
Radial Infall

$$v_r(r) = \frac{1 - \frac{2M}{r}}{\sqrt{1 - \frac{2M}{r_0}}}\sqrt{2M\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

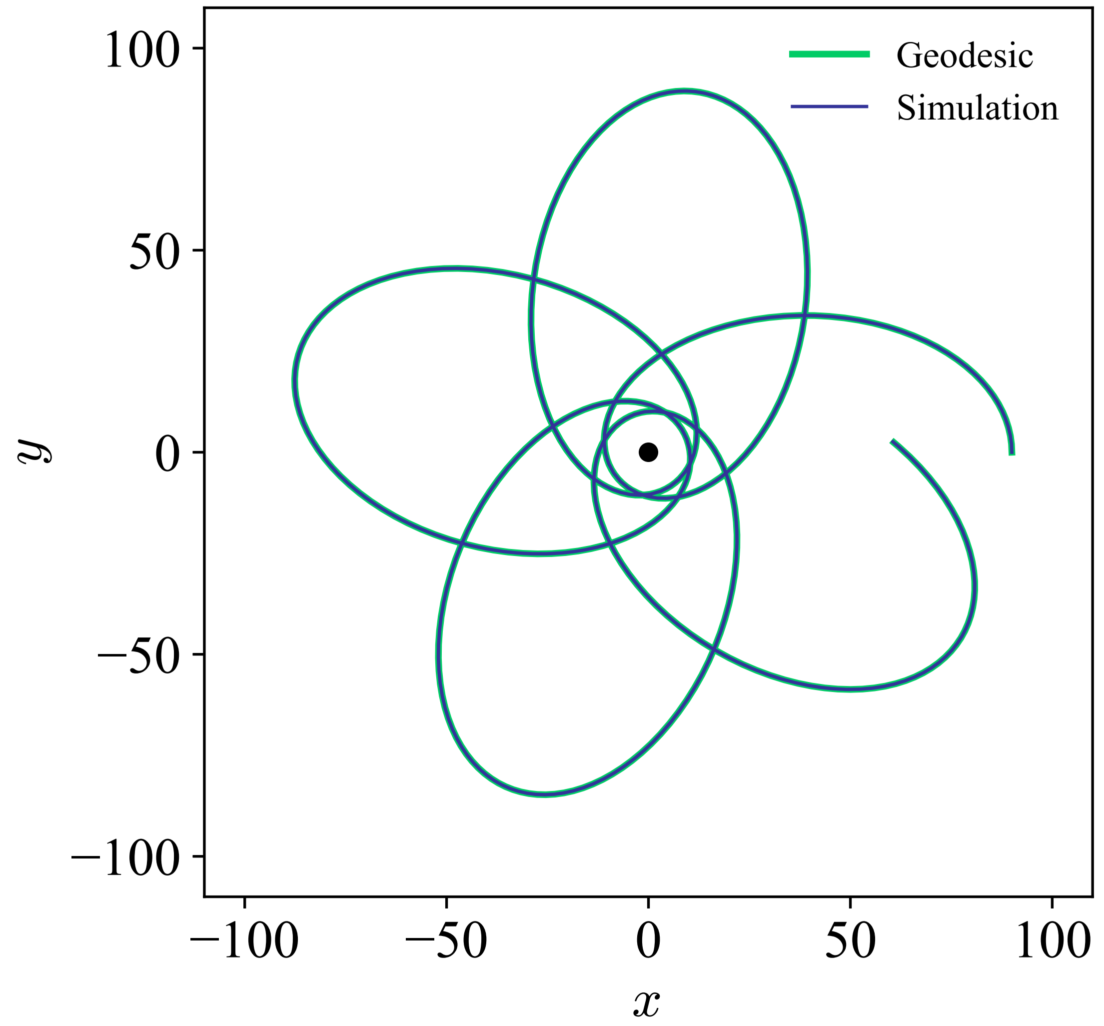


Circular orbit

$$\Omega = \frac{1}{r^{3/2}}$$

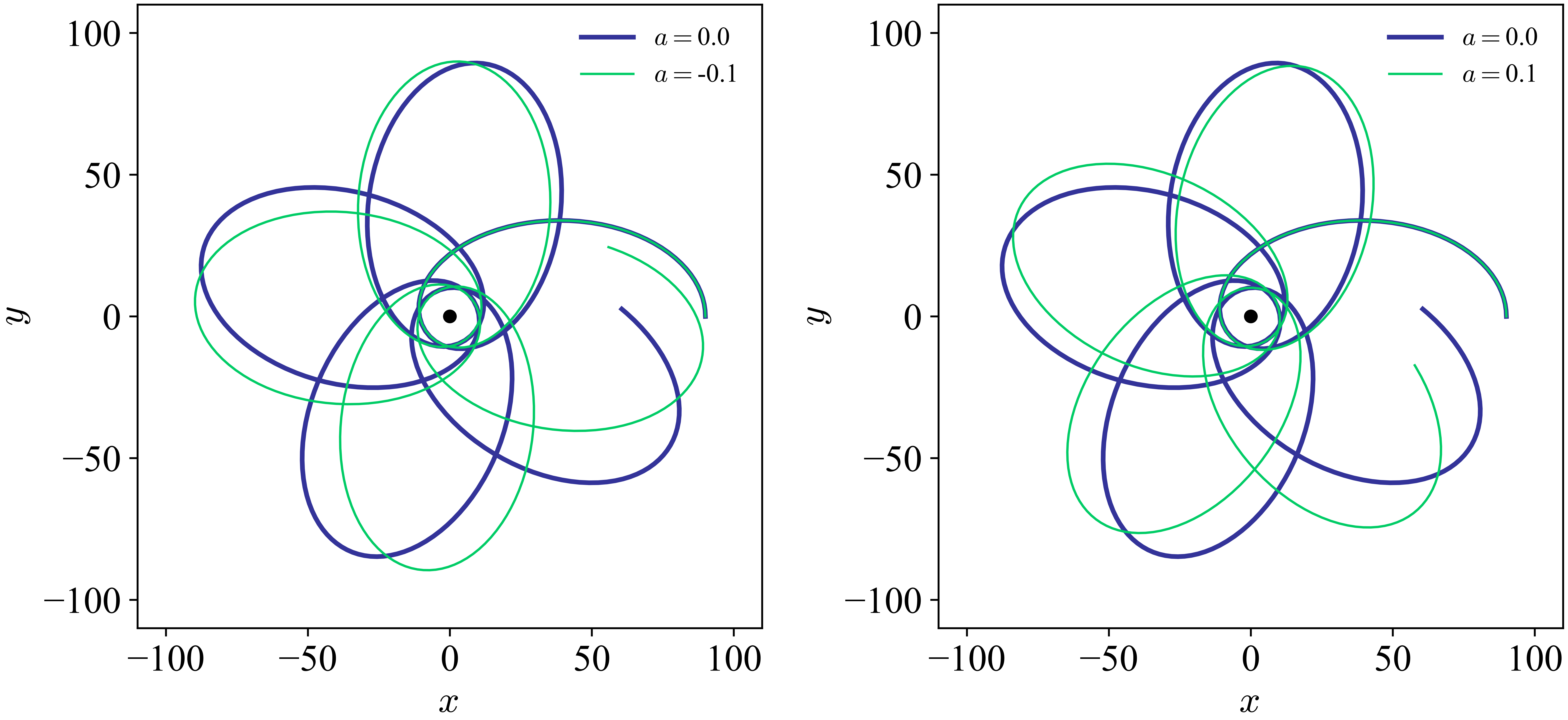


Precession



Tests: Kerr metric

Apsidal precession



Retrograde **Prograde**

Liptai and Price 2018 (In prep.)

Tests: Kerr metric

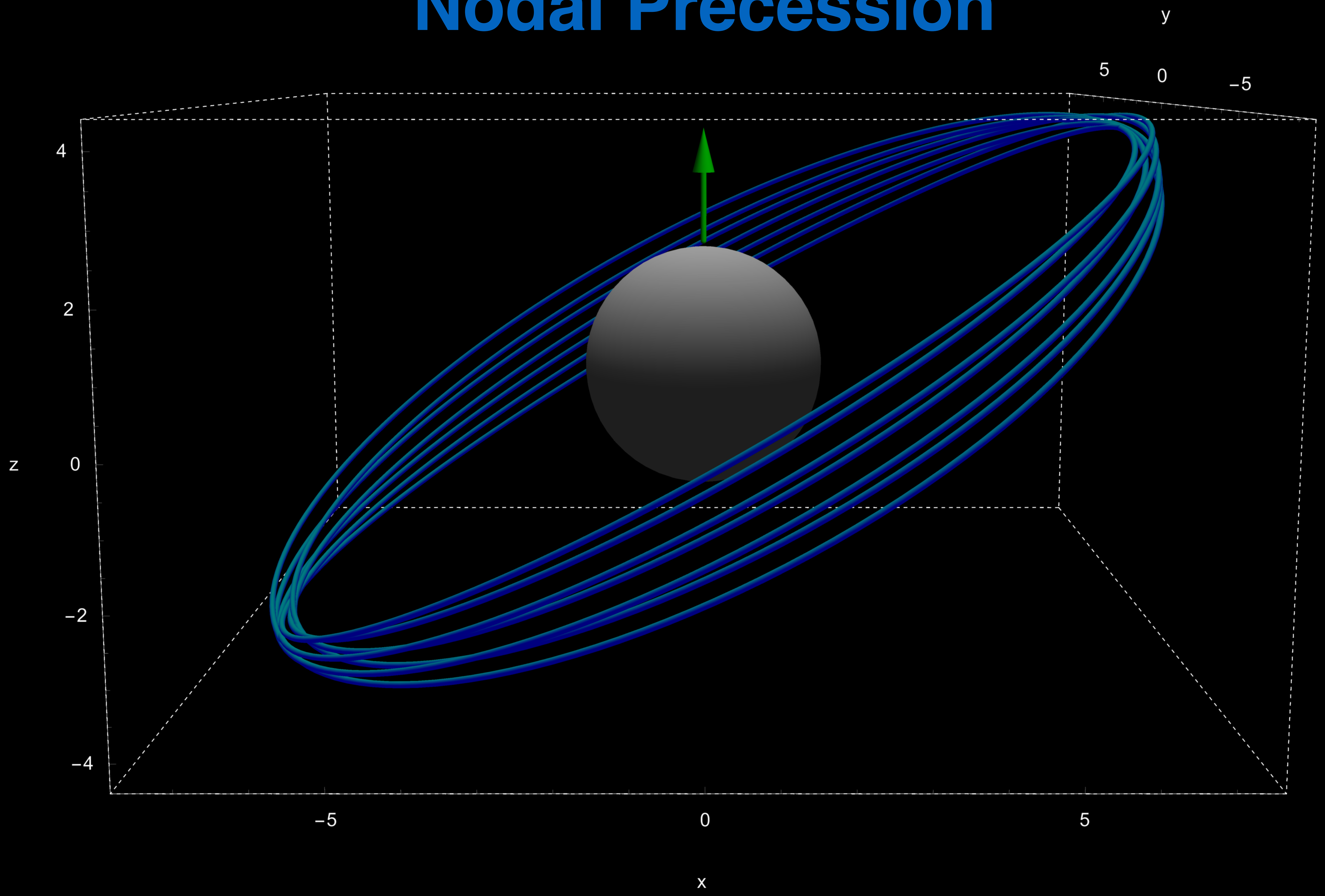
Epicyclic frequency

Vertical-oscillation frequency

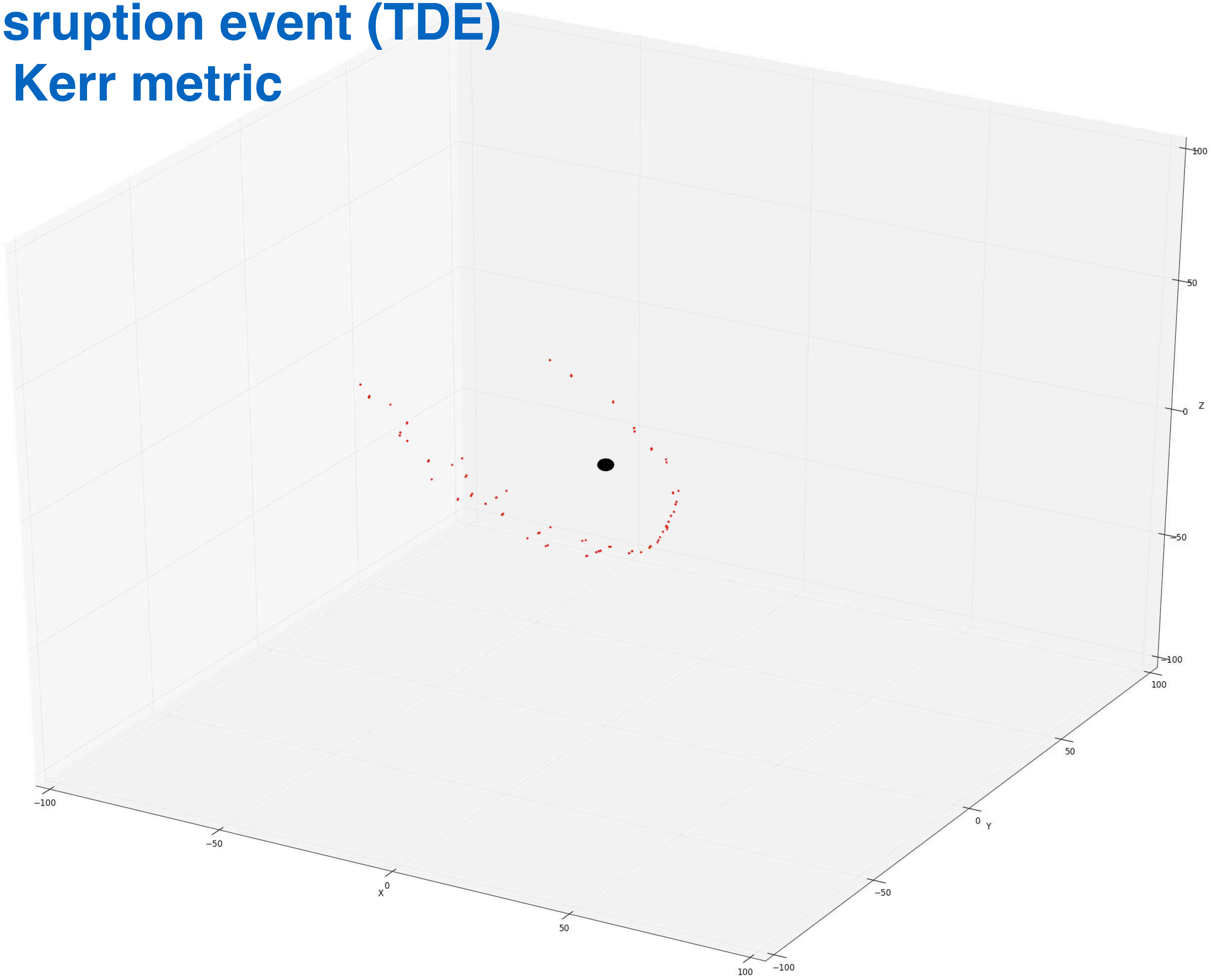
Liptai and Price 2018 (In prep.)

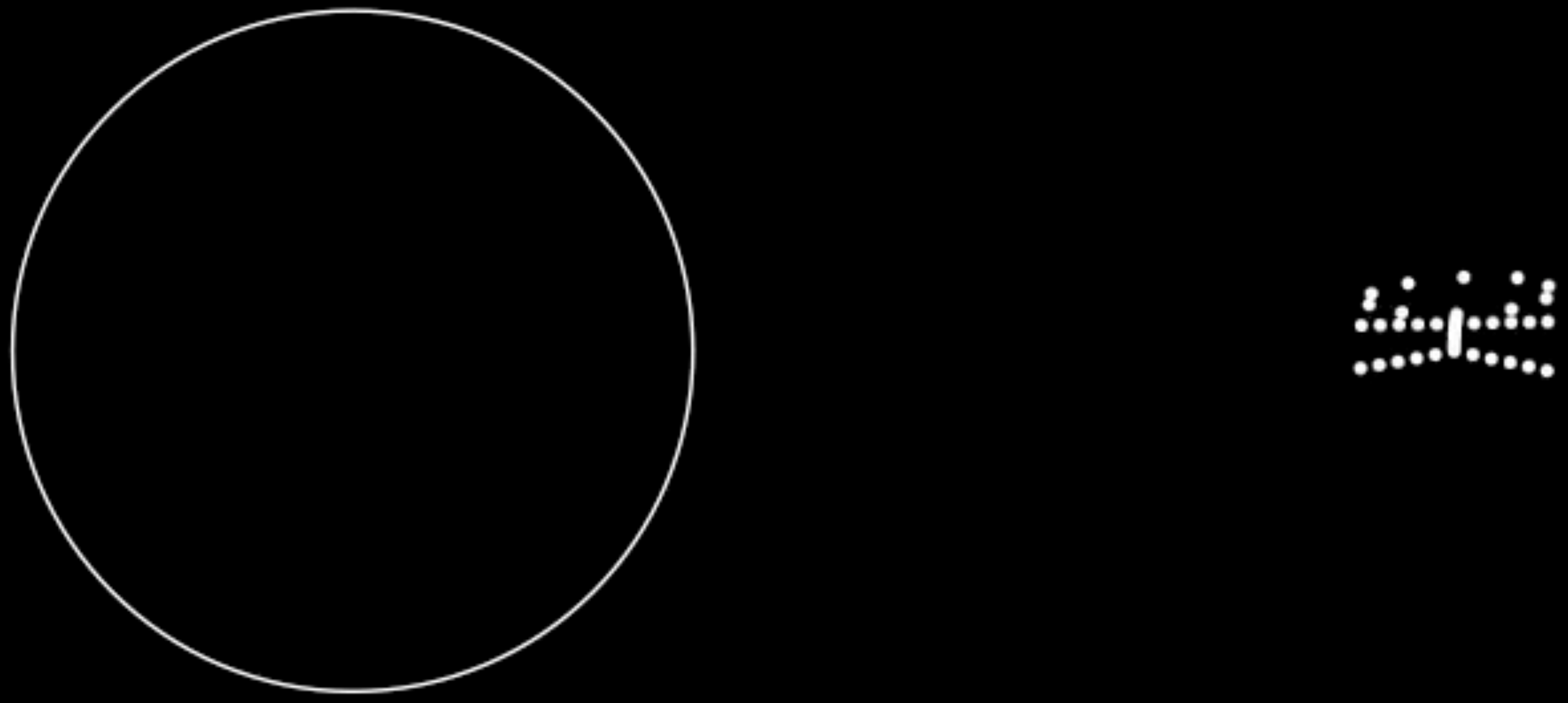
David Liptai 2016

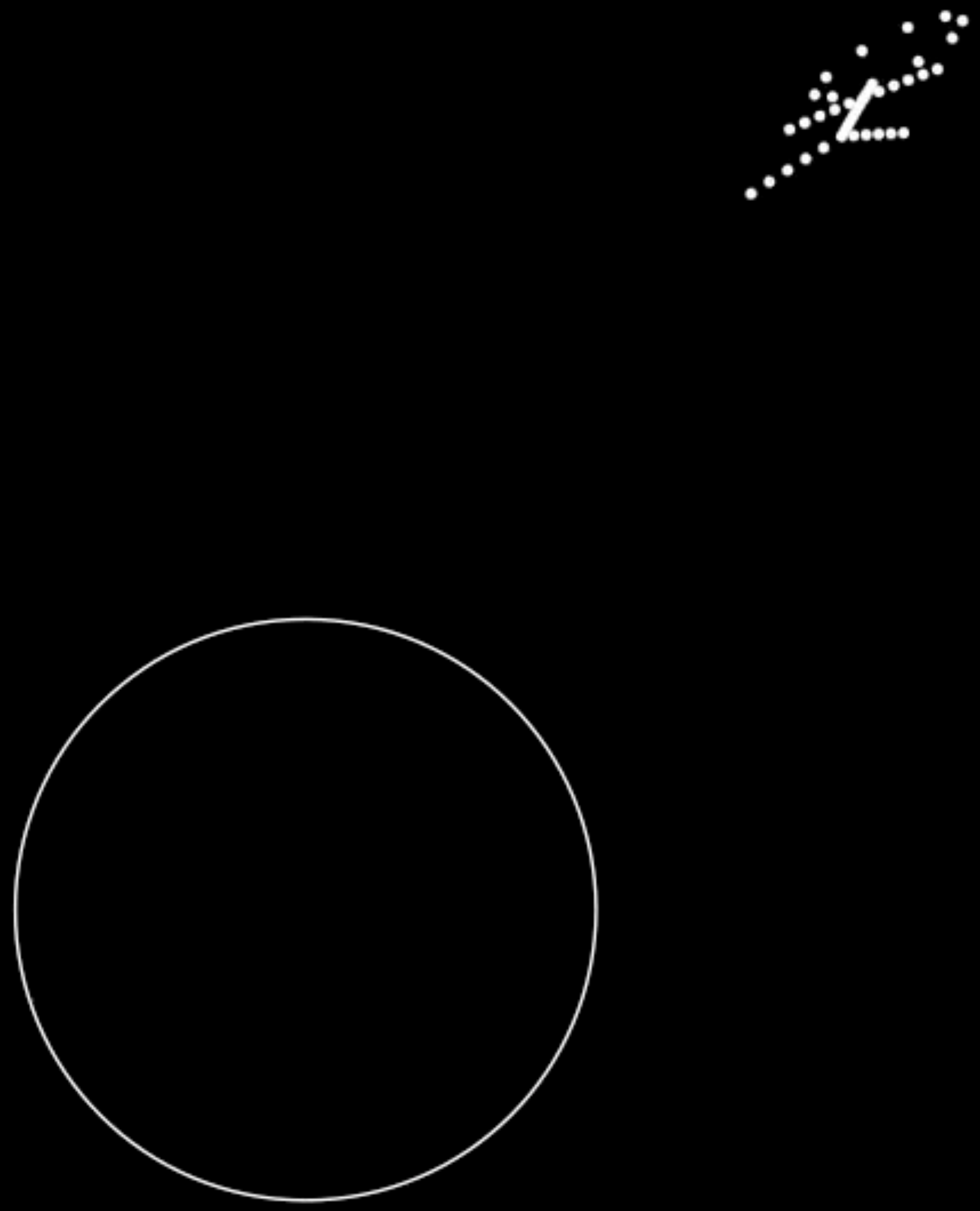
Nodal Precession



A mock tidal disruption event (TDE) in the Kerr metric







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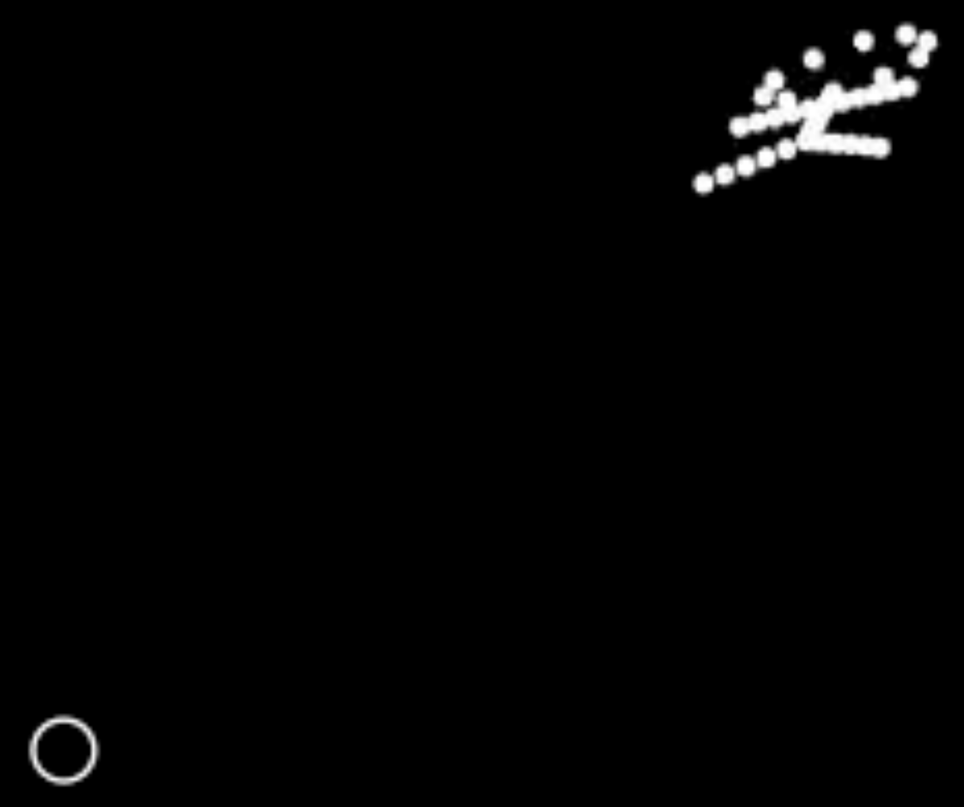


David Liptai 2016

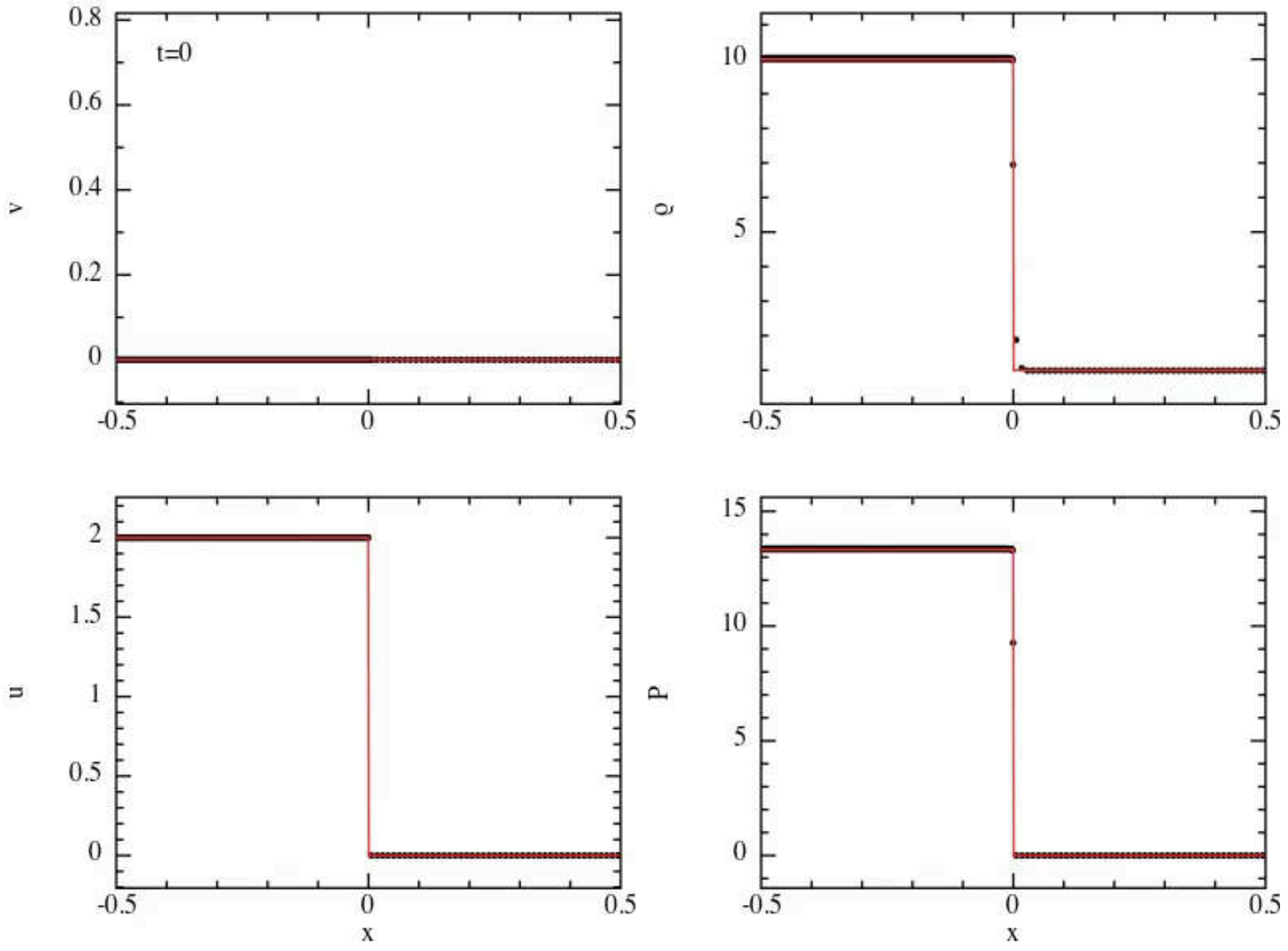
Tests: shock capturing

- 1D shock tubes
- Minkowski metric (special rel)

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\sum_b m_b \left[\frac{\sqrt{-g_a} P_a}{\Omega_a \rho_a^{*2}} \frac{\partial W_{ab}(h_a)}{\partial x^i} + \frac{\sqrt{-g_b} P_b}{\Omega_b \rho_b^{*2}} \frac{\partial W_{ab}(h_b)}{\partial x^i} \right] + \left(\frac{\mathrm{d}p_i}{\mathrm{d}t} \right)_{\mathrm{diss}}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\sum_b m_b \left[\frac{\sqrt{-g_a} P_a}{\Omega_a \rho_a^{*2}} v_b^i \frac{\partial W_{ab}(h_a)}{\partial x^i} + \frac{\sqrt{-g_b} P_b}{\Omega_b \rho_b^{*2}} v_a^i \frac{\partial W_{ab}(h_b)}{\partial x^i} \right] + \left(\frac{\mathrm{d}e}{\mathrm{d}t} \right)_{\mathrm{diss}}$$

What should we use?



Attempts at artificial dissipation in SR

Chow & Monaghan (1997)
Overly dissipative

Siegler & Riffert (2000)
No artificial conductivity

Controlling artificial conductivity

$$\left(\frac{\mathrm{d}\mathbf{p}_a}{\mathrm{d}t}\right)_{\mathrm{diss}} \sim \sum_b \frac{m_b}{\bar{\rho}_{ab}} v_{\mathrm{sig}}\ \hat{\mathbf{r}}_{ab} \cdot (\mathbf{p}_a - \mathbf{p}_b)\ \overline{\nabla W}_{ab}$$

$$\left(\frac{\mathrm{d}e_a}{\mathrm{d}t}\right)_{\mathrm{diss}} \sim \sum_b \frac{m_b}{\bar{\rho}_{ab}} v_{\mathrm{sig}}\ (e_a - e_b)\ \hat{\mathbf{r}}_{ab} \cdot \overline{\nabla W}_{ab}$$

Non-relativistic

$$e = \frac{1}{2} v^2 + u$$

$$e_a - e_b = \underbrace{\frac{1}{2}\alpha_{\mathrm{visc}} \left(v_a^2 - v_b^2\right)}_{\text{Viscosity}} + \underbrace{\alpha_{\mathrm{cond}} \left(u_a - u_b\right)}_{\text{Conductivity}}$$

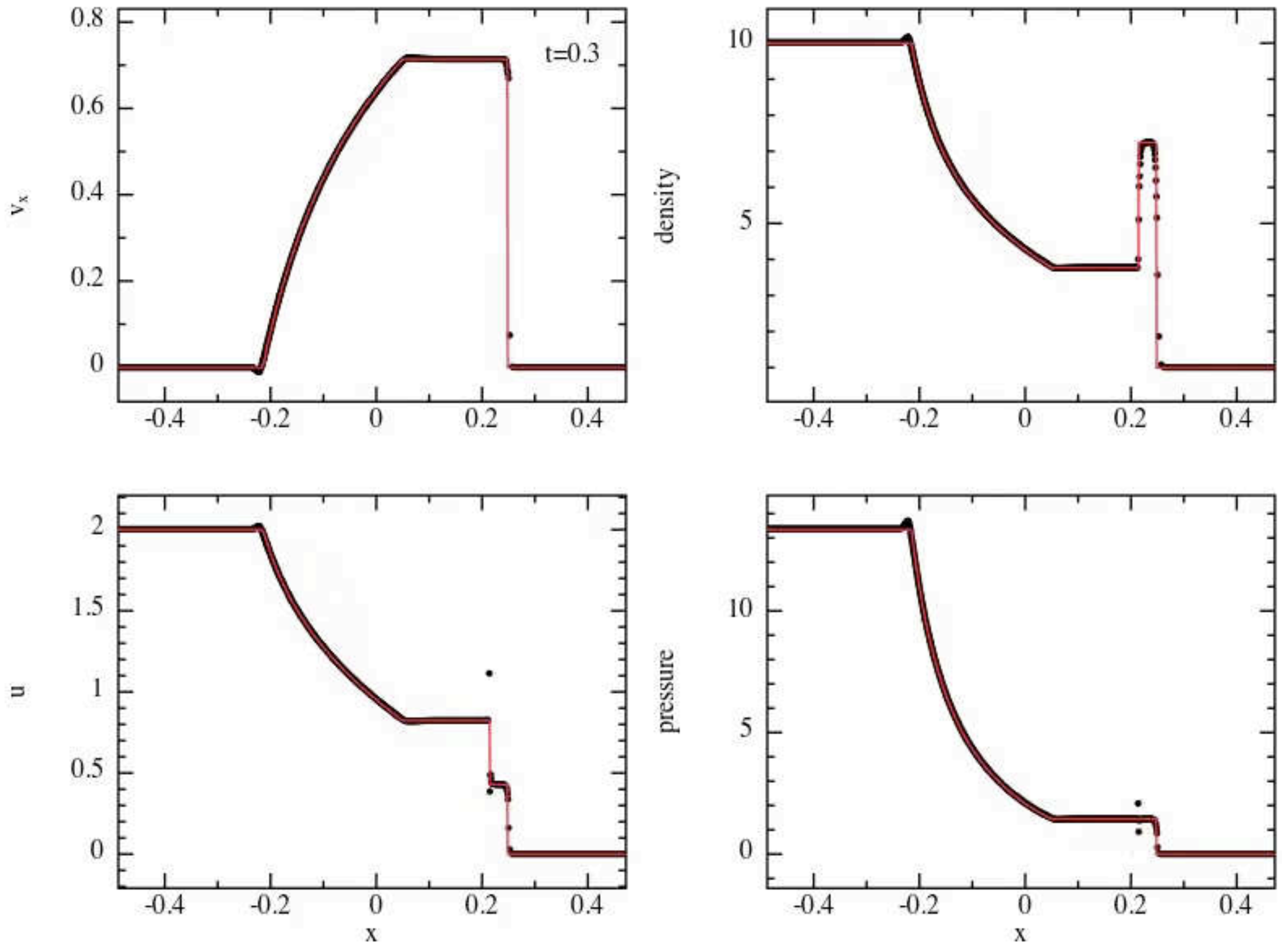
Relativistic

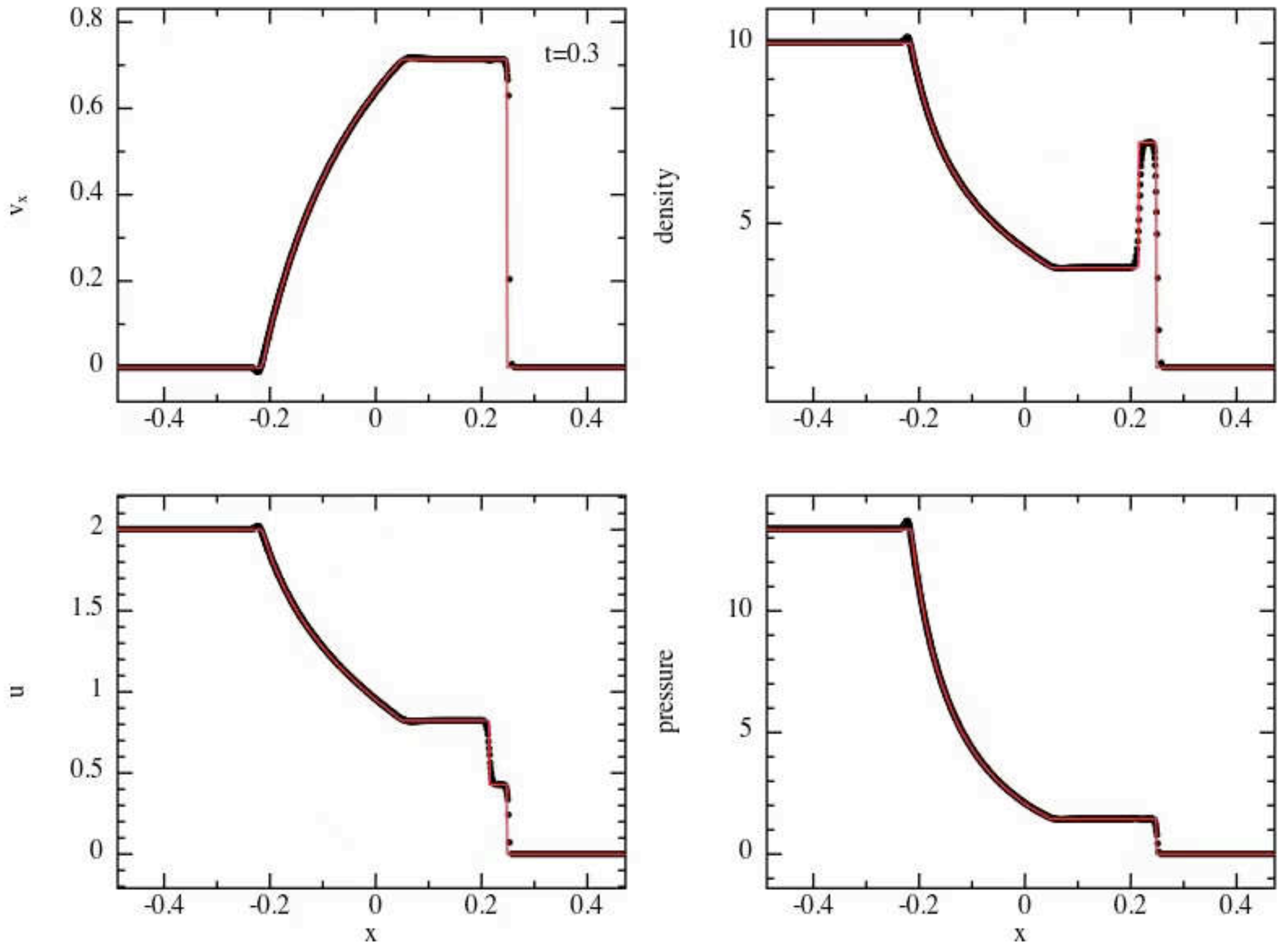
$$e = \frac{v^2}{\sqrt{1-v^2}}(1 + u + P/\rho) + \sqrt{1-v^2}(1+u)$$

$$e_a - e_b = \quad \ldots \quad ??? \quad \ldots$$

$$\boxed{\underbrace{\alpha_{\mathrm{visc}} \left[\varpi \left(\gamma_a v_a^2 - \gamma_b v_b^2\right) + \left(\frac{1}{\gamma_a} - \frac{1}{\gamma_b}\right)\right]}_{\text{Viscosity}} + \underbrace{\alpha_{\mathrm{cond}} \left[\frac{u_a}{\gamma_a} - \frac{u_b}{\gamma_b}\right]}_{\text{Conductivity}}}$$

1D special relativistic shock tubes





Artificial viscosity only

Artificial viscosity AND conductivity

Liptai and Price 2018 (In prep.)

1D special relativistic shock tubes

Ultra-relativistic 1D

Sine wave perturbation 1D

Liptai and Price 2018 (In prep.)

3D Hydrodynamics


Publications of the Astronomical Society of Australia (PASA)

doi: 10.1017/pas.2017.xxx.

PHANTOM: A smoothed particle hydrodynamics and magnetohydrodynamics code for astrophysics

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Abstract
We present PHANTOM, a fast, parallel, modular and low-memory smoothed particle hydrodynamics and magnetohydrodynamics code developed over the last decade for astrophysical applications in three dimensions. The code has been developed with a focus on stellar, galactic, planetary and high energy astrophysics and has already been used widely for studies of accretion discs and turbulence, from the birth of planets to how black holes accrete. Here we describe and test the core algorithms as well as modules for magnetohydrodynamics, self-gravity, sink particles, H_2 chemistry, dust-gas mixtures, physical viscosity, external forces including numerous galactic potentials as well as implementations of Lense-Thirring precession, Poynting-Robertson drag and stochastic turbulent driving. PHANTOM is hereby made publicly available.

Keywords: hydrodynamics — methods: numerical — magnetohydrodynamics (MHD) — accretion, accretion discs — ISM: general

arXiv:1702.03930v1 [astro-ph.IM] 13 Feb 2017

1 Introduction

Numerical simulations are the 'third pillar' of astrophysics, standing alongside observations and analytic theory. Since it is difficult to perform laboratory experiments in the relevant physical regimes and over the correct range of length and time-scales involved in most astrophysical problems, we turn instead to 'numerical experiments' in the computer for understanding and insight. As algorithms and simulation codes become ever more sophisticated, the public availability of simulation codes has become crucial to ensure that these experiments can be both verified and reproduced.

PHANTOM is a smoothed particle hydrodynamics (SPH) code, written in Fortran 90, developed over the last decade. It has been used widely for studies of accretion (Lodato & Price, 2010; Nixon et al., 2012a; Rosotti et al., 2012; Nixon, 2012; Nixon et al., 2012b; Facchini et al., 2013; Nixon et al., 2013; Martin et al., 2014a,b; Nixon & Lubow, 2015; Coughlin & Nixon, 2015; Forgan et al., 2017) and turbulence (Kitsionas et al., 2009; Price & Federrath, 2010; Price et al., 2011; Price, 2012b; Tricco et al., 2016b) as well as for studies of the Galaxy

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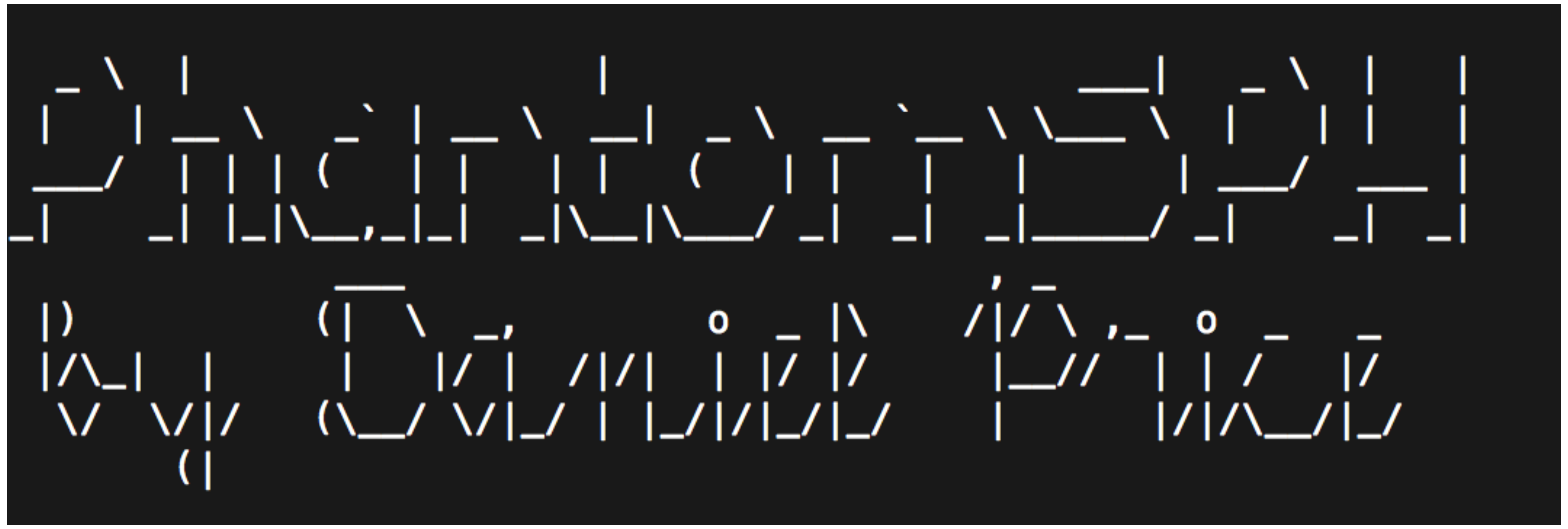


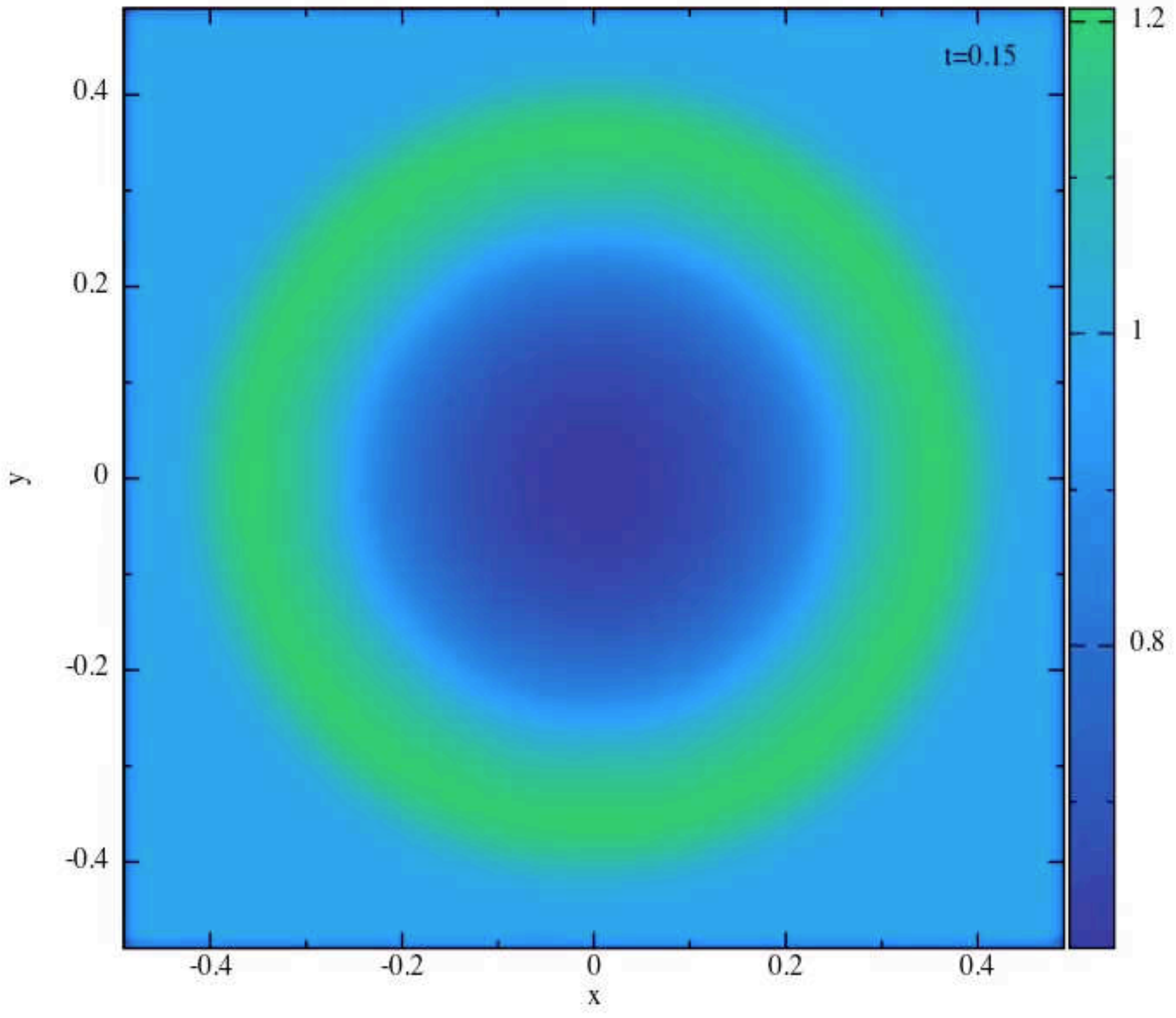
3D Special relativistic shocktubes

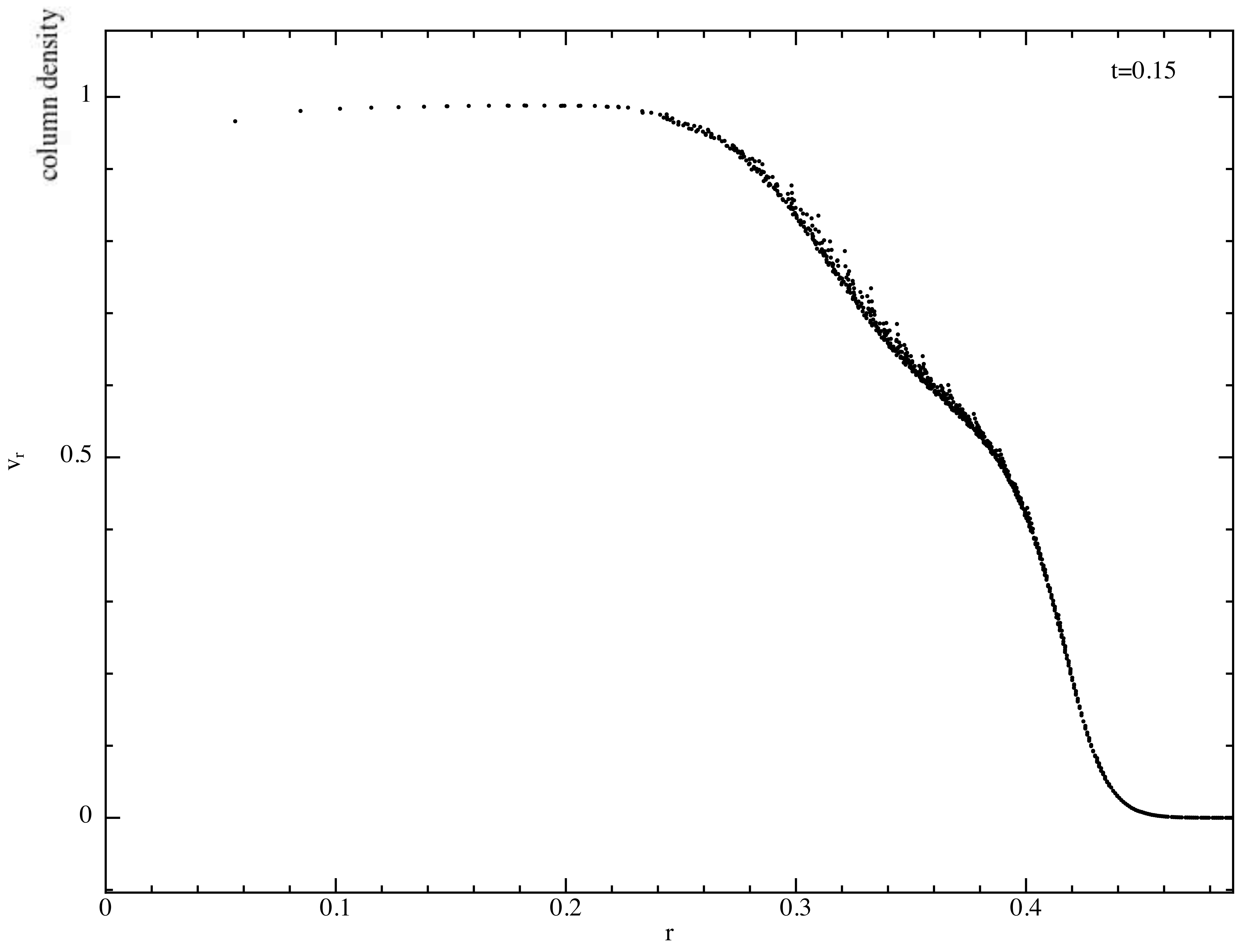
Mildly-relativistic 3D

Ultra-relativistic 3D

Liptai and Price 2018 (In prep.)

3D spherical blast wave

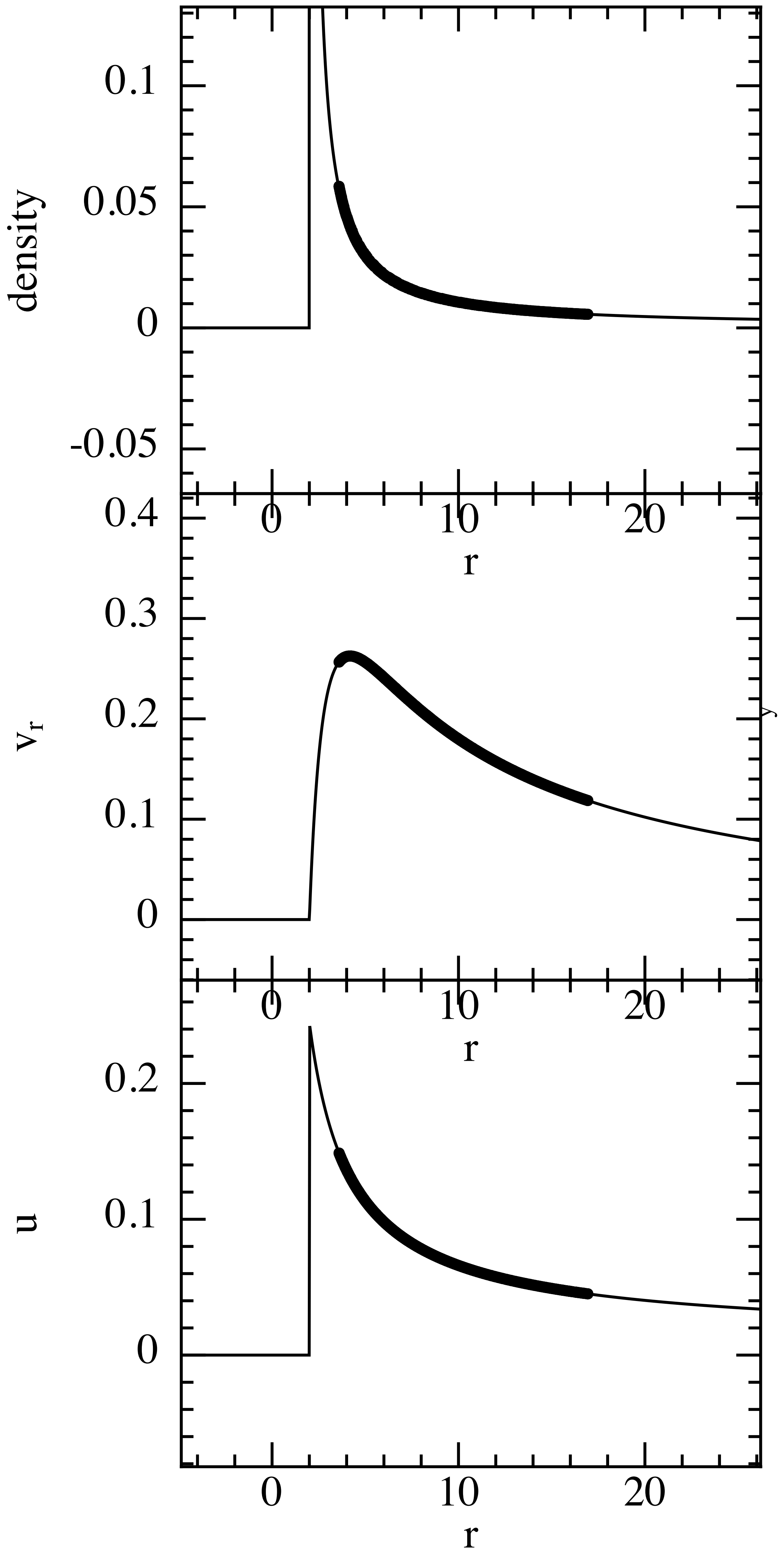


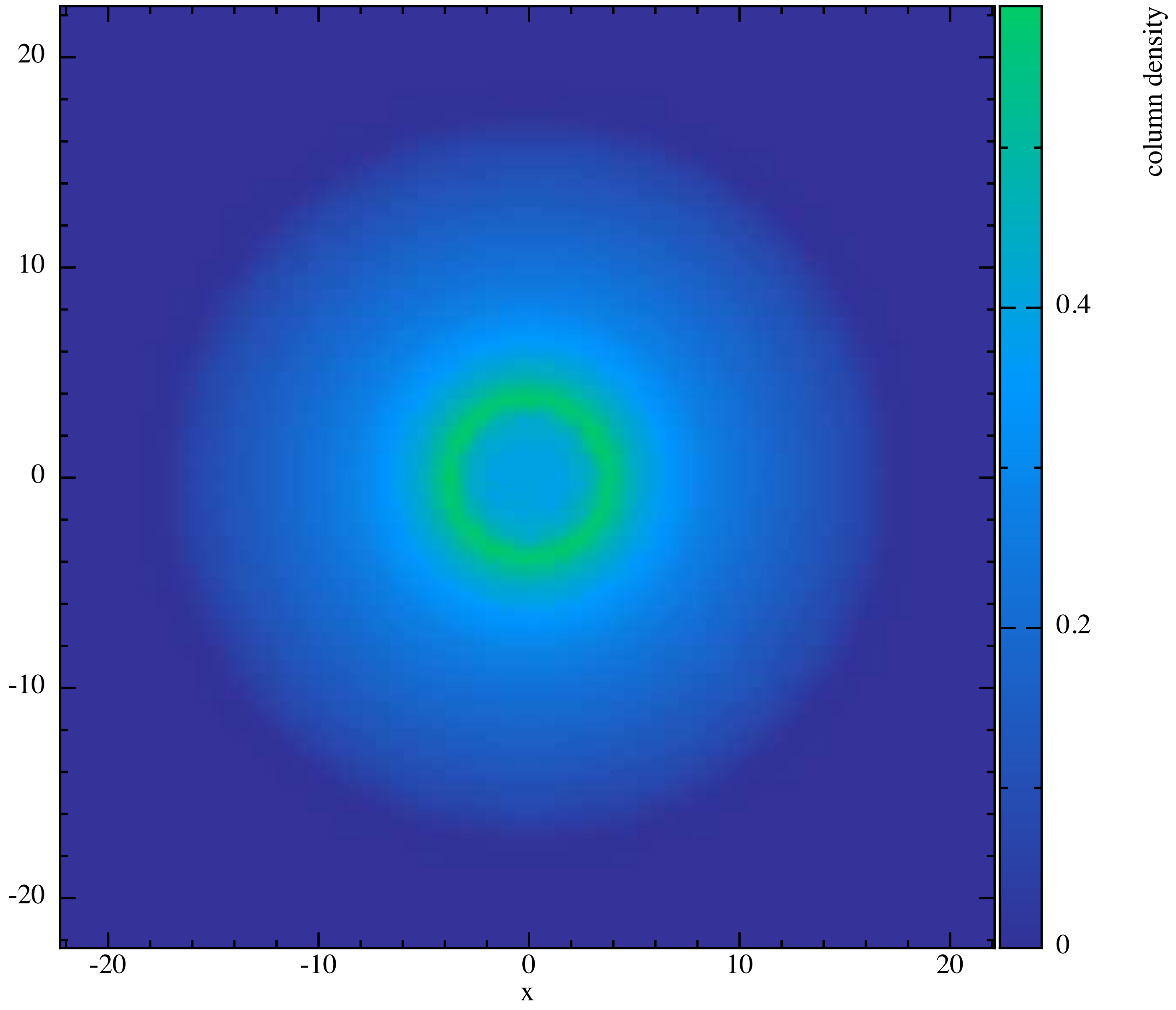


Maximum Lorentz factor ~6.4

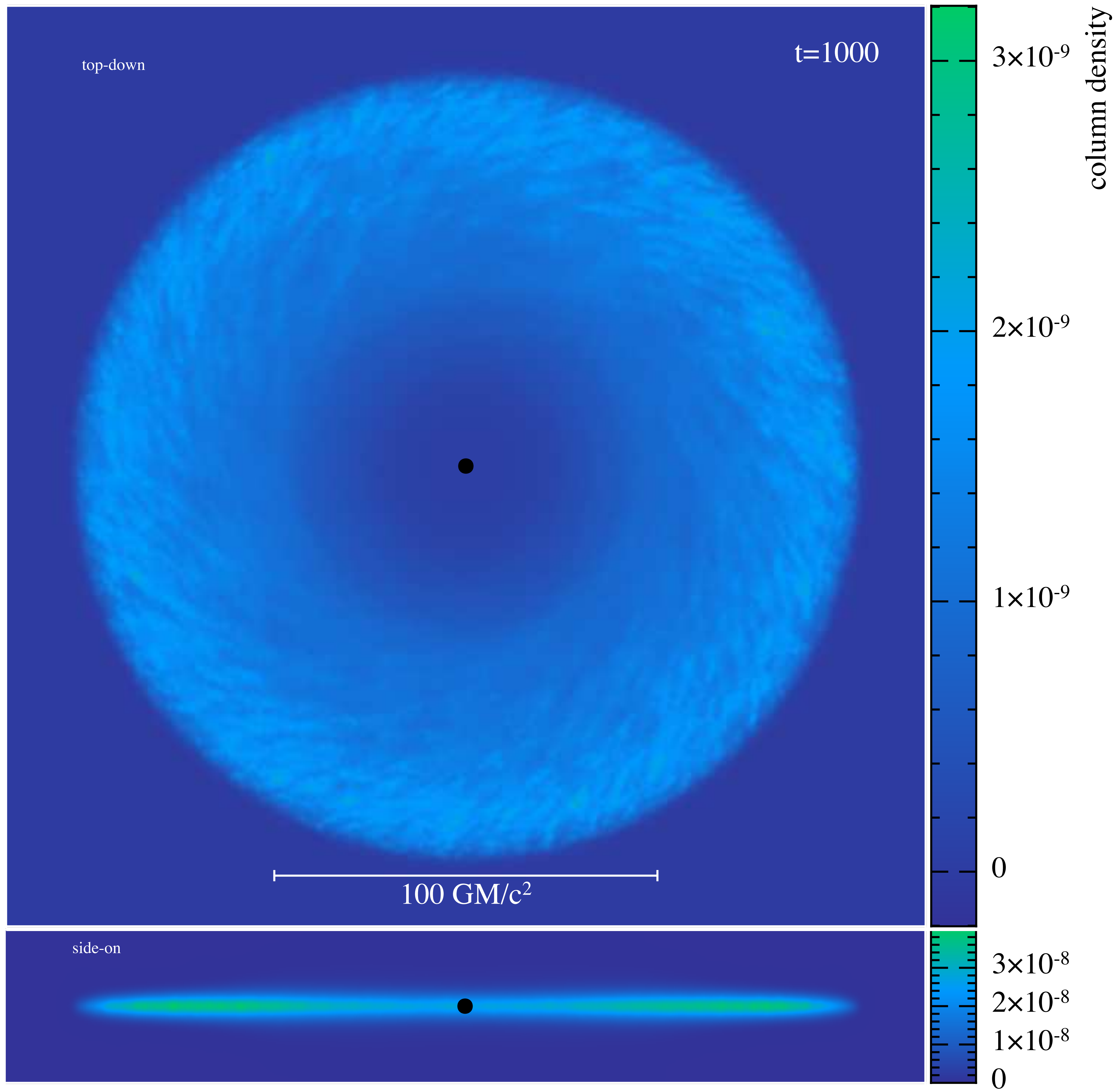
Other 3D Hydro

Generalised spherically symmetric Bondi accretion (Schwarzschild)





Accretion disc around a Schwarzschild black hole



Liptai and Price 2018 (In prep.)

Conclusions

- **Orbital tests** (Schwarzschild AND Kerr) are in excellent agreement with theory
- We can handle **relativistic shocks** very well
- We have **split artificial dissipation** into viscosity and conductivity
- Merged with **PHANTOM** to do full **3D-GRSPH** simulations

Matthew McConaughey