Ionising Stellar Feedback with Phantom and CMacIonize

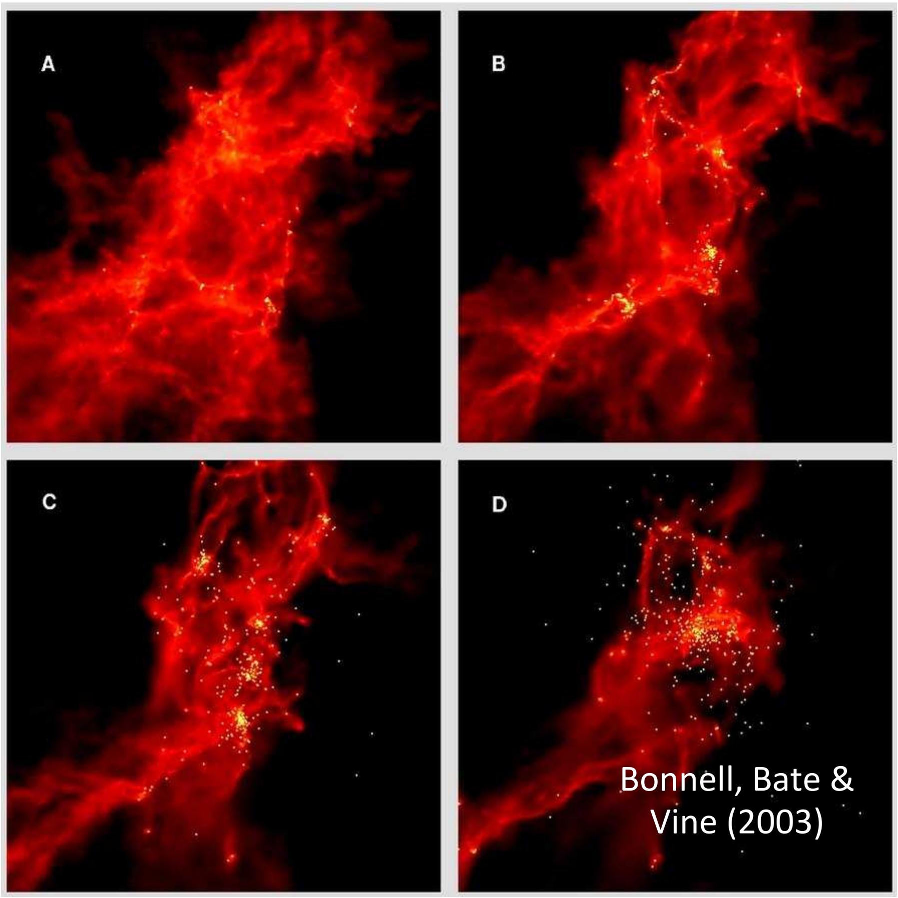
Maya Petkova

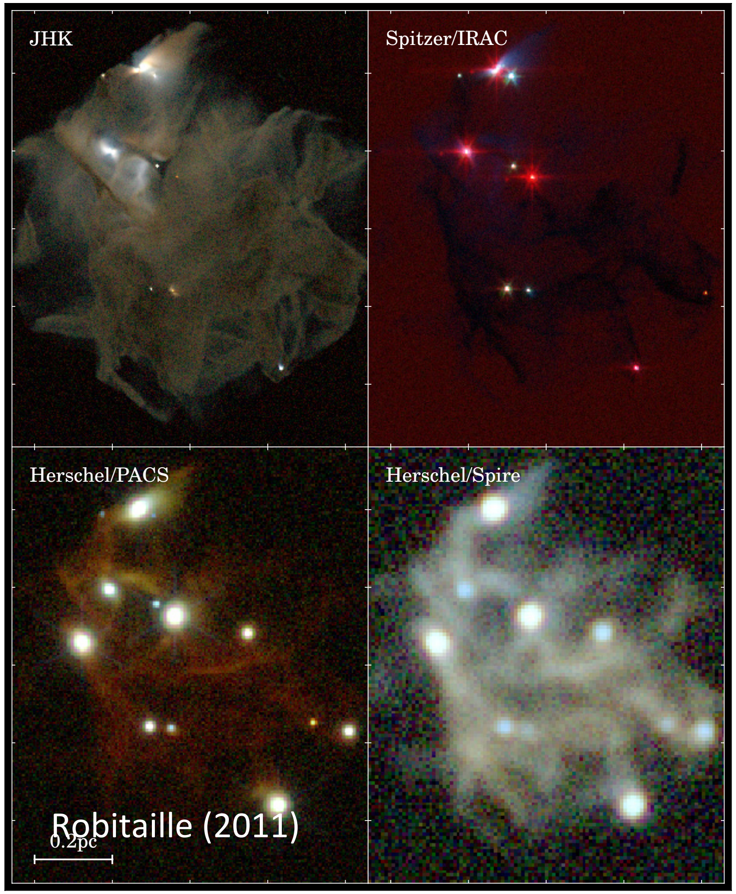
Supervisor: Ian Bonnell

Collaborators: Guillaume Laibe, Bert Vandenbroucke, Jim Dale


The Heart & Soul Nebulae © 2014 Terry Hancock

SPH and MCRT

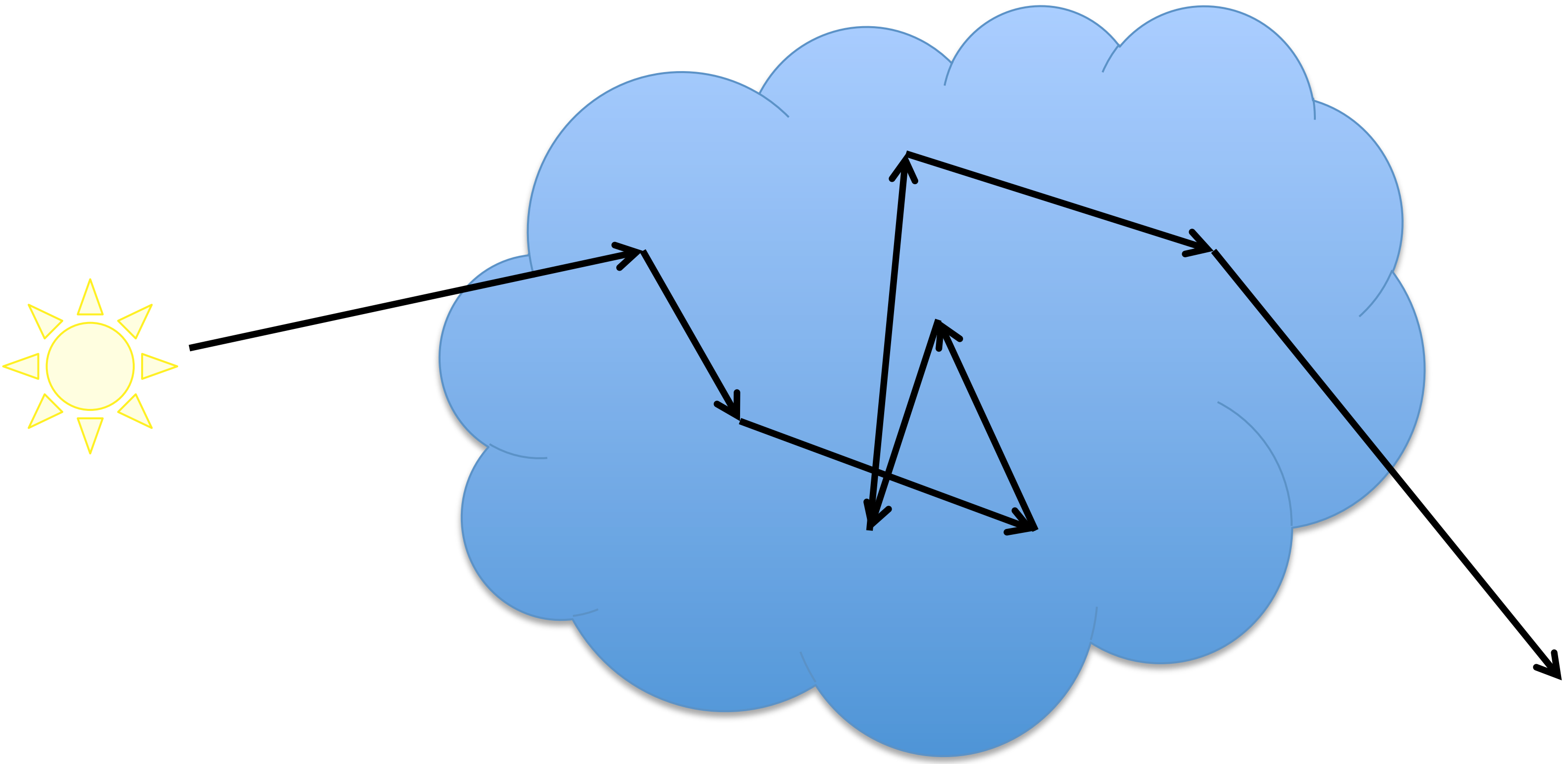




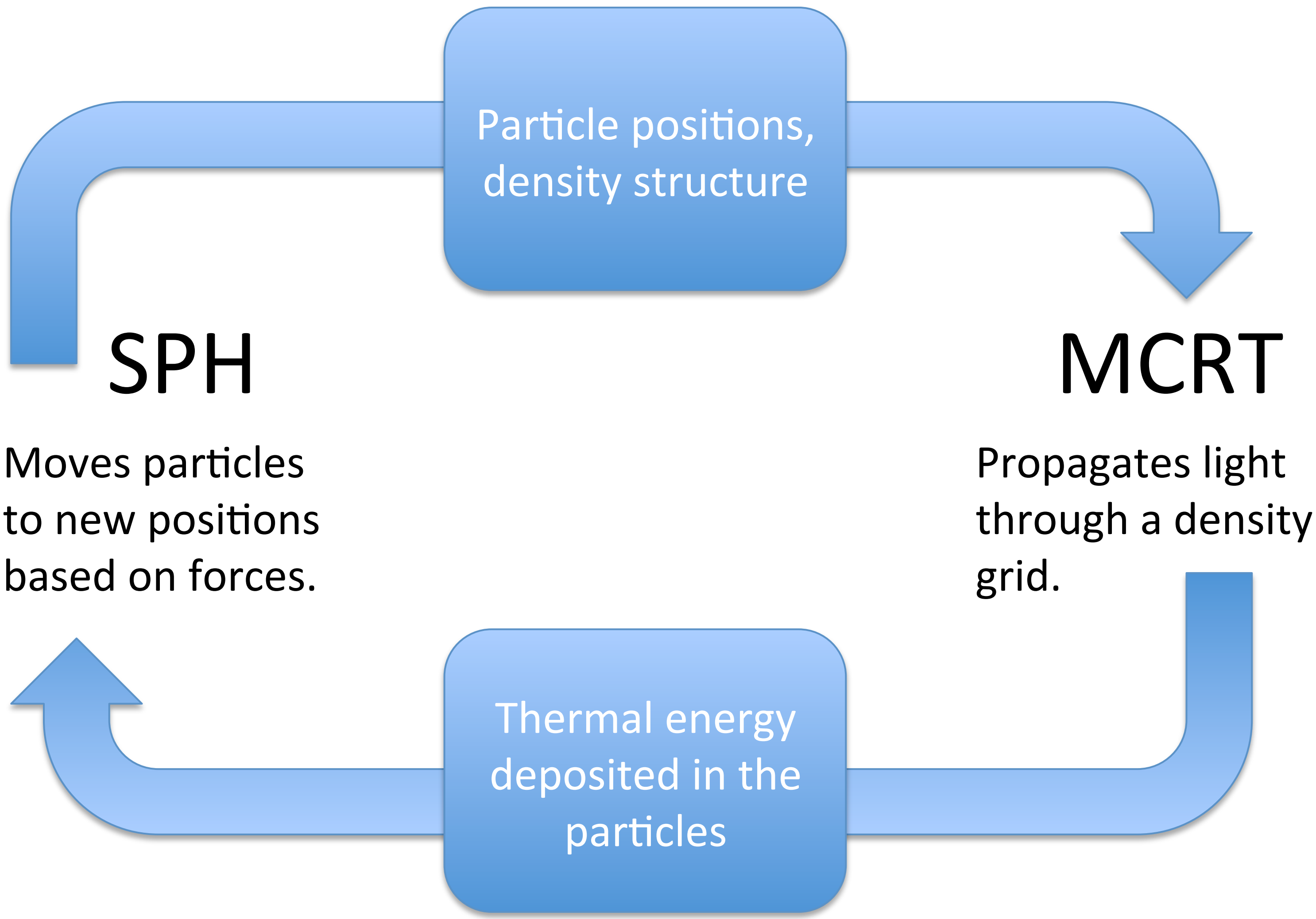
Smoothed Particle
Hydrodynamics

Monte Carlo
Radiative Transfer

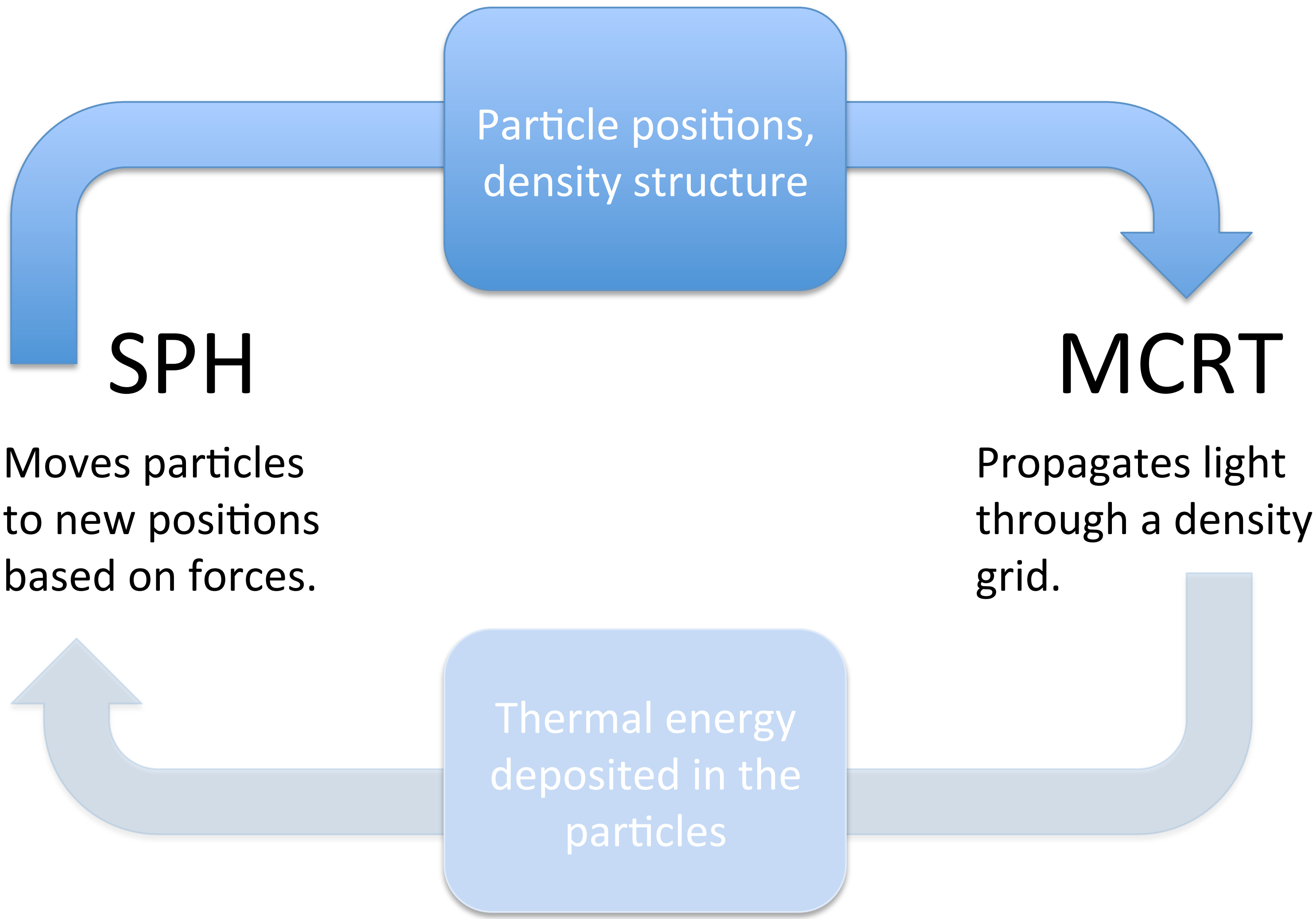
MCRT Recap



SPH and MCRT


Particle positions, density structure
SPH
Moves particles to new positions based on forces.
MCRT
Propagates light through a density grid.
Thermal energy deposited in the particles

SPH and MCRT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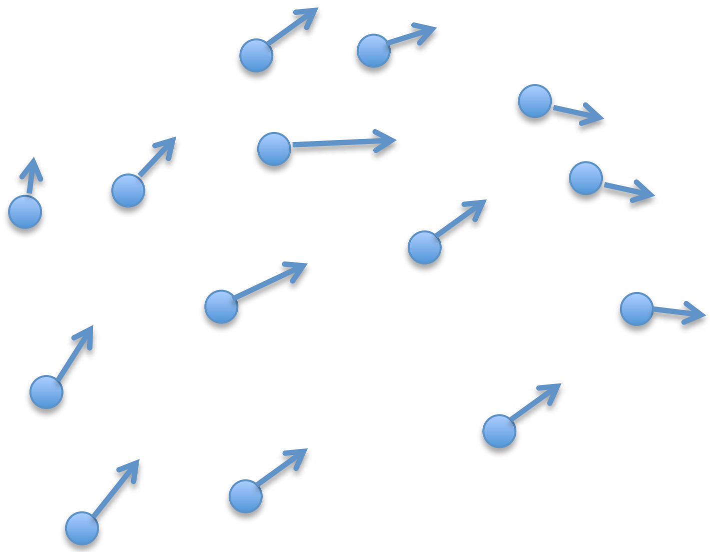
Lagrangian vs Eulerian Cloud



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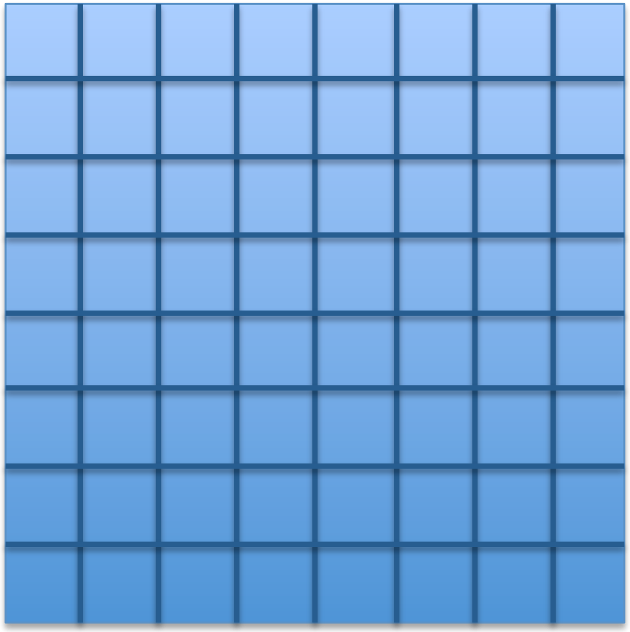
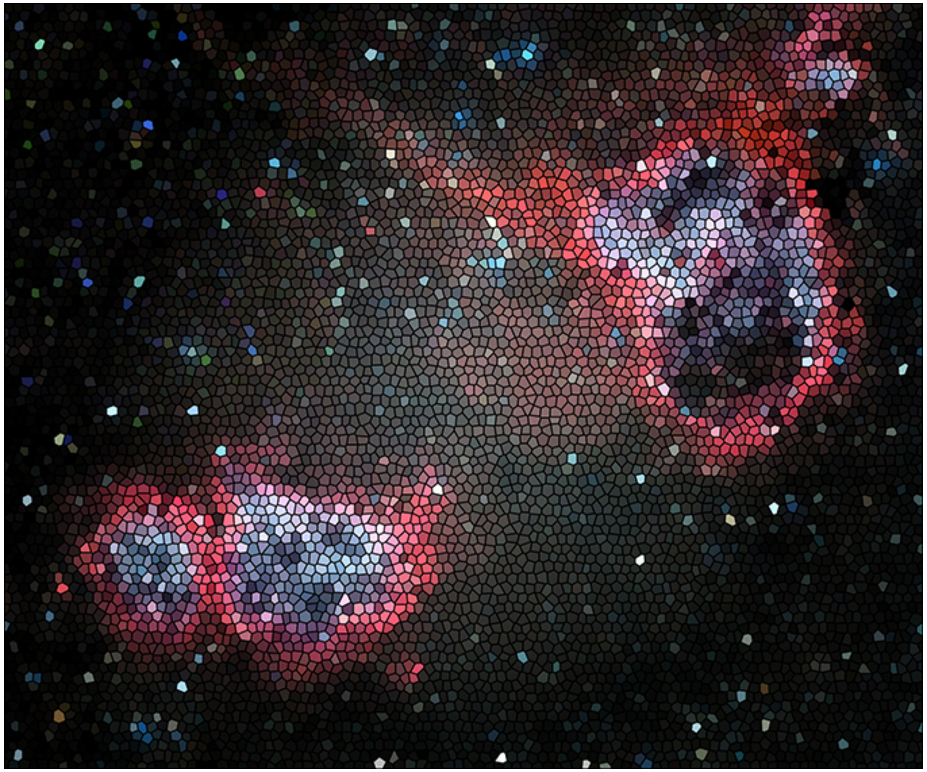


MCRT

Voronoi tessellation



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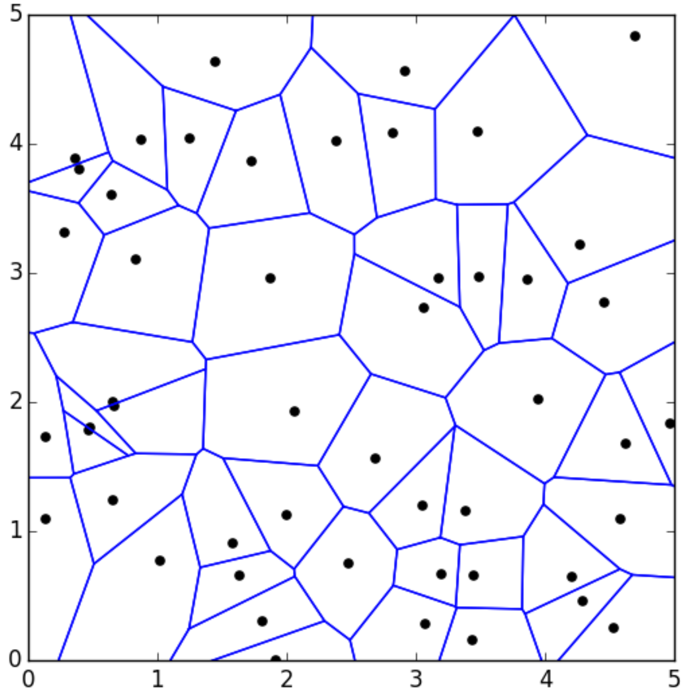
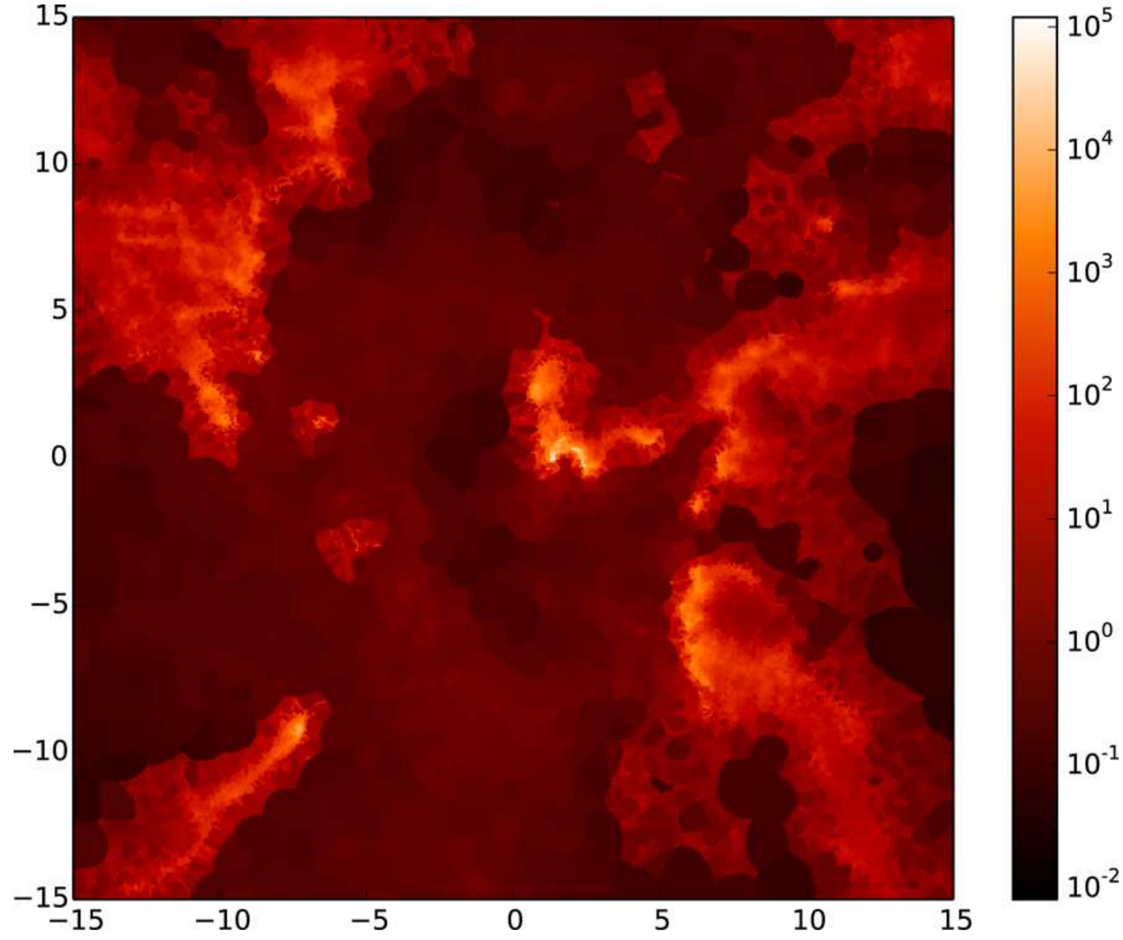
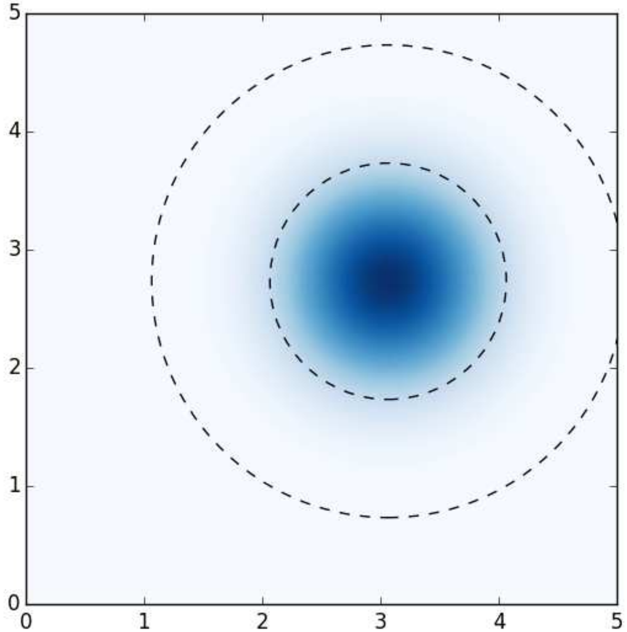


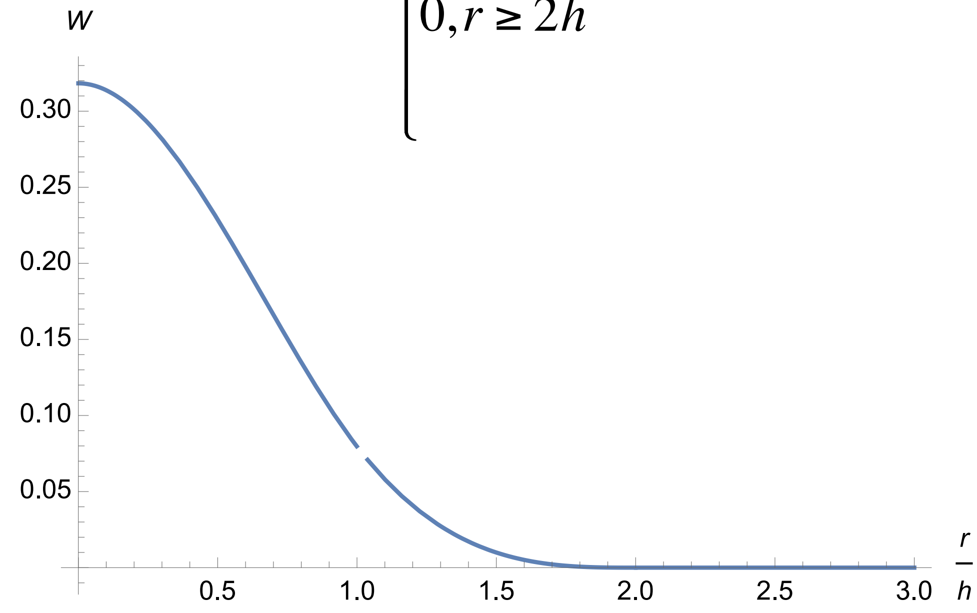


Hubber, Ercolano & Dale (2016)

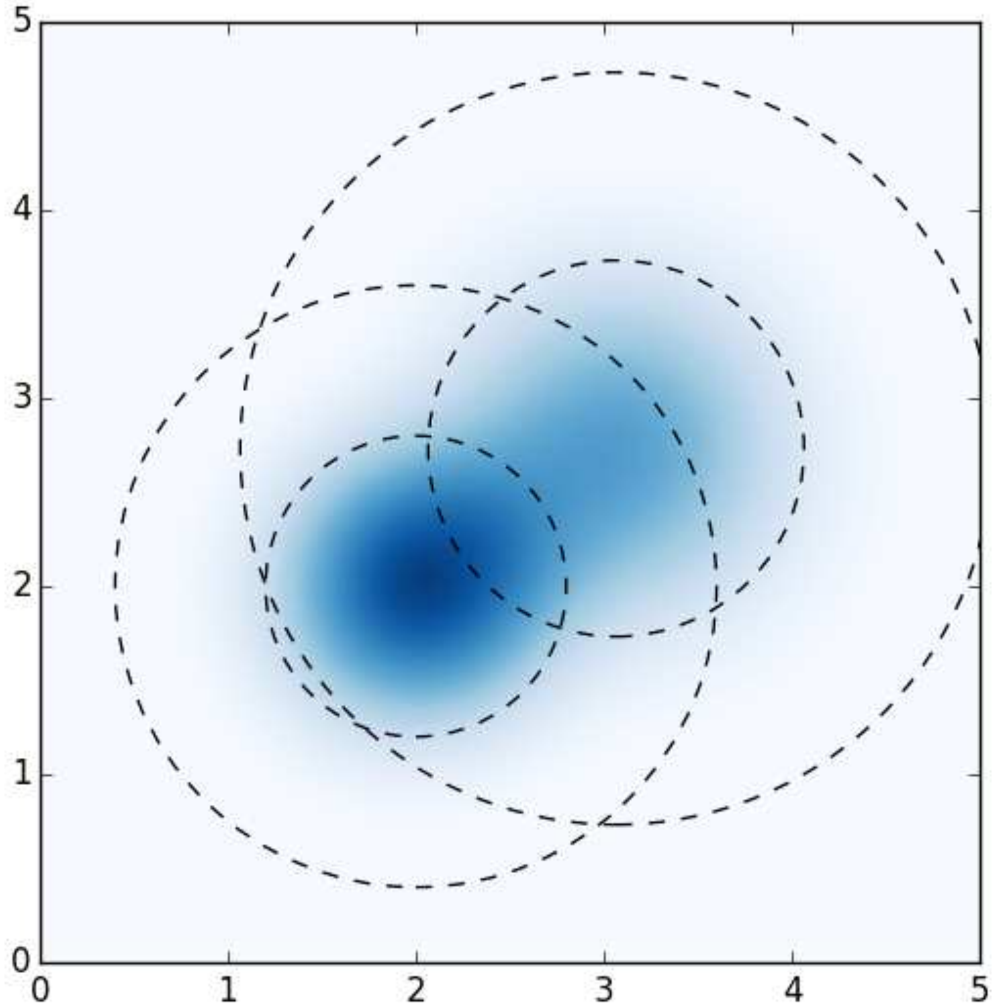
An SPH particle and its kernel



$$W(r,h) = \frac{1}{h^3\pi}\begin{cases} 1 - 1.5\left(\frac{r}{h}\right)^2 + 0.75\left(\frac{r}{h}\right)^3, r \le h \\ 0.25\left(2 - \frac{r}{h}\right)^3, h \le r \le 2h \\ 0, r \ge 2h \end{cases}$$

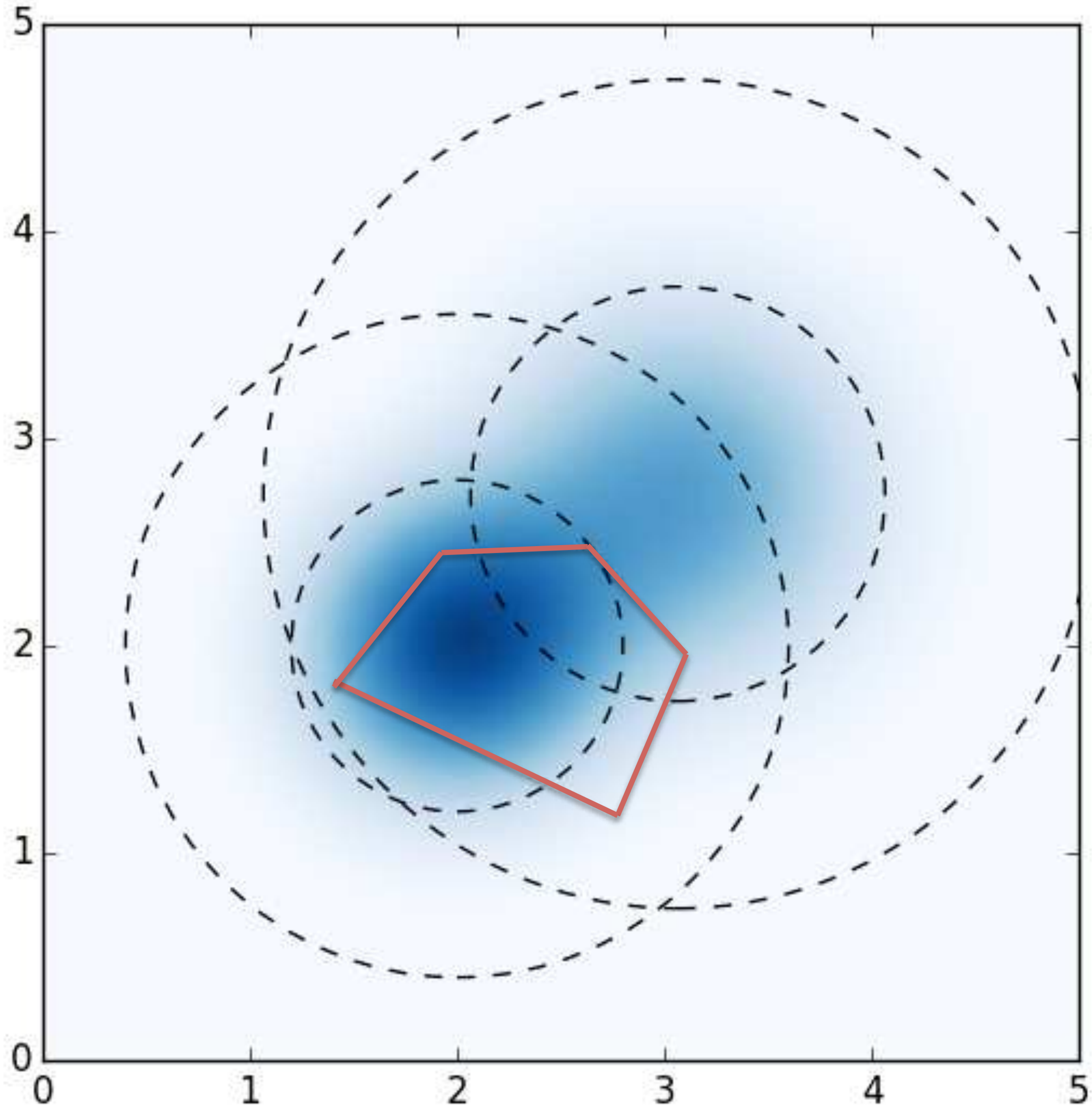


SPH density sum



$$\rho(\vec{r}) = \sum_{i=1}^{N} m_i W(\vec{r} - \vec{r}_i, h)$$

Voronoi cell density



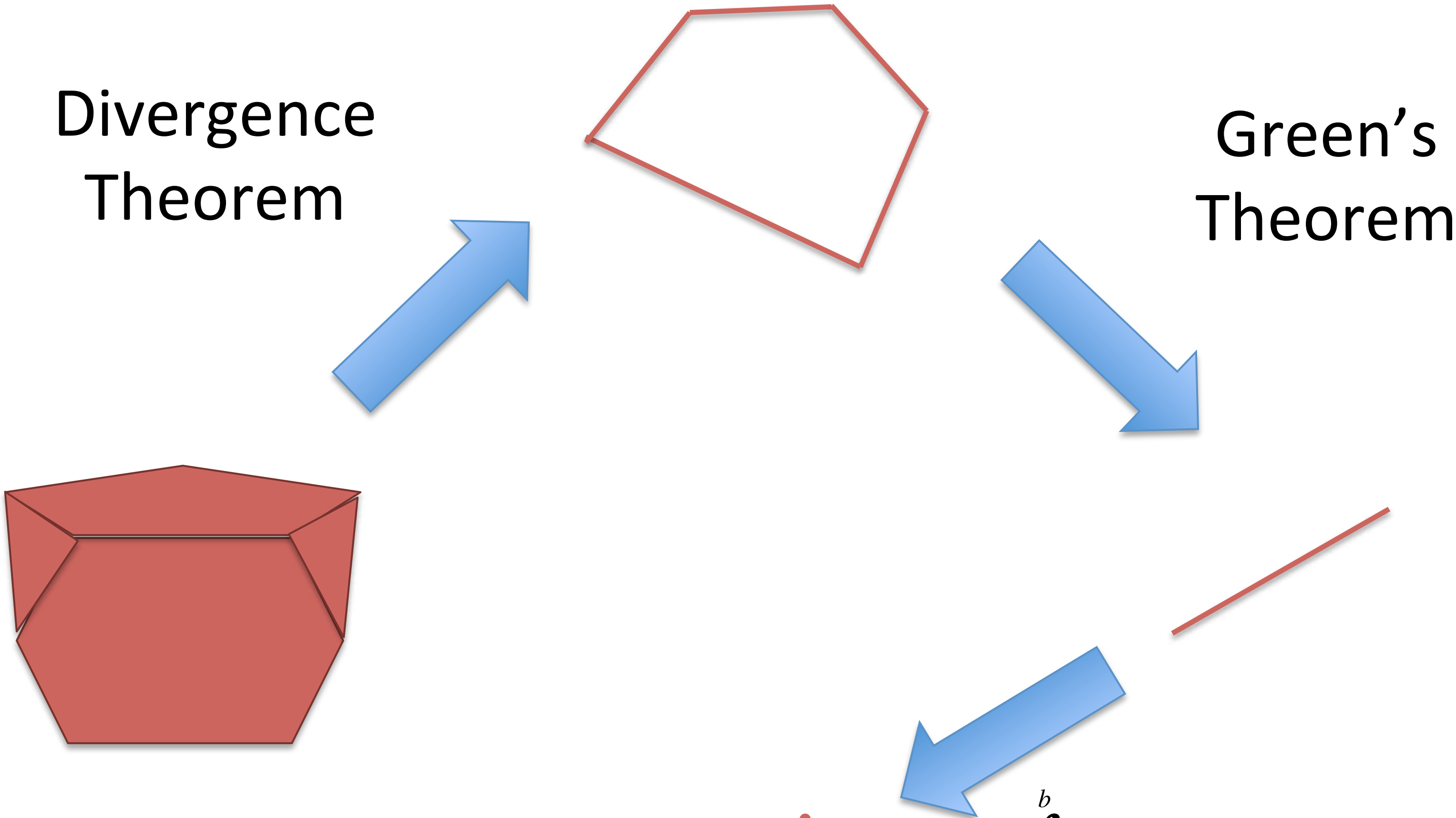
How do we integrate a spherically symmetric function over the volume of any random polyhedron?

Can

~~How do~~ we integrate a spherically symmetric function over the volume of any random polyhedron?

(analytically)

Yes. And this is how.

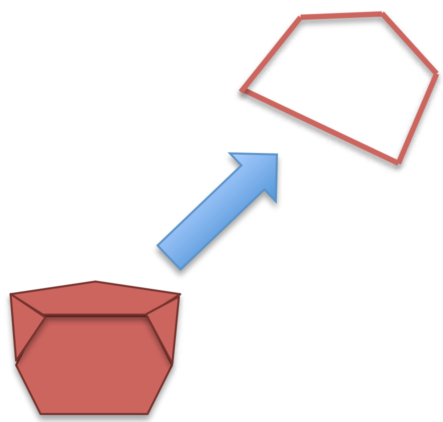


$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Divergence Theorem

$$\int_V \nabla \cdot \vec{F}\, dV = \int_{\partial V} \vec{F} \cdot \hat{n}\, dA$$

$$\int_V W\, dV = \int_V \nabla \cdot \vec{F}\, dV$$



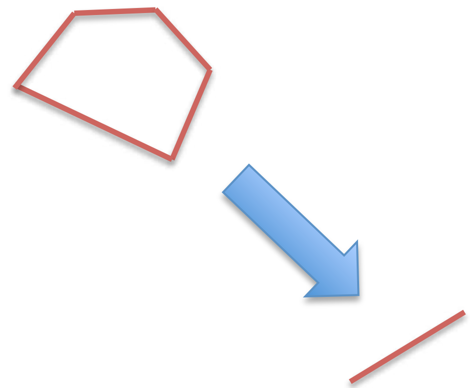
$$W(r) = \frac{1}{h^3\pi}\begin{cases} 1 - 1.5\left(\frac{r}{h}\right)^2 + 0.75\left(\frac{r}{h}\right)^3, r \le h \\ 0.25\left(2 - \frac{r}{h}\right)^3, h \le r \le 2h \\ 0, r \ge 2h \end{cases}$$

$$\vec{F} = F_r \hat{r}$$

$$F_r = \frac{1}{r^2}\int r^2 W(r)\, dr = \frac{1}{r^2}\frac{1}{h^3\pi}\begin{cases} \frac{1}{3}r^3 - \frac{3}{10h^2}r^5 + \frac{1}{8h^3}r^6, r \le h \\ \frac{1}{4}\left(\frac{8}{3}r^3 - \frac{3}{h}r^4 + \frac{6}{5h^2}r^5 - \frac{1}{6h^3}r^6 - \frac{h^3}{15}\right), h \le r \le 2h \\ \frac{h^3}{4}, r \ge 2h \end{cases}$$

Green's Theorem

$$\int_A \nabla \cdot \vec{H}\, dA = \oint_{\partial A} \vec{H} \cdot \hat{m}\, dl$$



$$\int_{\partial V} \vec{F} \cdot \hat{n}\, dA = \int_A \nabla \cdot \vec{H}\, dA$$

$$\vec{H} = H_R \vec{R}$$

$$\mu = \cos\theta = \frac{r_0}{r}$$

$$H_R = \frac{1}{R}\int F_r \sin\theta\, dR = \frac{1}{R}\frac{r_0^3}{h^3\pi}\begin{cases} \frac{1}{6}\mu^{-2} - \frac{3}{40}\left(\frac{r_0}{h}\right)^2 \mu^{-4} - \frac{1}{40}\left(\frac{r_0}{h}\right)^3 \mu^{-5} + \frac{B_1}{r_0^3}, \mu \ge \frac{r_0}{h} \\ \frac{1}{4}\left(\frac{4}{3}\mu^{-2} - \left(\frac{r_0}{h}\right)\mu^{-3} + \frac{3}{10}\left(\frac{r_0}{h}\right)^2 \mu^{-4} - \frac{1}{30}\left(\frac{r_0}{h}\right)^3 \mu^{-5} + \frac{1}{15}\left(\frac{r_0}{h}\right)^{-3} \mu\right) + \frac{B_2}{r_0^3}, \frac{r_0}{2h} \le \mu \le \frac{r_0}{h} \\ -\frac{1}{4}\left(\frac{r_0}{h}\right)^{-3} \mu + \frac{B_3}{r_0^3}, \mu \le \frac{r_0}{2h} \end{cases}$$

Final solution

$$B_1 = \frac{r_0^3}{4}\left(-\frac{2}{3}+\frac{3}{10}\left(\frac{r_0}{h}\right)^2-\frac{1}{10}\left(\frac{r_0}{h}\right)^3\right)$$

$$B_2 = \frac{r_0^3}{4}\begin{cases}-\frac{2}{3}+\frac{3}{10}\left(\frac{r_0}{h}\right)^2-\frac{1}{10}\left(\frac{r_0}{h}\right)^3-\frac{1}{5}\left(\frac{r_0}{h}\right)^{-2}, r_0 \le h \\ -\frac{4}{3}+\left(\frac{r_0}{h}\right)-\frac{3}{10}\left(\frac{r_0}{h}\right)^2+\frac{1}{30}\left(\frac{r_0}{h}\right)^3-\frac{1}{15}\left(\frac{r_0}{h}\right)^{-3}, h \le r_0 \le 2h\end{cases}$$

$$B_3 = \frac{r_0^3}{4}\begin{cases}-\frac{2}{3}+\frac{3}{10}\left(\frac{r_0}{h}\right)^2-\frac{1}{10}\left(\frac{r_0}{h}\right)^3+\frac{7}{5}\left(\frac{r_0}{h}\right)^{-2}, r_0 \le h \\ -\frac{4}{3}+\left(\frac{r_0}{h}\right)-\frac{3}{10}\left(\frac{r_0}{h}\right)^2+\frac{1}{30}\left(\frac{r_0}{h}\right)^3-\frac{1}{15}\left(\frac{r_0}{h}\right)^{-3}+\frac{8}{5}\left(\frac{r_0}{h}\right)^{-2}, h \le r_0 \le 2h \\ \left(\frac{r_0}{h}\right)^{-3}, r_0 \ge 2h\end{cases}$$

$$I_0 = \varphi + C$$

$$I_1 = -\sin^{-1}\left(\sqrt{\frac{1+\frac{r_0^2}{R_0^2}\cos^2\varphi}{1+\frac{r_0^2}{R_0^2}}}\right)+C$$

$$I_{-2} = \varphi + \frac{r_0^2}{R_0^2}\tan\varphi + C$$

$$I_{-4} = \varphi + 2\frac{r_0^2}{R_0^2}\tan\varphi + \frac{1}{3}\frac{r_0^4}{R_0^4}\tan\varphi\left(\sec^2\varphi+2\right)+C$$

$$\alpha = \frac{R_0}{r_0}$$

$$\mu = \frac{\frac{r_0}{R_0}\cos\varphi}{\sqrt{1+\frac{r_0^2}{R_0^2}\cos^2\varphi}}$$

$$u = \sqrt{1-(1+\alpha^2)\mu^2}$$

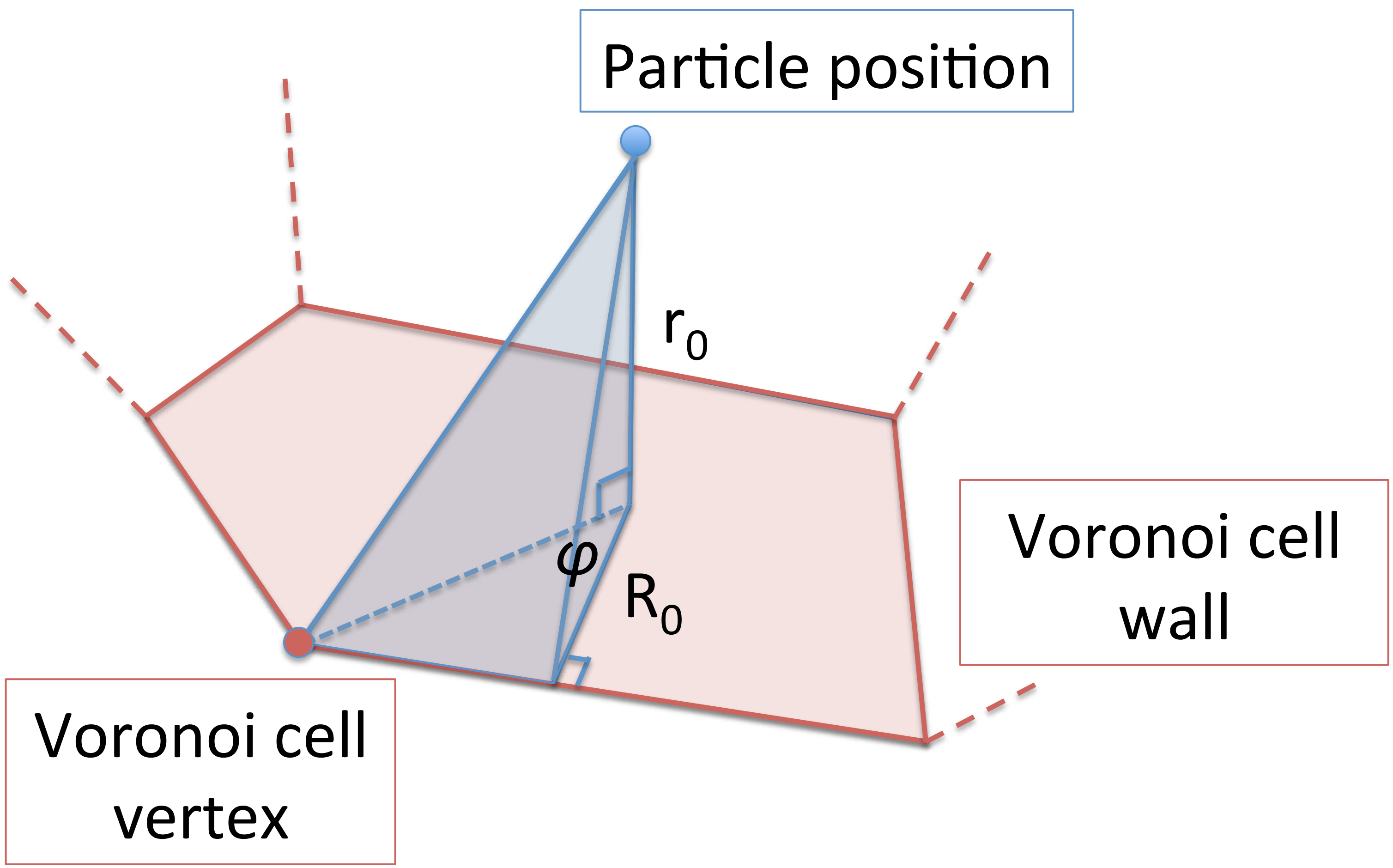
$$I_{-3} = \frac{\alpha(1+\alpha^2)}{4}\left(\frac{2u}{1-u^2}+\log(1+u)-\log(1-u)\right)+\frac{\alpha}{2}\left(\log(1+u)-\log(1-u)\right)+\tan^{-1}\left(\frac{u}{\alpha}\right)+C$$

$$I_{-5} = \frac{\alpha(1+\alpha^2)^2}{16}\left(\frac{10u-6u^3}{(1-u^2)^2}+3\left(\log(1+u)-\log(1-u)\right)\right)+\frac{\alpha(1+\alpha^2)}{4}\left(\frac{2u}{1-u^2}+\log(1+u)-\log(1-u)\right)+\frac{\alpha}{2}\left(\log(1+u)-\log(1-u)\right)+\tan^{-1}\left(\frac{u}{\alpha}\right)+C$$

$$\int H_R R\, d\varphi = \frac{r_0^3}{h^3\pi}\begin{cases}\frac{1}{6}I_{-2}-\frac{3}{40}\left(\frac{r_0}{h}\right)^2 I_{-4}-\frac{1}{40}\left(\frac{r_0}{h}\right)^3 I_{-5}+\frac{B_1}{r_0^3}I_0, \mu \ge \frac{r_0}{h} \\ \frac{1}{4}\left(\frac{4}{3}I_{-2}-\left(\frac{r_0}{h}\right)I_{-3}+\frac{3}{10}\left(\frac{r_0}{h}\right)^2 I_{-4}-\frac{1}{30}\left(\frac{r_0}{h}\right)^3 I_{-5}+\frac{1}{15}\left(\frac{r_0}{h}\right)^{-3} I_1\right)+\frac{B_2}{r_0^3}I_0, \frac{r_0}{2h} \le \mu \le \frac{r_0}{h} \\ -\frac{1}{4}\left(\frac{r_0}{h}\right)^{-3} I_1+\frac{B_3}{r_0^3}I_0, \mu \le \frac{r_0}{2h}\end{cases}$$

Graphic Representation of the Solution

Petkova et al. 2018



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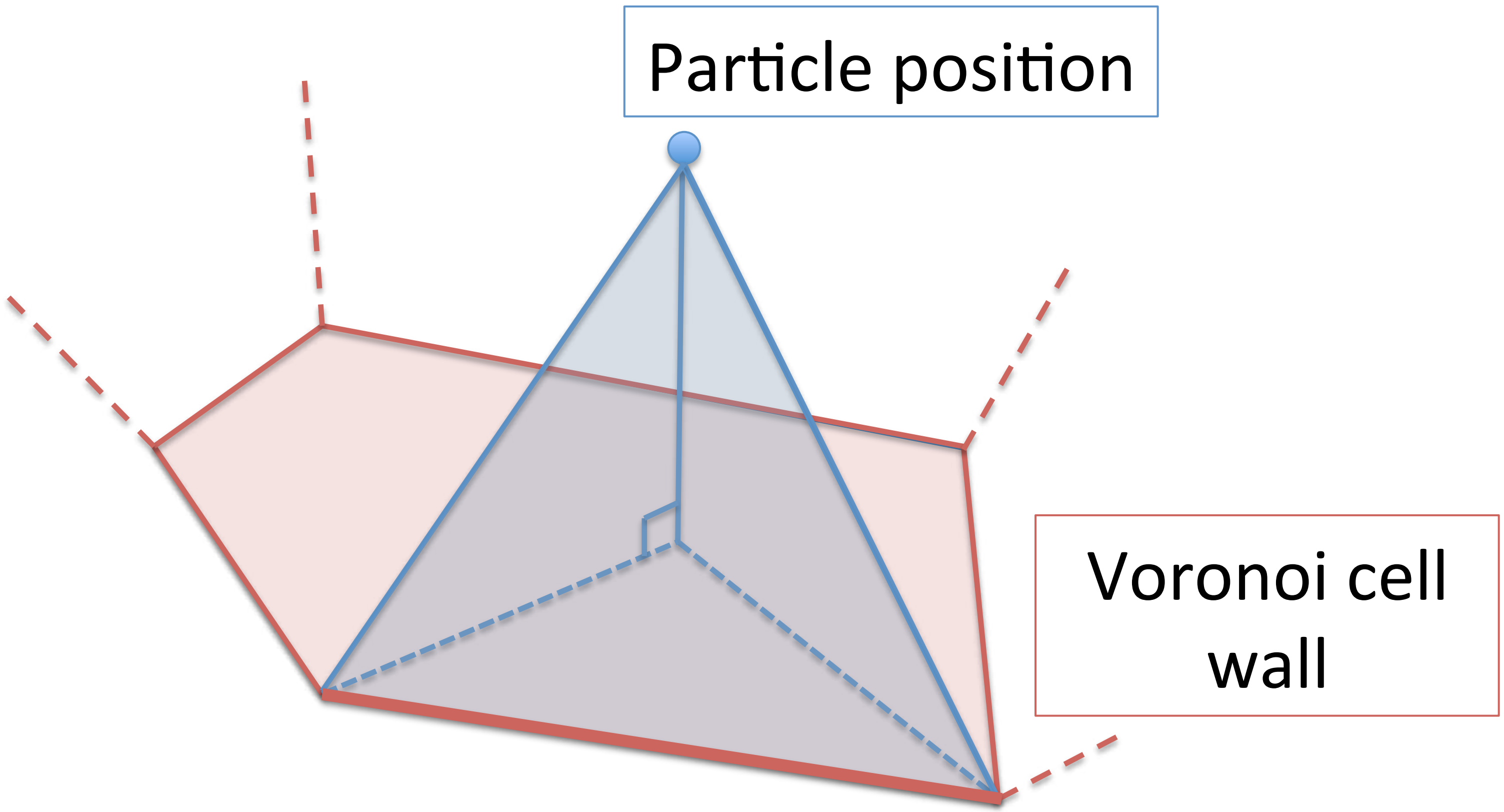

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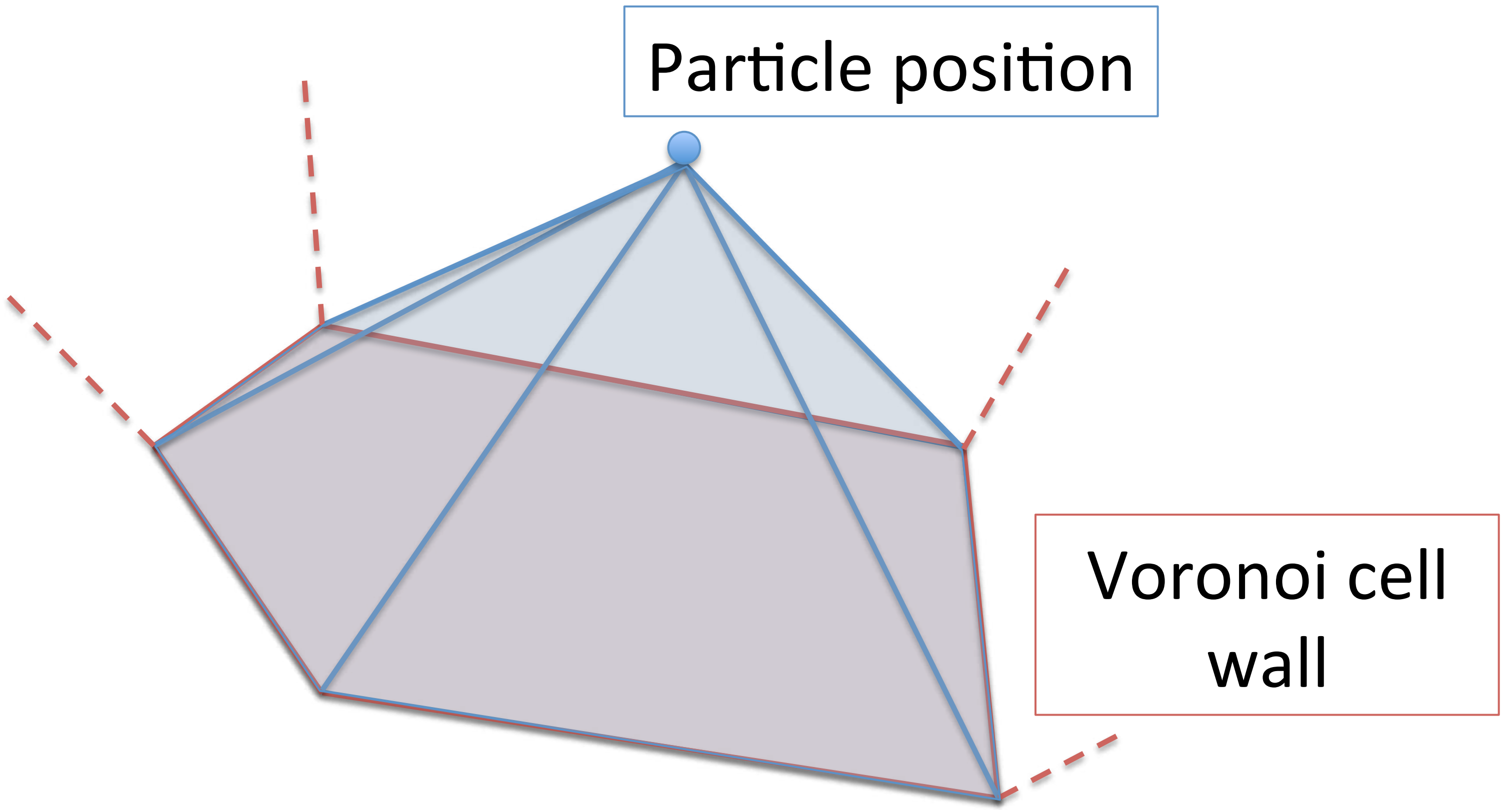
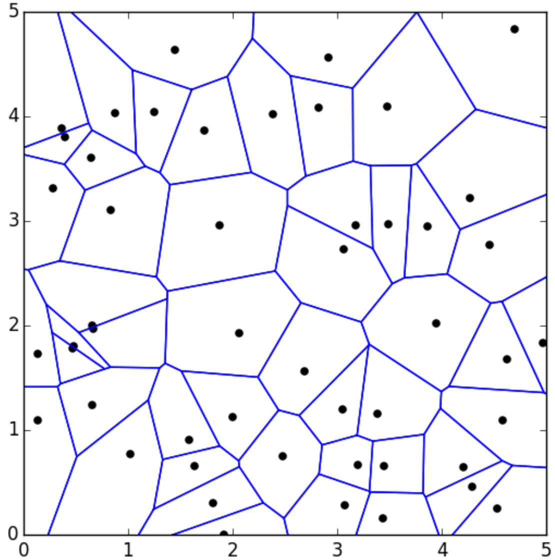
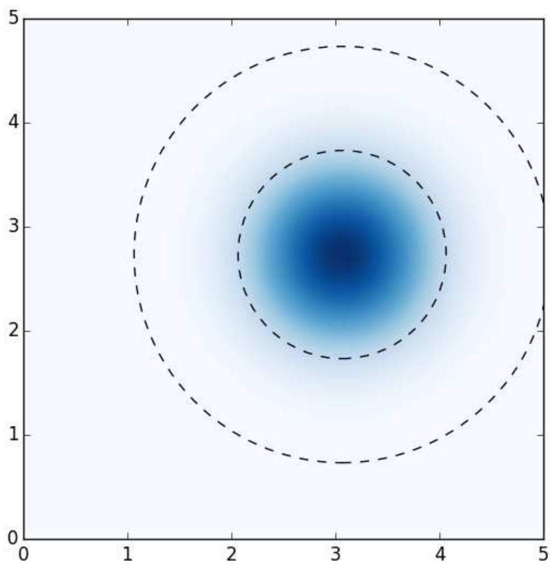
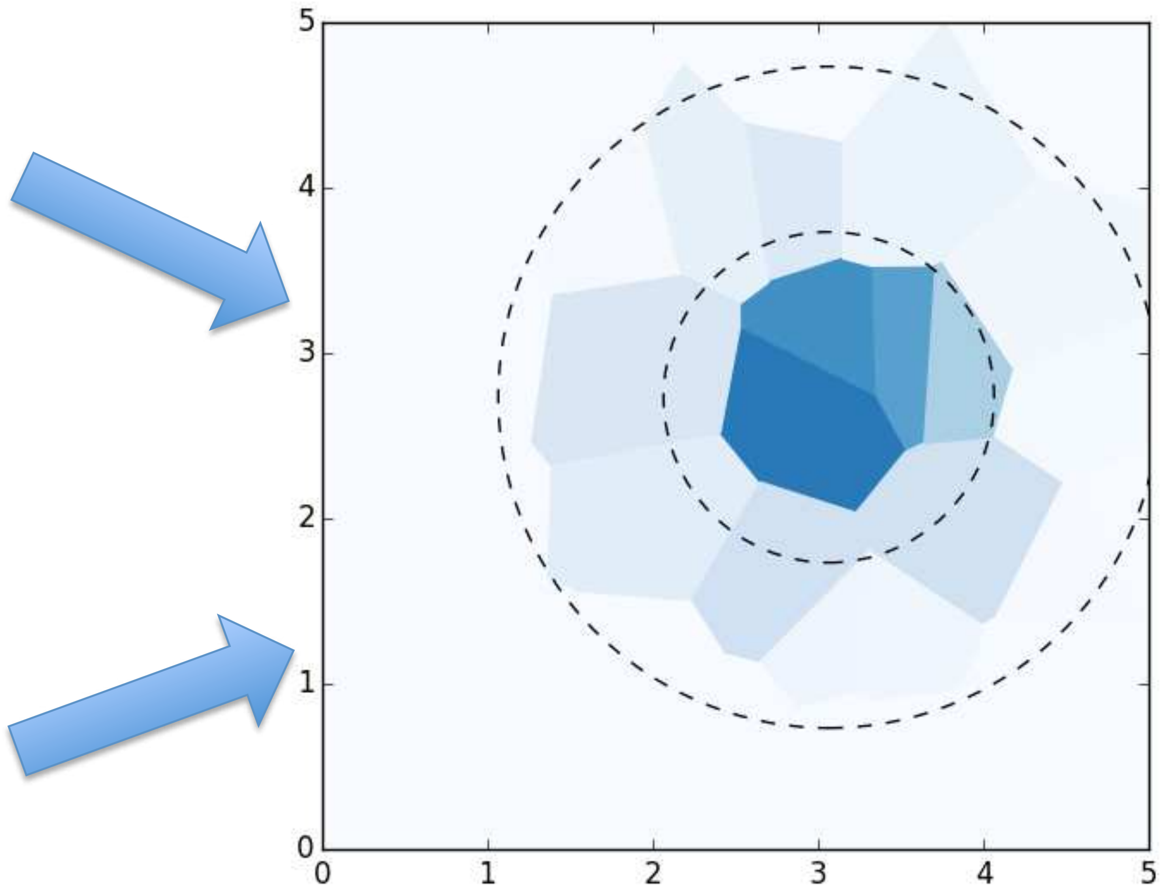


Kernel Integration in 2D



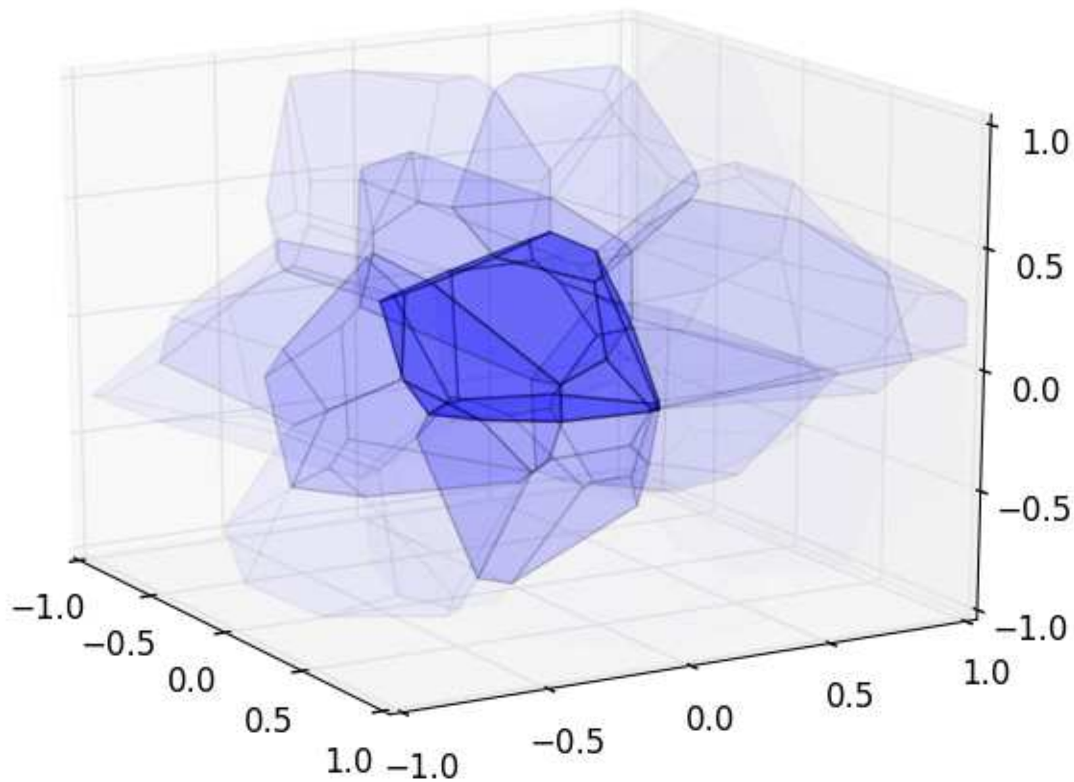


Petkova et al. 2018



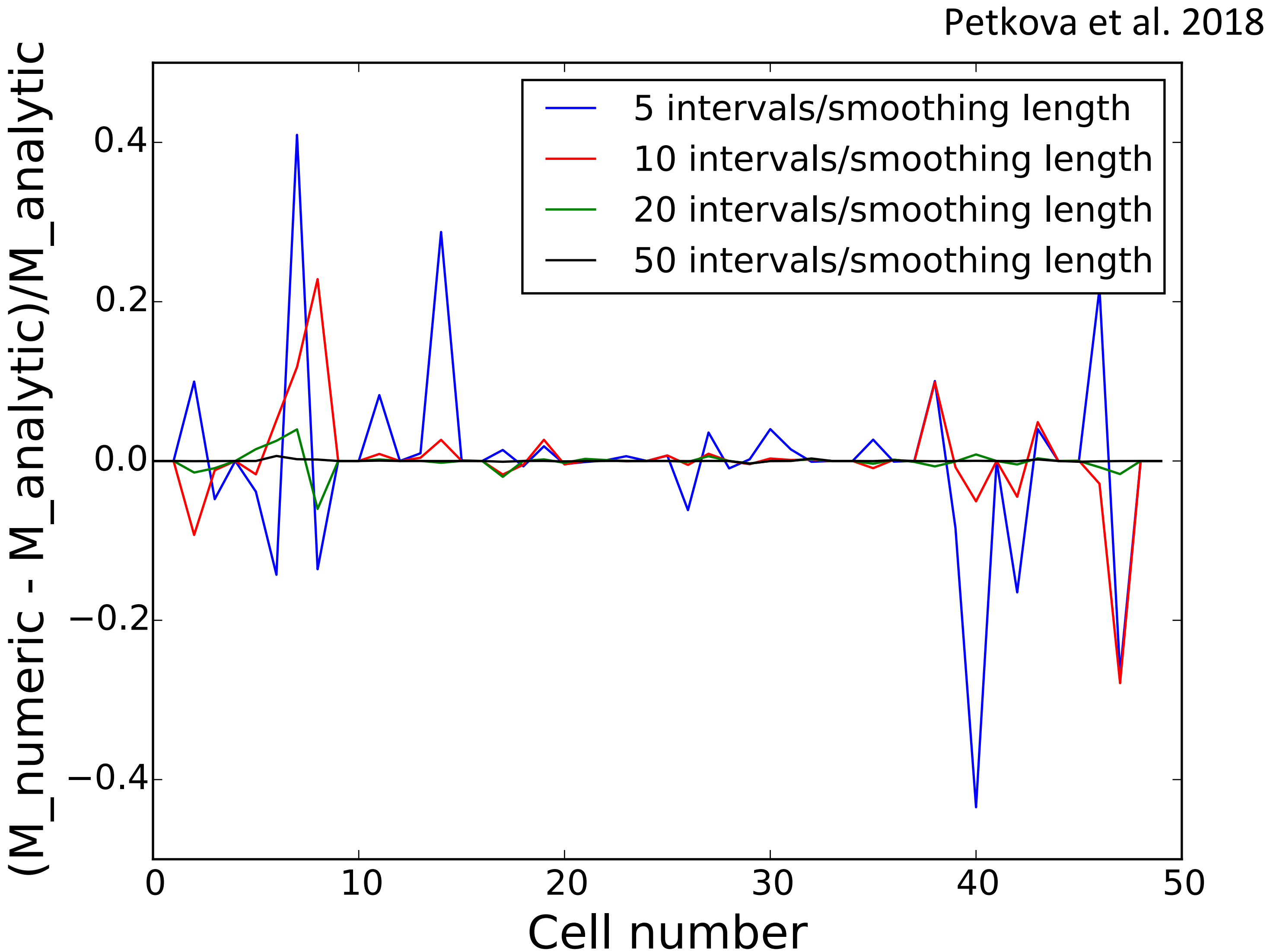
Kernel Integration in 3D

Petkova et al. 2018



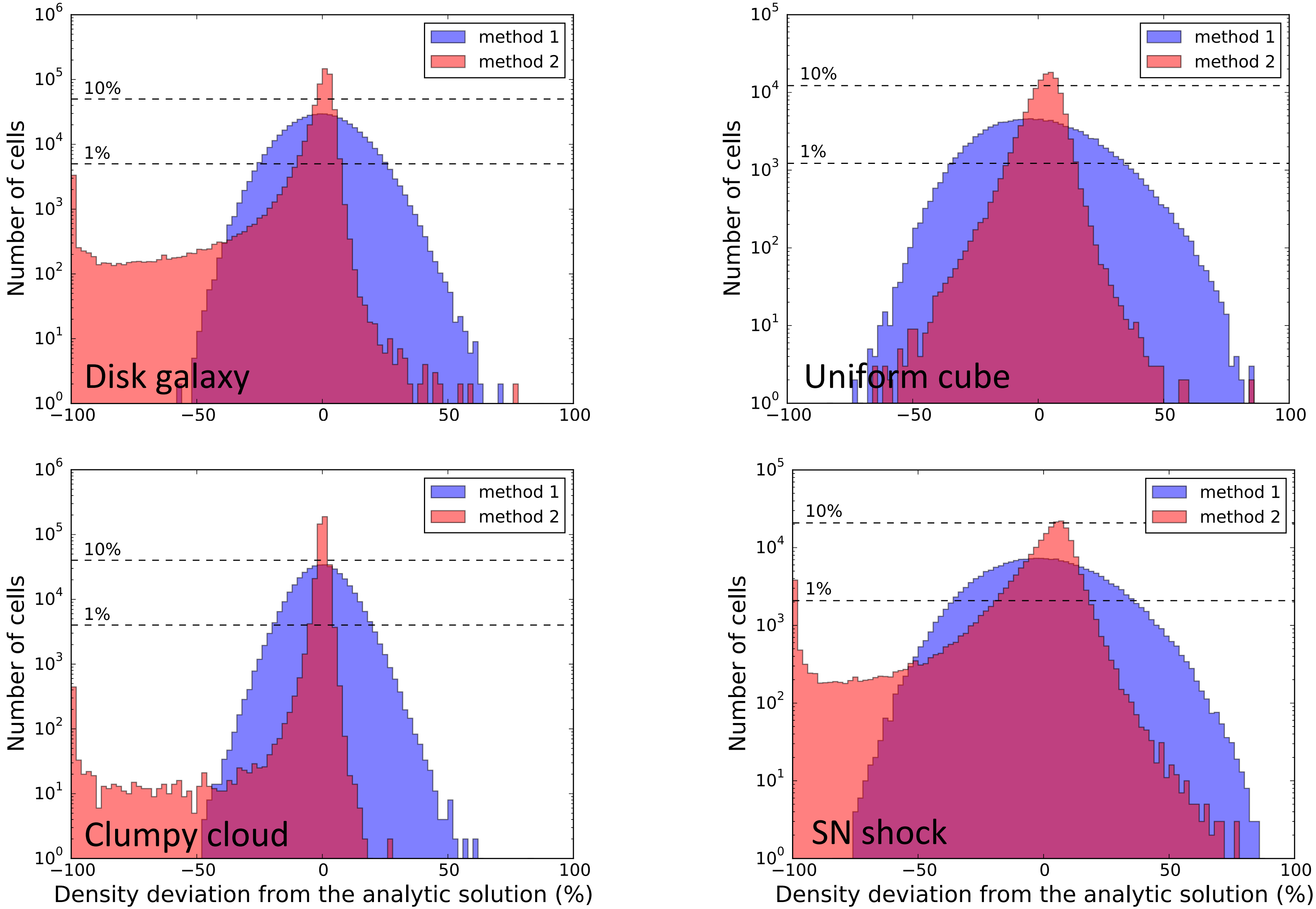
https://github.com/mapetkova/kernel-integration

Numerical Tests



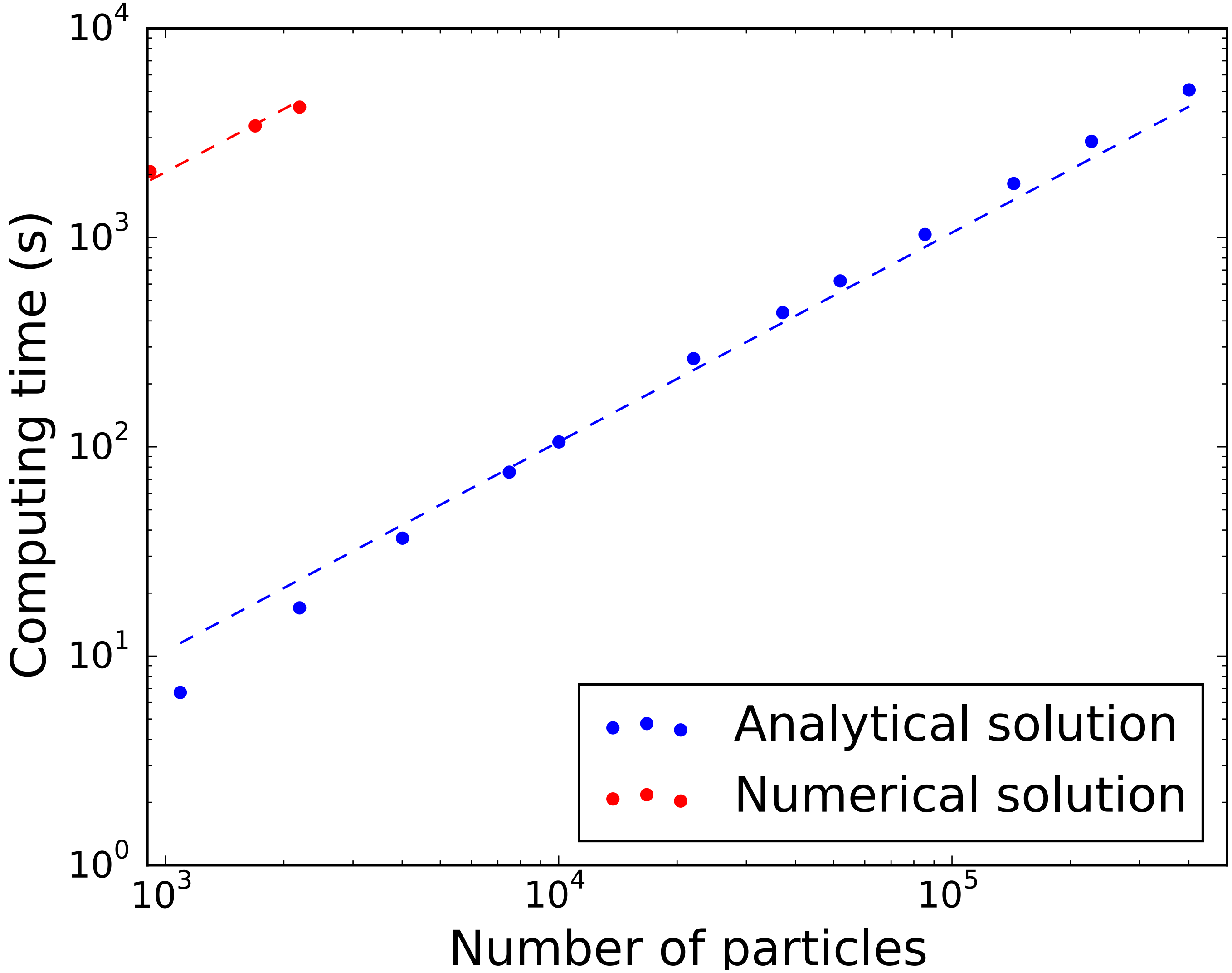
Comparison with the Common Density Calculation Methods

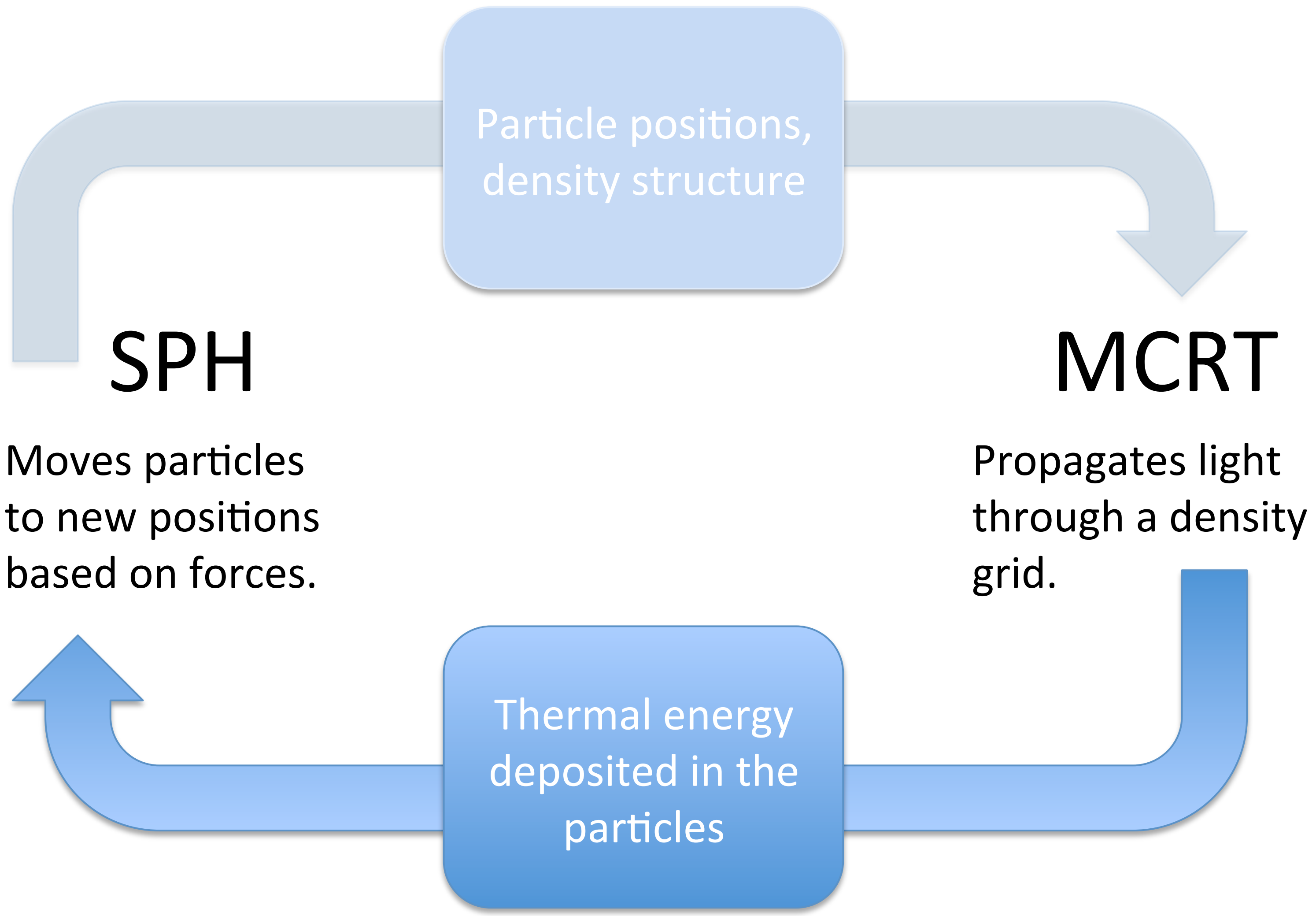
Petkova et al. 2018



Density Calculation Timing Tests

Petkova et al. 2018

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Moves particles to new positions based on forces.
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Live radiation hydrodynamics

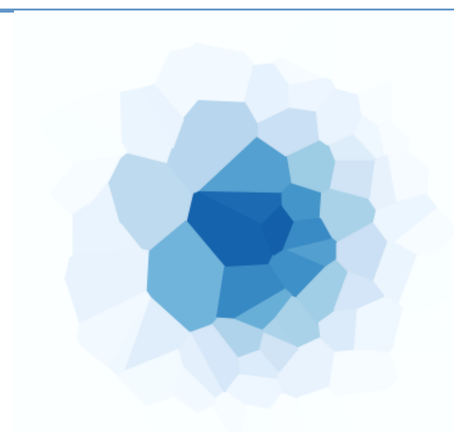
SPH: Phantom (Price et al. 2017)

+

MCRT: CMacIonize (Vandenbroucke & Wood, in press)

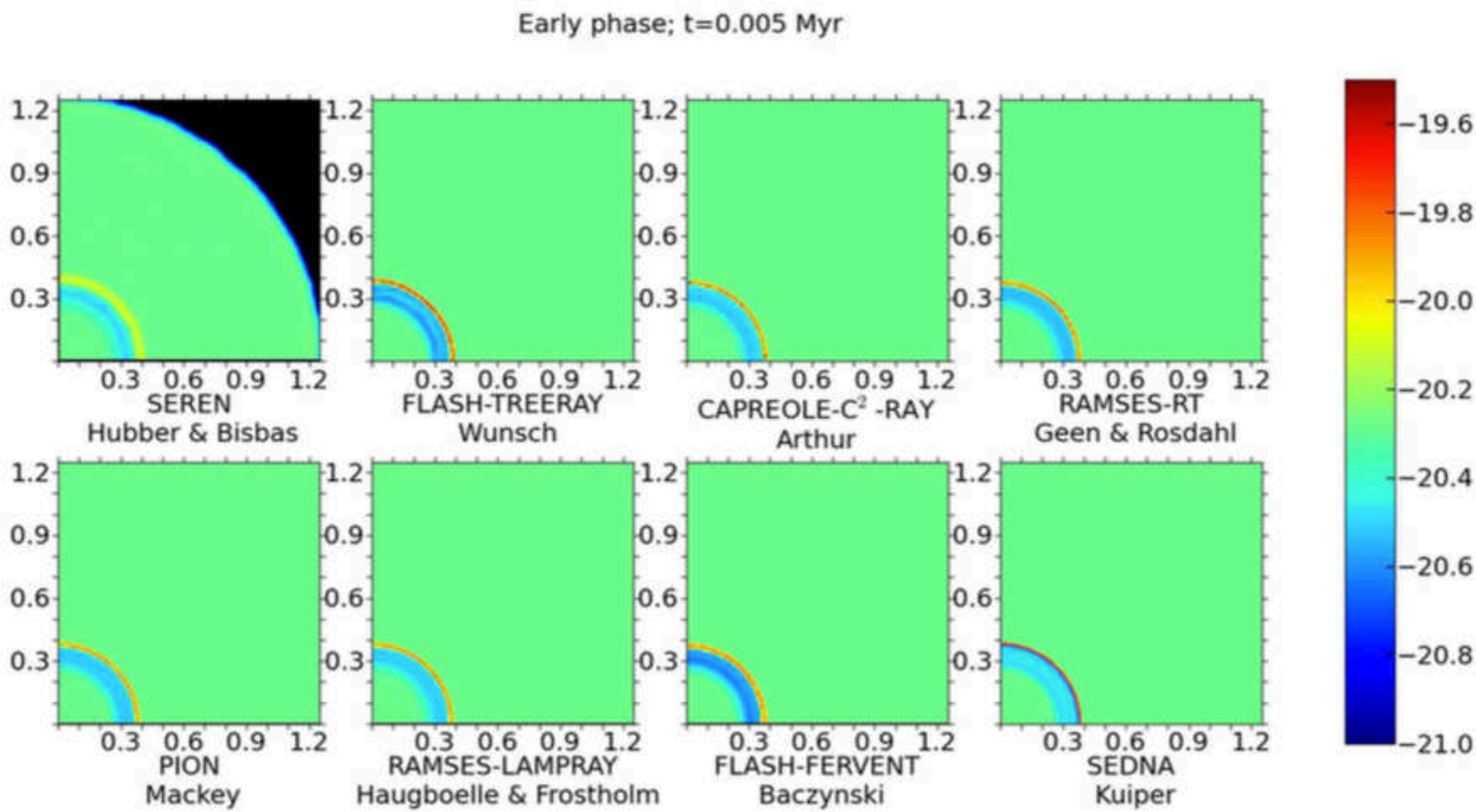
+

Density mapping: Petkova et al. 2018



Live radiation hydrodynamics (test): D-type expansion of an H II region

Bisbas et al. 2015



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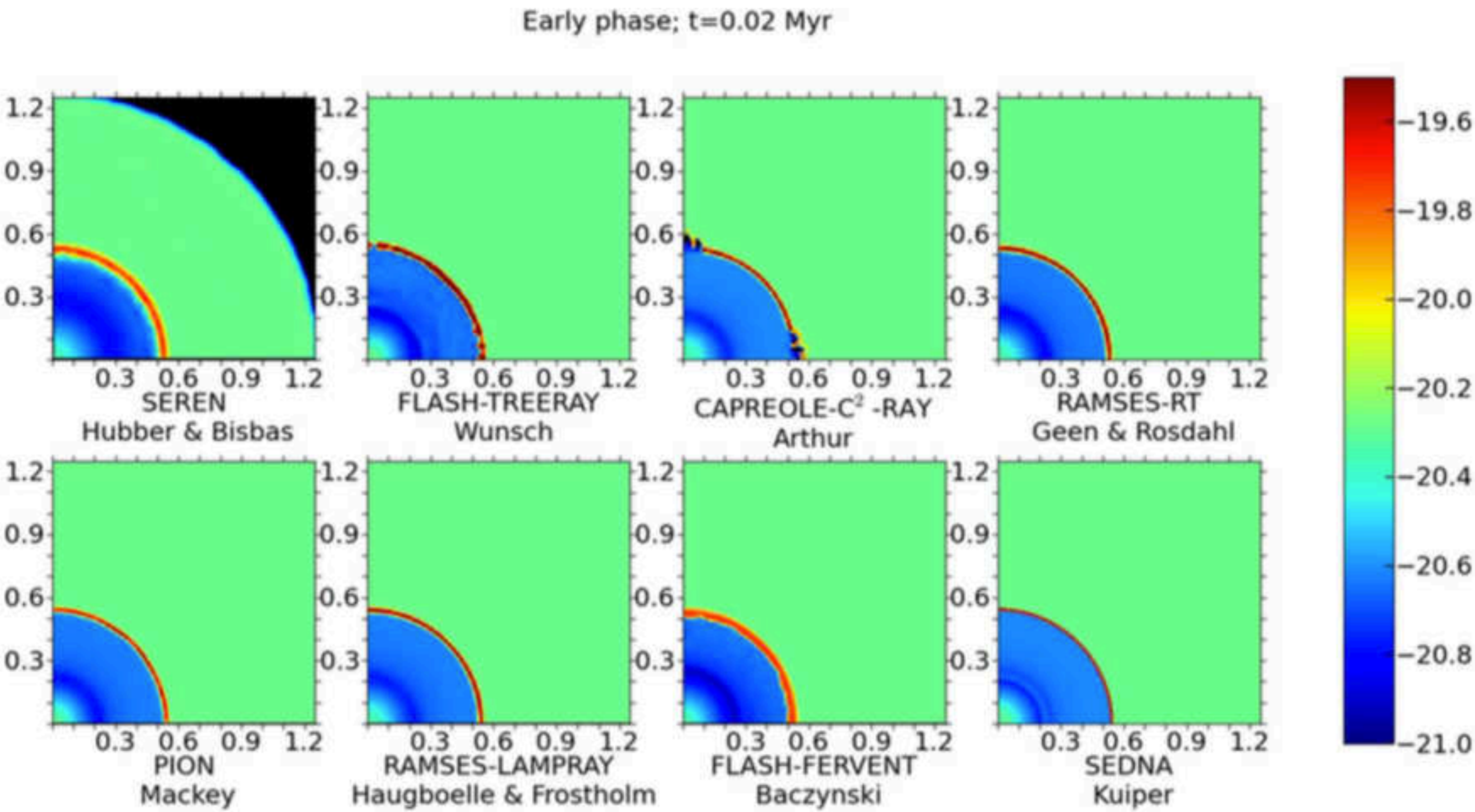

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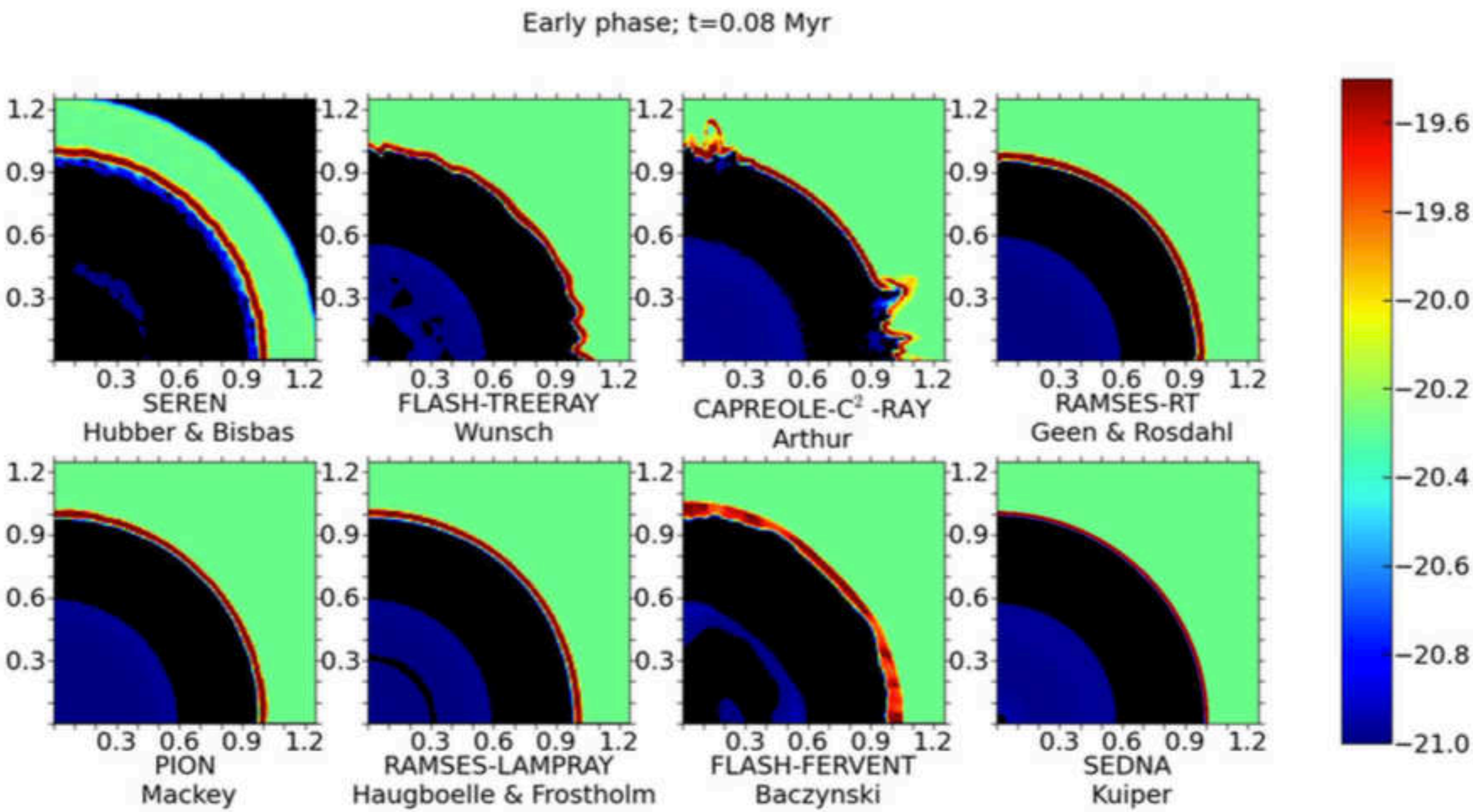

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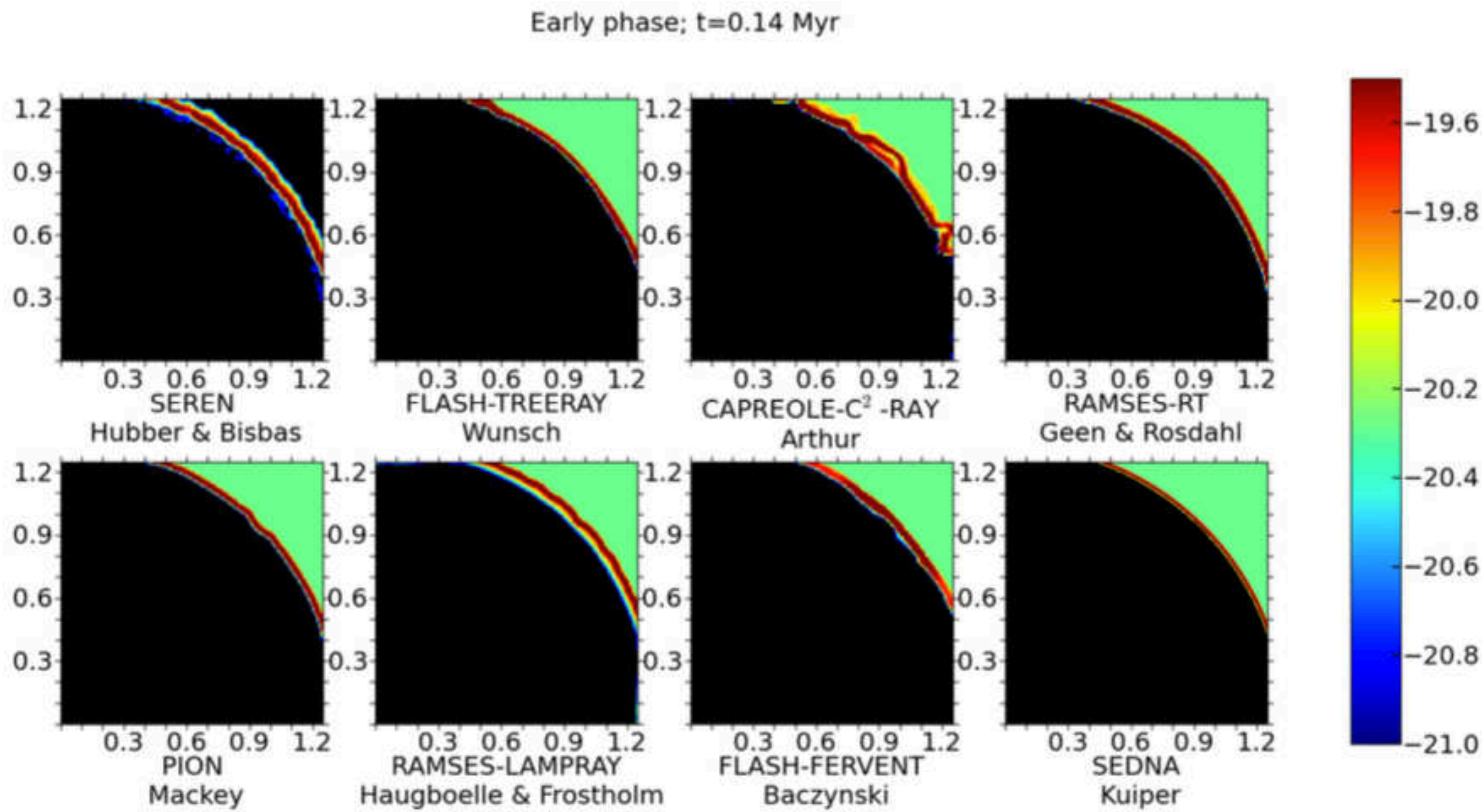
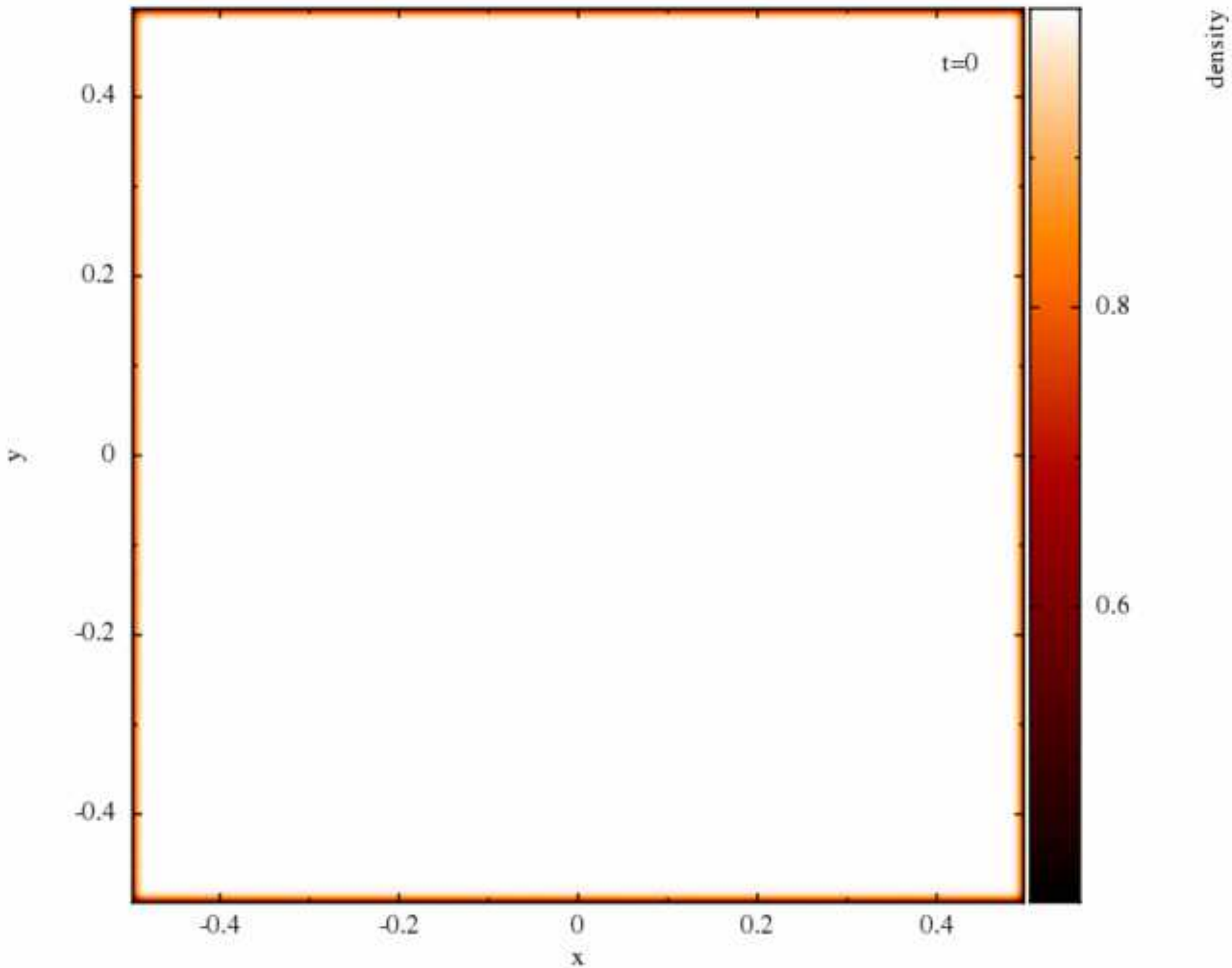


Live radiation hydrodynamics (test)



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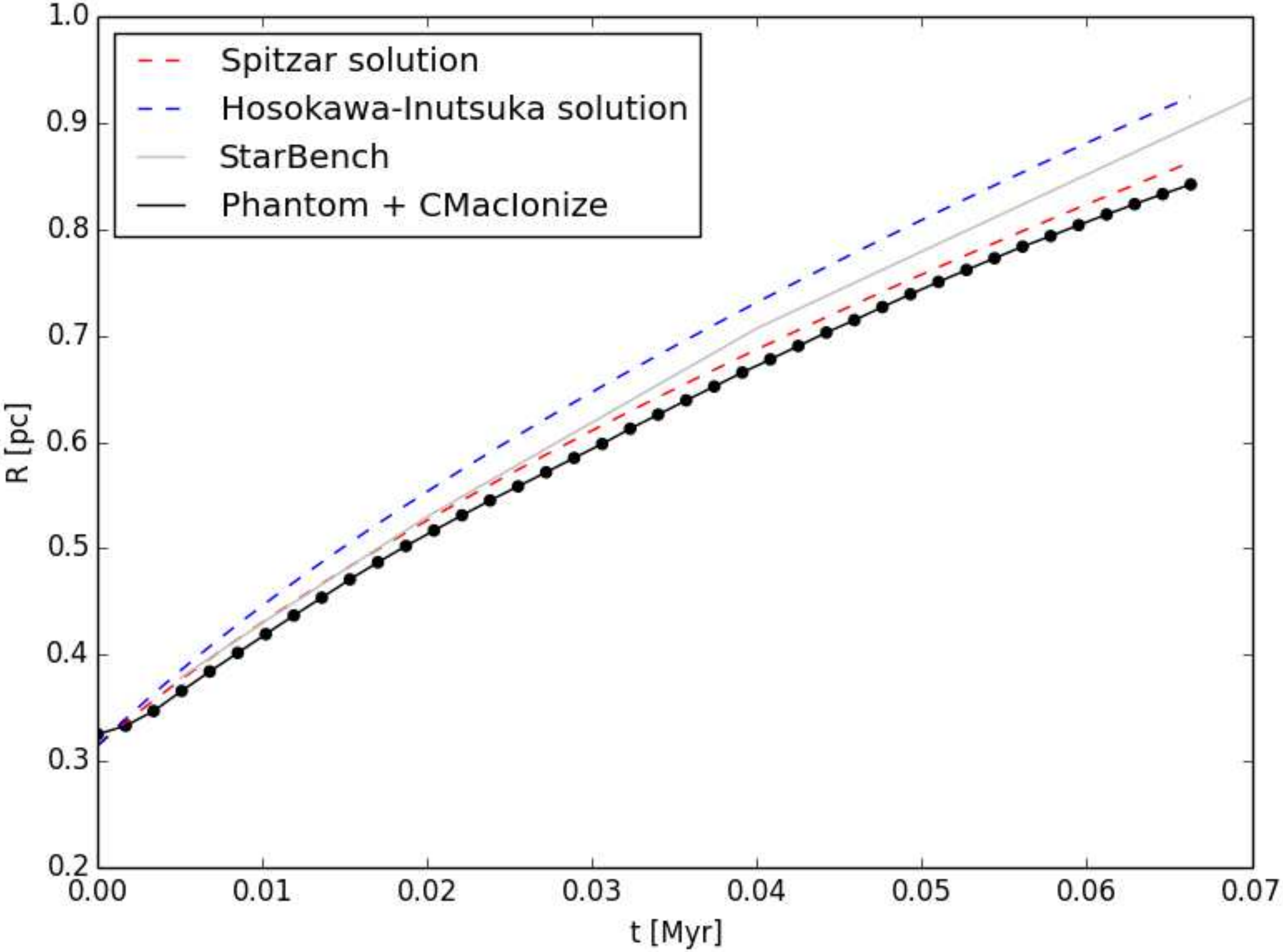
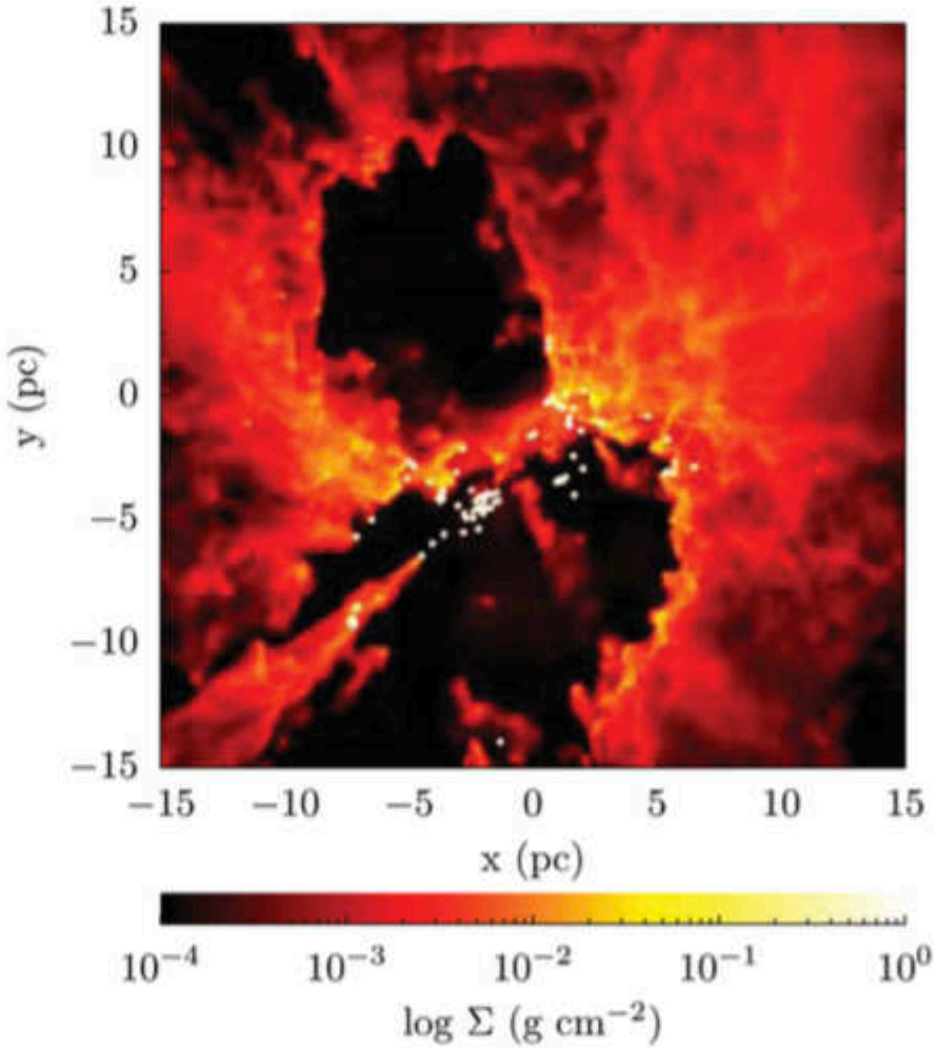
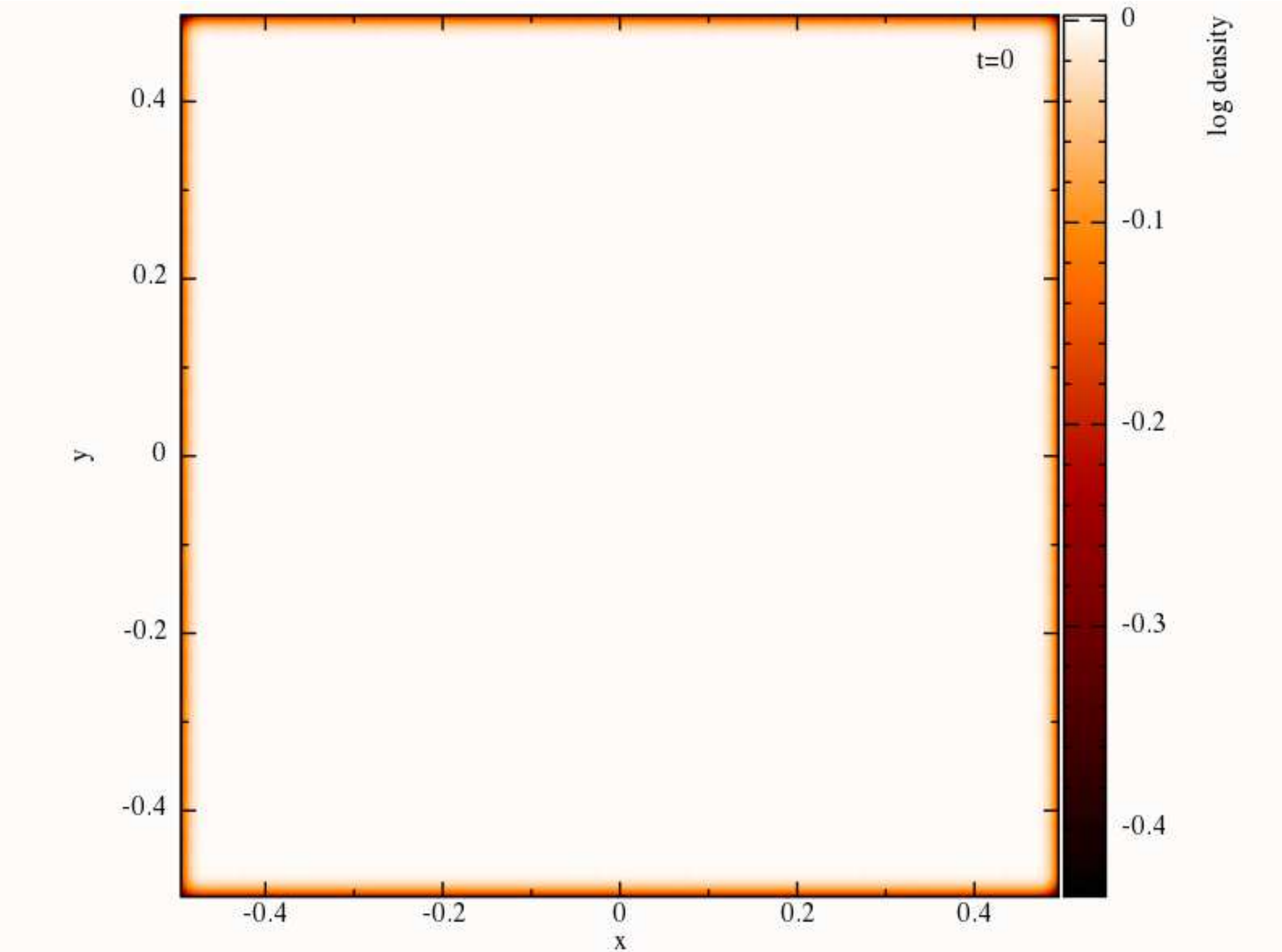


Soon to come…

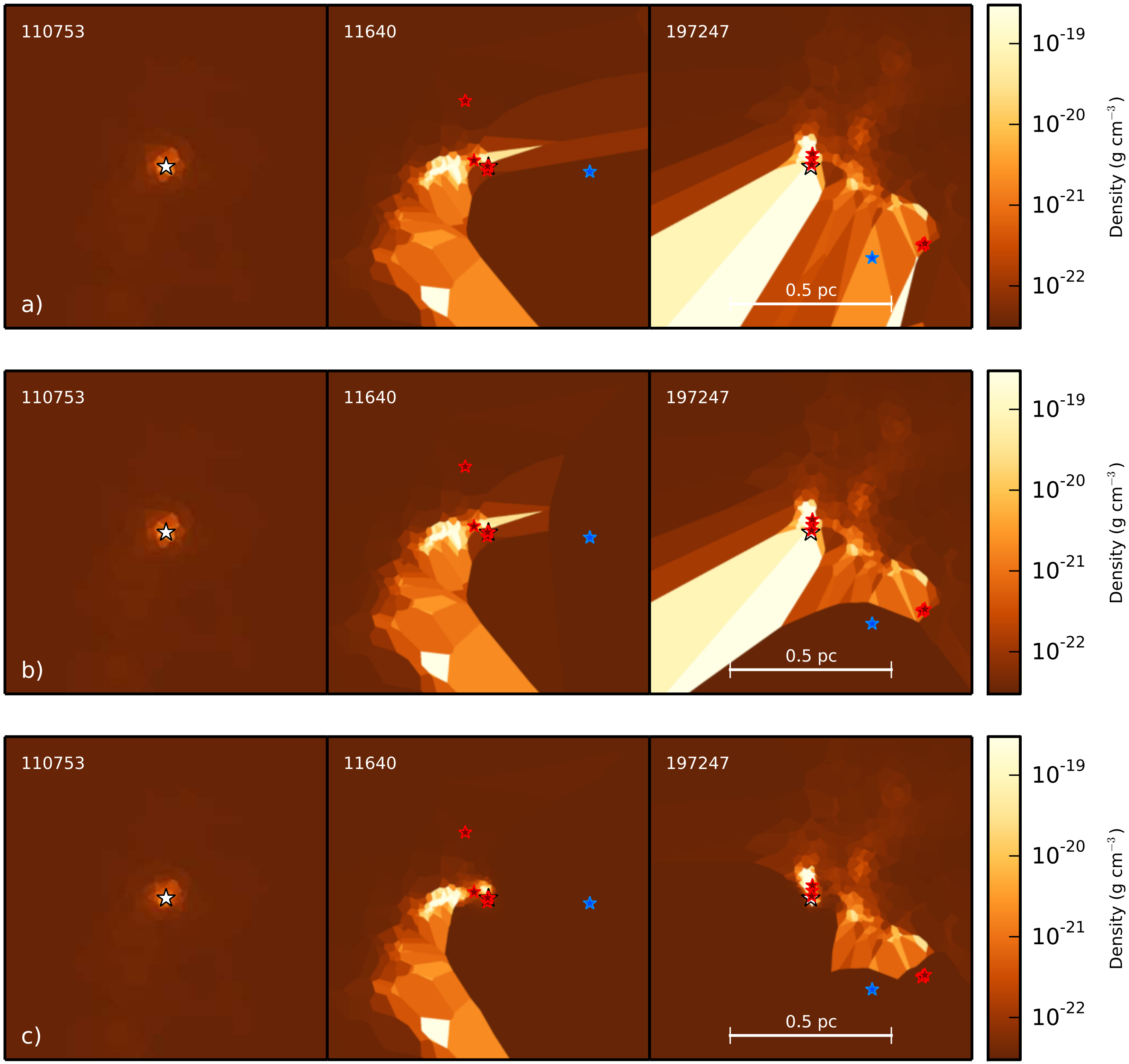


Dale et al. 2012

Multiple sources

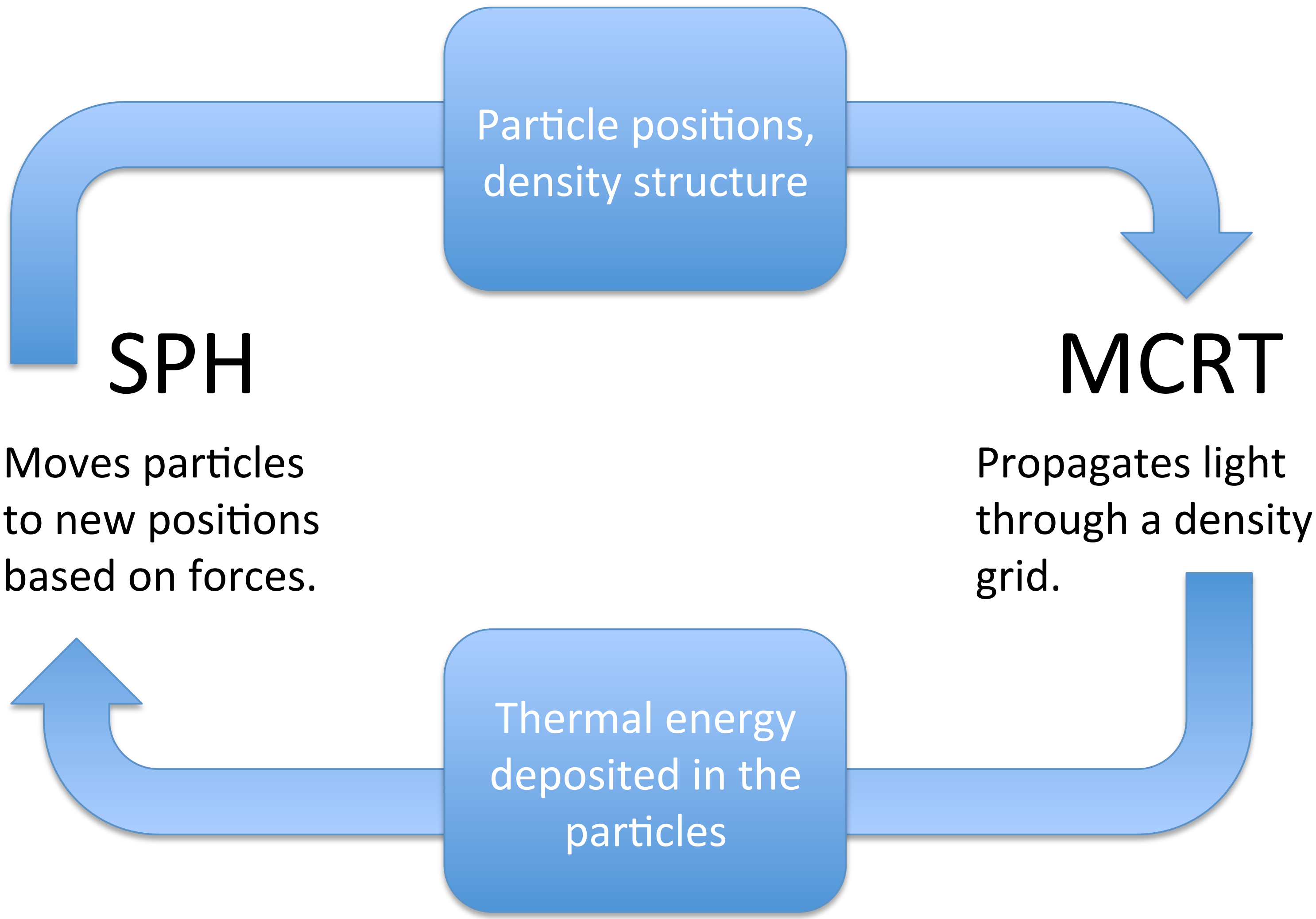


Open Question: Resolution



Koepferl et. al (2016)

Summary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