The Kelvin-Helmholtz Instability in SPH

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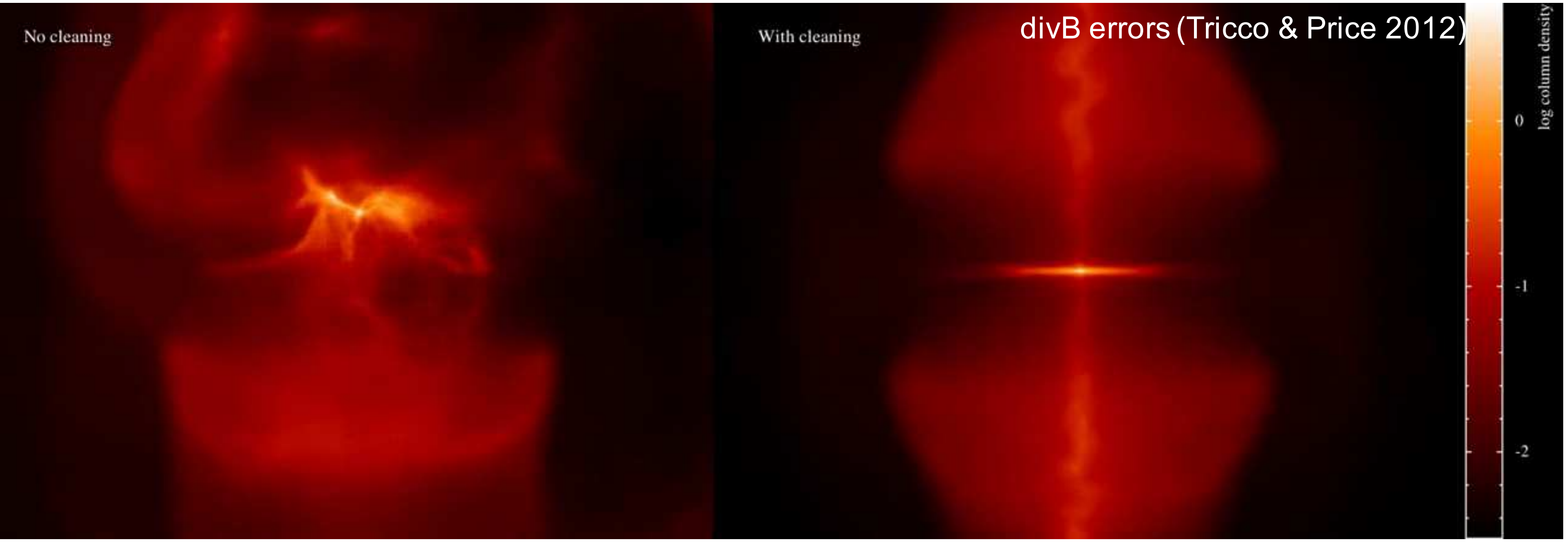
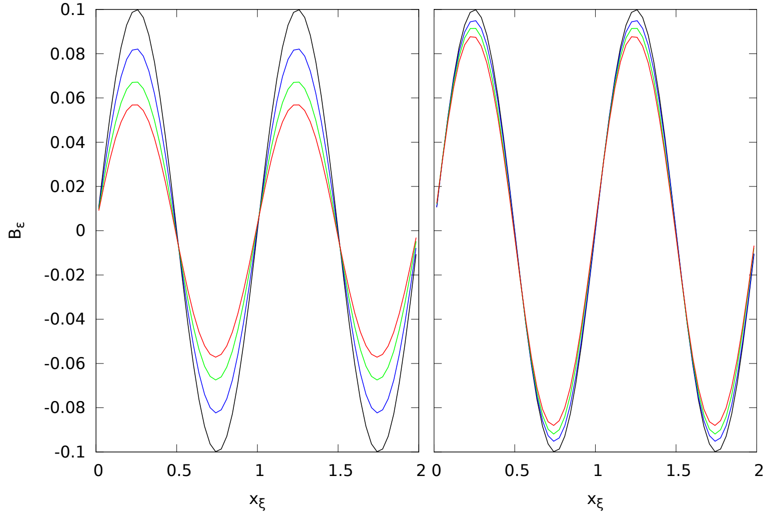
Accuracy and Correctness of SPH and SPMHD

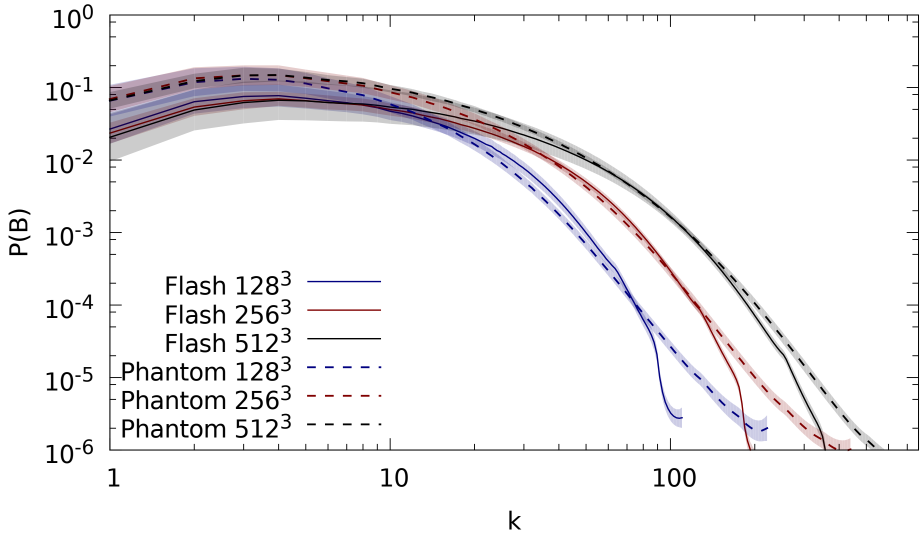


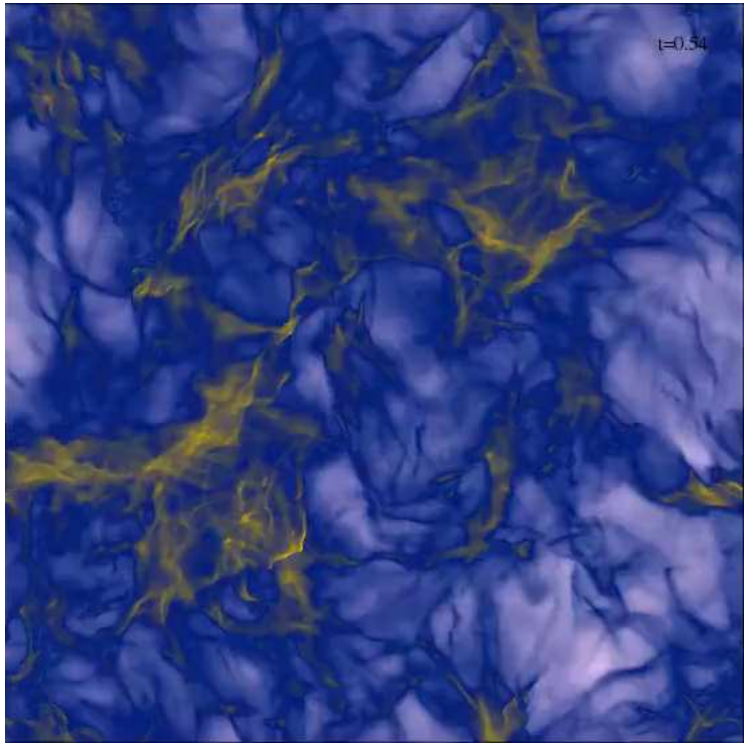
divB errors (Tricco & Price 2012)

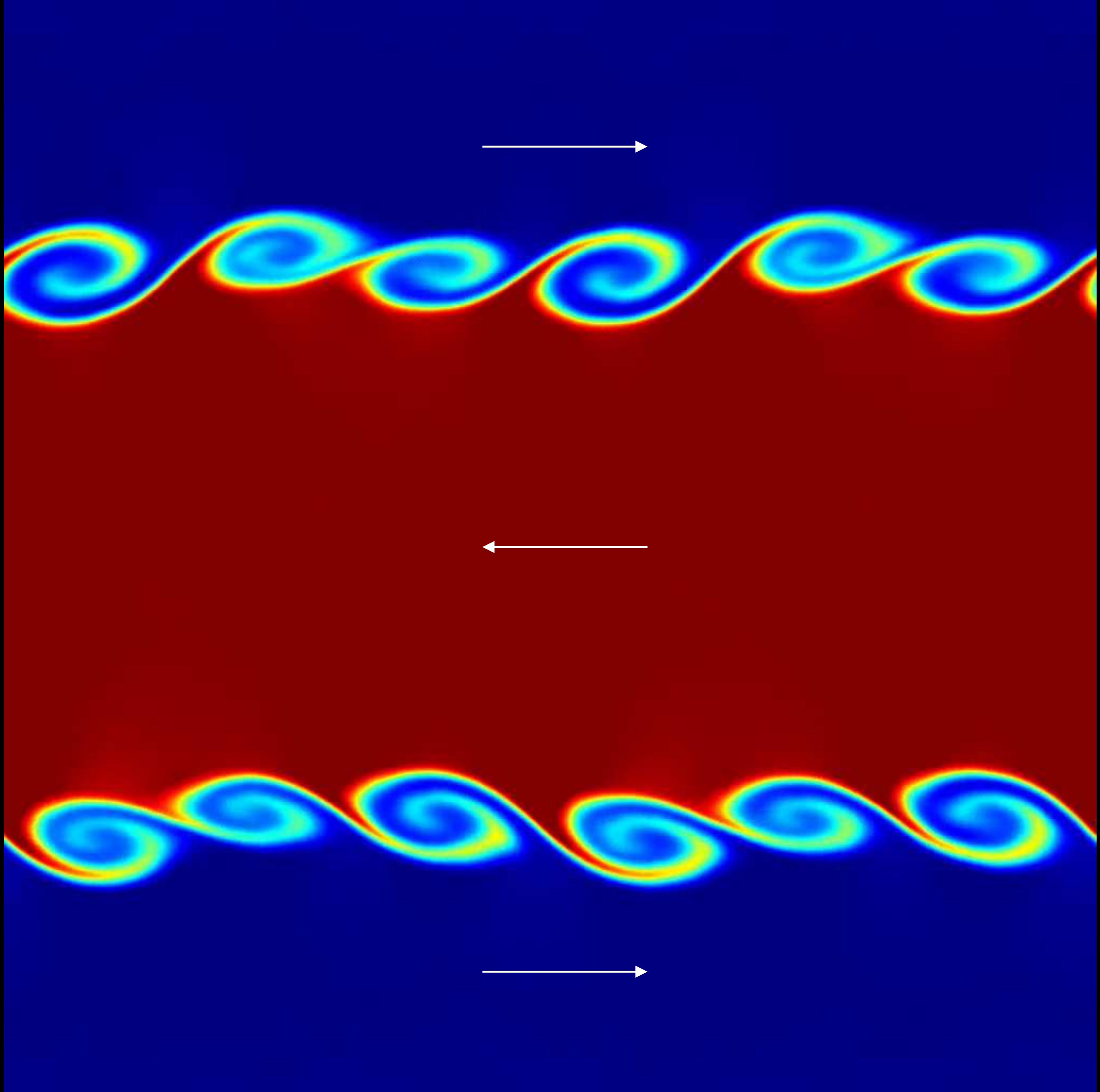
Propagation of magnetic waves (Tricco & Price 2013)



Spectra of magnetic energy in turbulence (Tricco, Price & Federrath 2016)







The KH Instability in SPH

Mon. Not. R. Astron. Soc. **380**, 963–978 (2007) doi:10.1111/j.1365-2966.2007.12183.x

Fundamental differences between SPH and grid methods

Oscar Agertz,[1★] Ben Moore,[1] Joachim Stadel,[1] Doug Potter,[1] Francesco Miniati,[2] Justin Read,[1] Lucio Mayer,[2] Artur Gawryszczak,[3] Andrey Kravtsov,[4] Åke Nordlund,[5] Frazer Pearce,[6] Vicent Quilis,[7] Douglas Rudd,[4] Volker Springel,[8] James Stone,[9] Elizabeth Tasker,[10] Romain Teyssier,[11] James Wadsley[12] and Rolf Walder[13]

between the two main techniques for simulating fluids. **While grid codes are able to resolve and treat dynamical instabilities and mixing, these processes are poorly or not at all resolved by the current SPH techniques.** We show that the reason for this is that SPH, at

Agertz et al (2007)

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Agertz et al (2007)

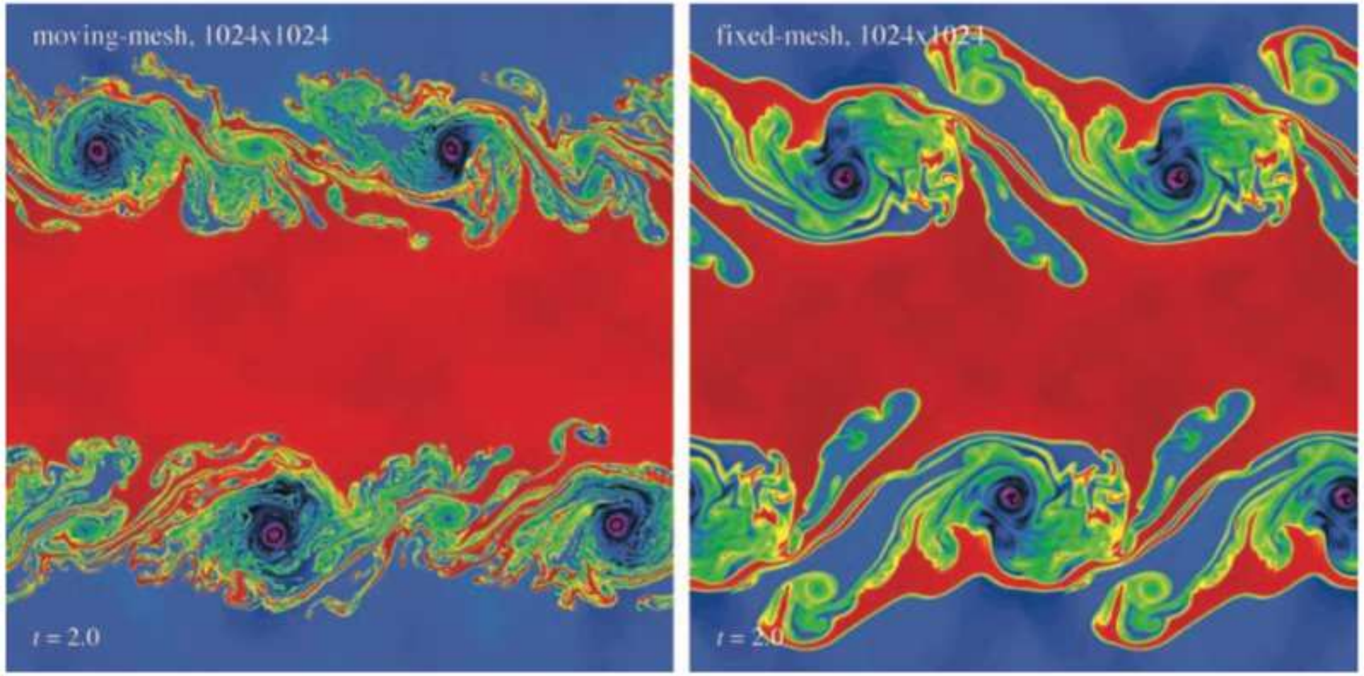
However, it is well known that 'traditional' SPH (TSPH) algorithms have a number of problems. They suppress certain fluid-mixing instabilities (e.g. Kelvin–Helmholtz, KH, instabilities;

Hopkins (2015)

be incorrect. As we have discussed, unsolved conceptual problems with the accuracy of SPH and its convergence rate remain even in the most recent incarnations of the proposed improved versions of SPH. We therefore think that more accurate numerical techniques, such as our moving-mesh approach, should clearly be preferred

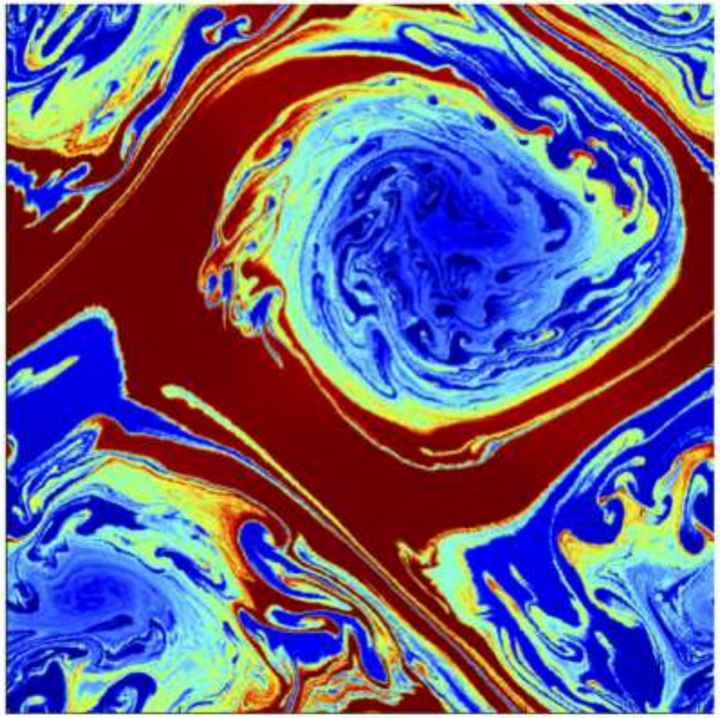
Hayward et al (2014)

All the Mixing (Moving mesh / meshless finite volume)



a resolution of 1024^2. The moving-mesh preserves much more fine detail in the flow. This is because contact discontinuities between different phases can be advected with large speeds without being necessarily mixed. We think this is a very interesting difference, which makes the moving-mesh code particularly attractive for the study of multi-phase media.

Springel (2010)



complicated contact discontinuities. In the late non-linear phases, it is truly remarkable how much fine-structure is captured by the MFV runs, given the relatively low resolution used. In these stages,

(1024^2), showing the same character and the exceptional degree of resolved sub-structure and small-scale modes. Very similar results

Hopkins (2015)

Mo’ mixing, Mo’ problems

- Robertson et al (2010), McNally et al (2012), Lecoanet et al (2016) introduce KH tests with well-posed initial conditions that demonstrate convergence

> (e.g. Springel 2010; Hopkins 2015). Presumedly, more small-scale structure implies less numerical dissipation, and therefore greater accuracy. We find in the current paper that this intuition can, in some cases, lead to false conclusions. Mocz et al. (2015) show

Lecoanet et al (2016)

Mo’ mixing, Mo’ problems

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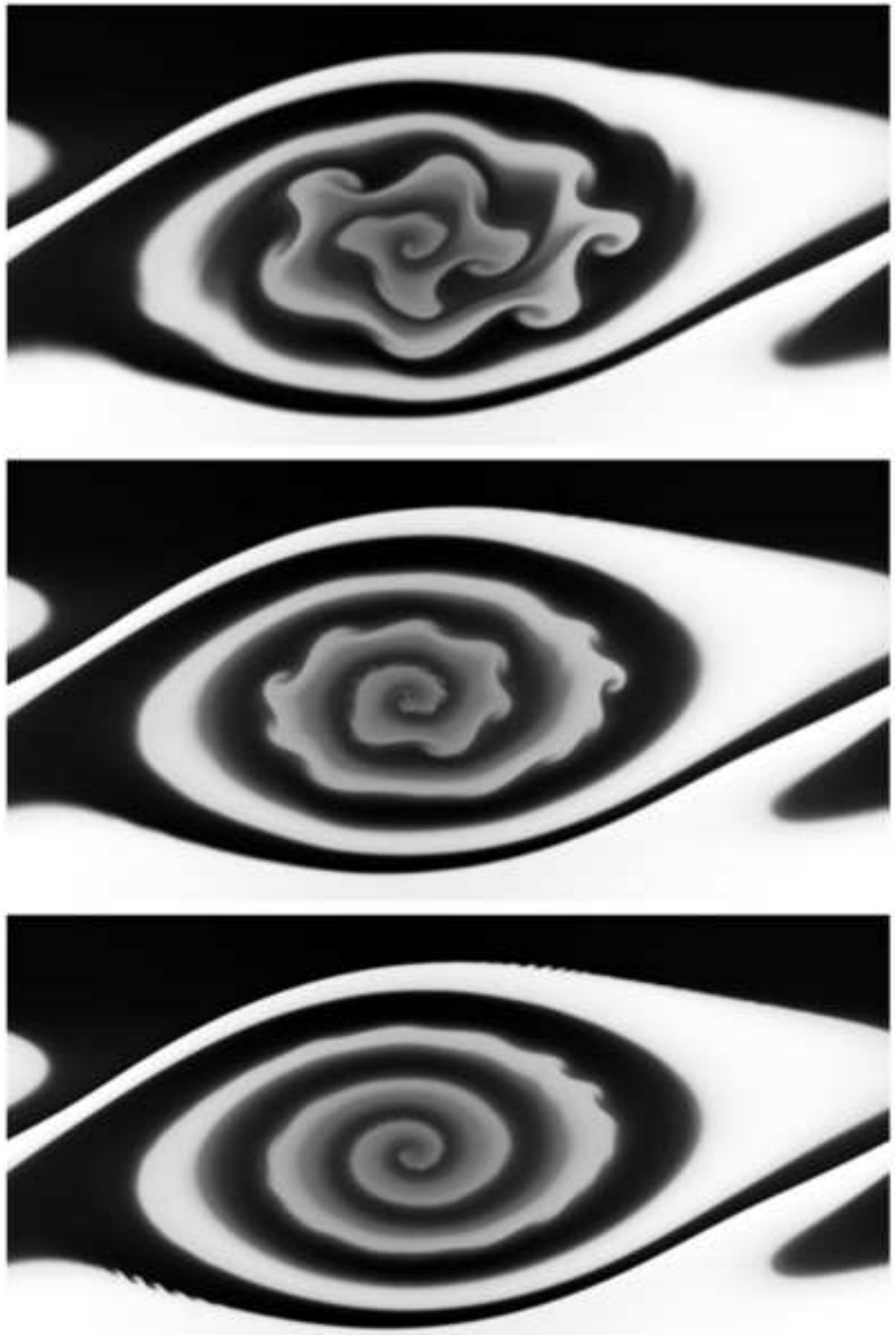
Not all new instabilities seen as resolution is increased when solving the discretized Euler equations are physically real. New numerical instabilities can reveal themselves as resolution is increased, as the flow can enter into new regimes where it is more sensitive to the inevitable numerical noise in a method.

the exact details of the setup used. However, we can show that for our problem that secondary instabilities that do develop are of a purely numerical origin. This strongly suggests that the secondary billows seen in Springel (2011) are a numerical artifact, so the observation that a fixed grid codes does not develop these on the same problem does not imply that the fixed grid code is too diffusive to support the modes.

McNally et al (2012)



The KH Tests of Lecoanet et al (2016)

- Two-dimensional tests with well-posed initial conditions
- Introduce a scalar "colour" field to measure degree of mixing
- Include physical dissipation, that is Navier-Stokes viscosity and thermal conductivity (also colour diffusion!) – dissipation is numerically independent!
- Lecoanet et al (2016) show converged solutions between grid (Athena) and spectral methods (Dedalus) in the non-linear regime

Initial Conditions

$$\rho = 1 + \frac{\Delta\rho}{\rho_0}\frac{1}{2}\left[\tanh\left(\frac{z-z_1}{a}\right) - \tanh\left(\frac{z-z_2}{a}\right)\right],$$

$$u_x = u_{\text{flow}}\left[\tanh\left(\frac{z-z_1}{a}\right) - \tanh\left(\frac{z-z_2}{a}\right) - 1\right],$$

$$u_z = A\sin(2\pi x)\left[\exp\left(-\frac{(z-z_1)^2}{\sigma^2}\right) + \exp\left(-\frac{(z-z_2)^2}{\sigma^2}\right)\right],$$

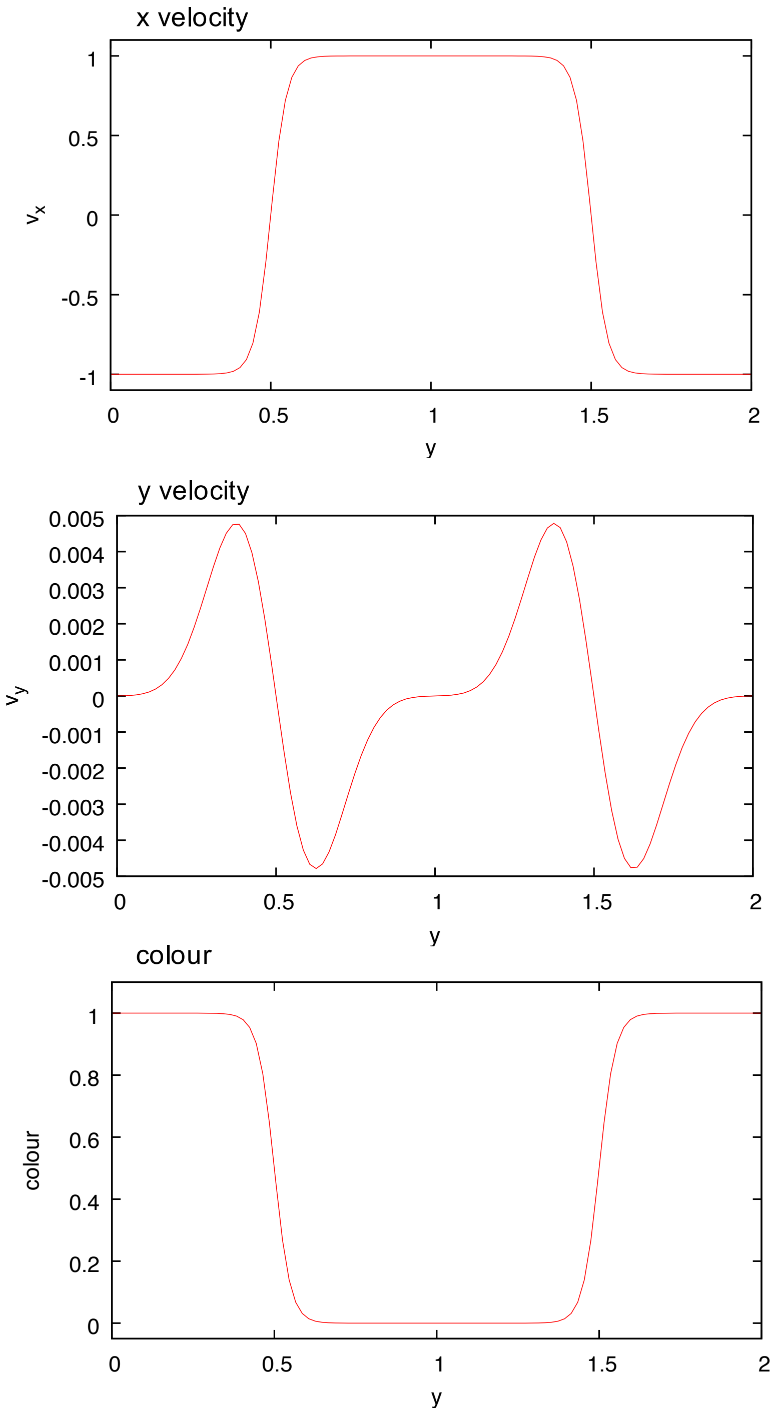
$$P = P_0,$$

$$c = \frac{1}{2}\left[\tanh\left(\frac{z-z_2}{a}\right) - \tanh\left(\frac{z-z_1}{a}\right) + 2\right],$$

$a = 0.05$ $u_{flow} = 1$ $z_1 = 0.5$

$\sigma = 0.2$ $P_0 = 10$ $z_2 = 1.5$

$A = 0.01$



Initial Conditions

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I am using $\Delta\rho/\rho = 0$ (uniform density)

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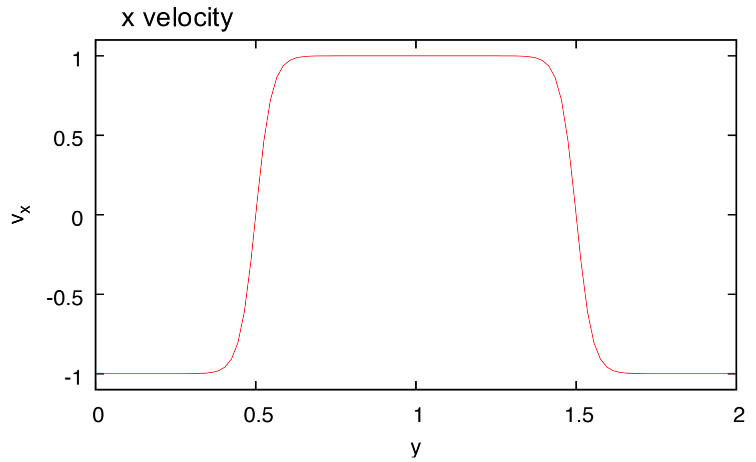
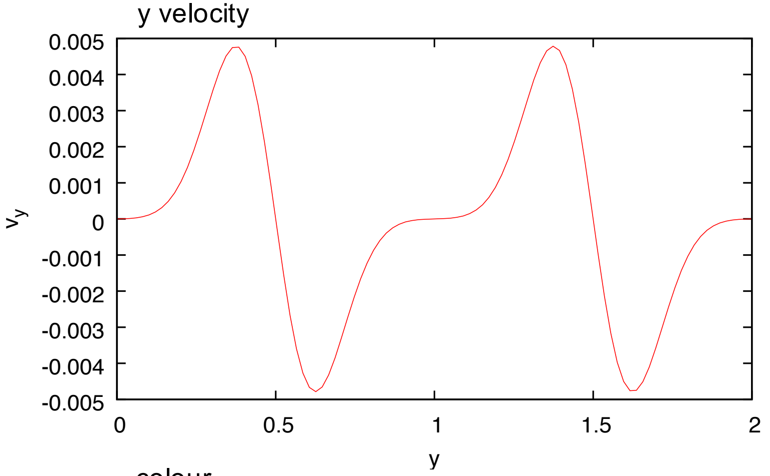
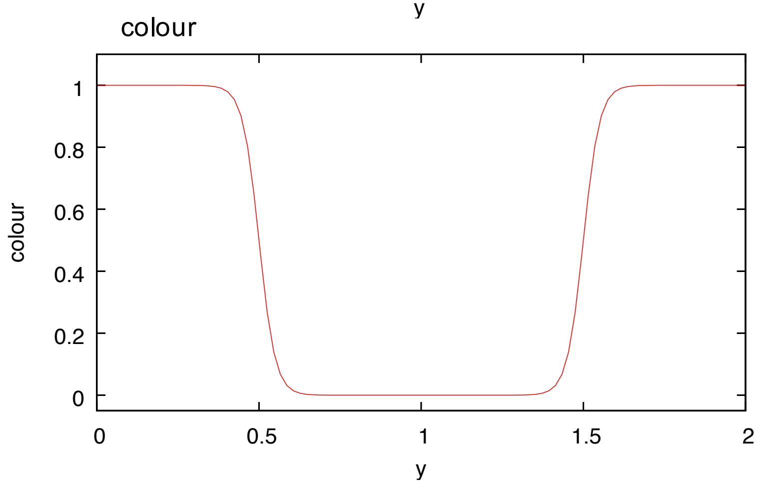
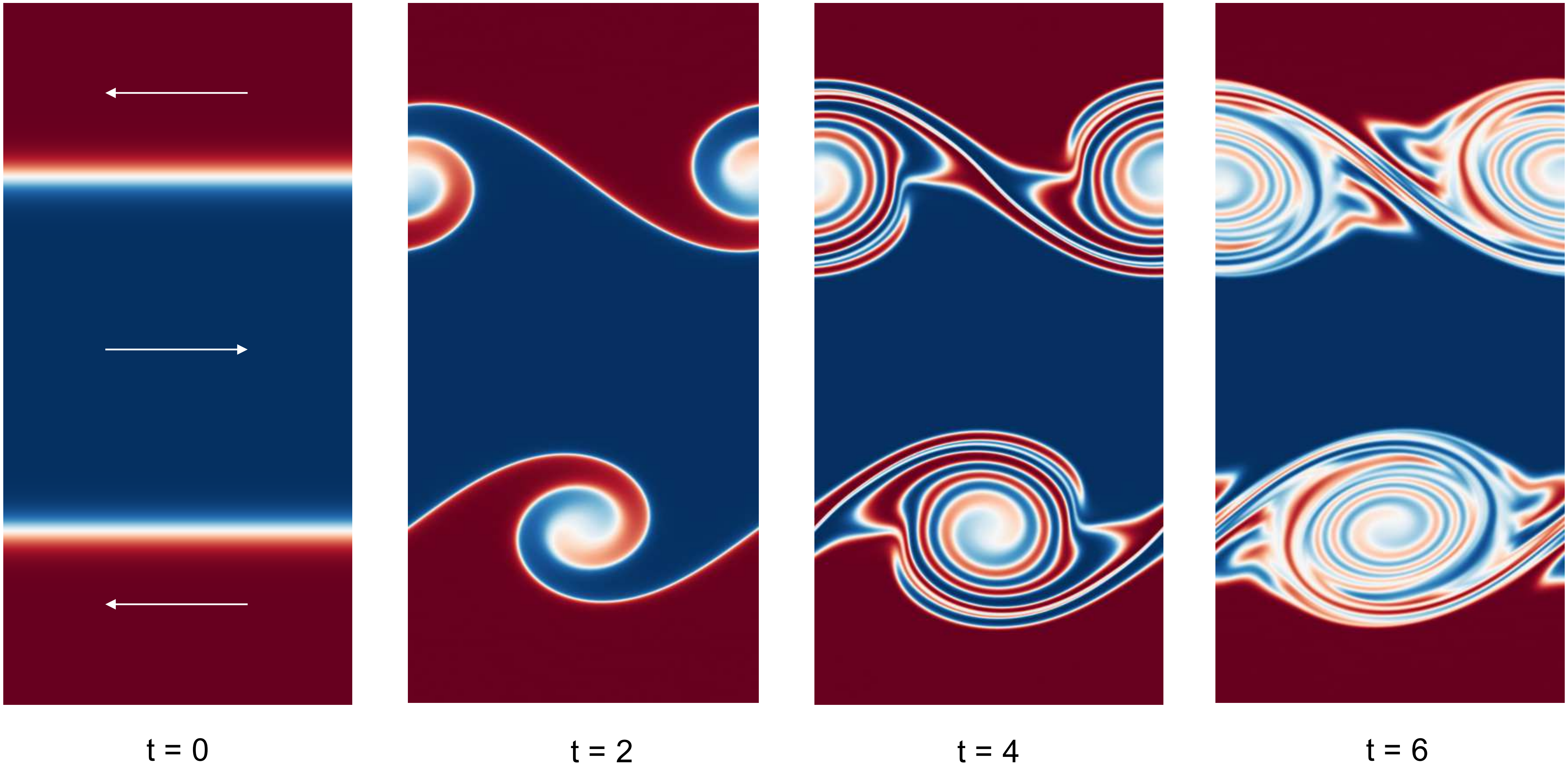


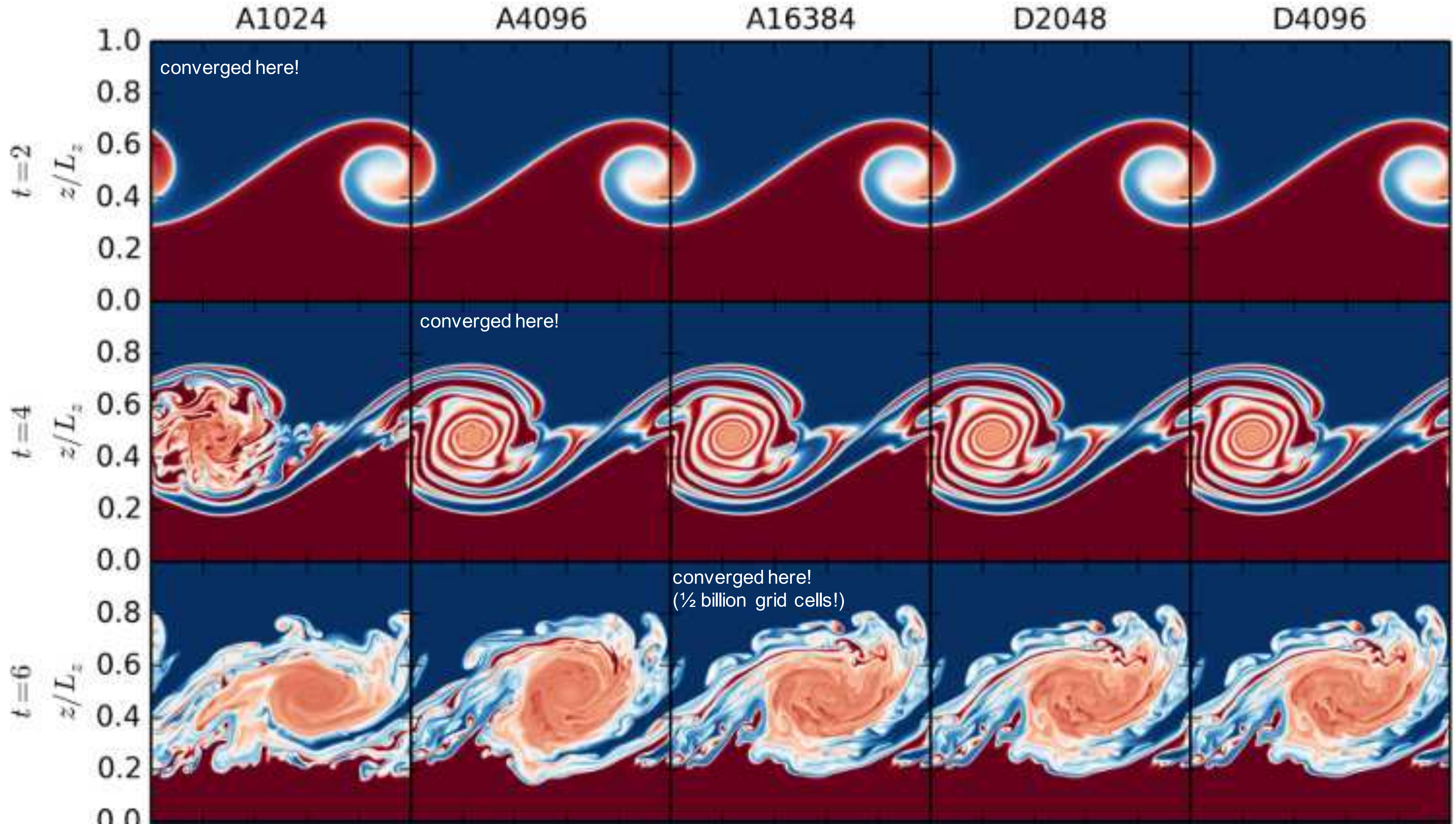




Results of Lecoanet et al (2016)



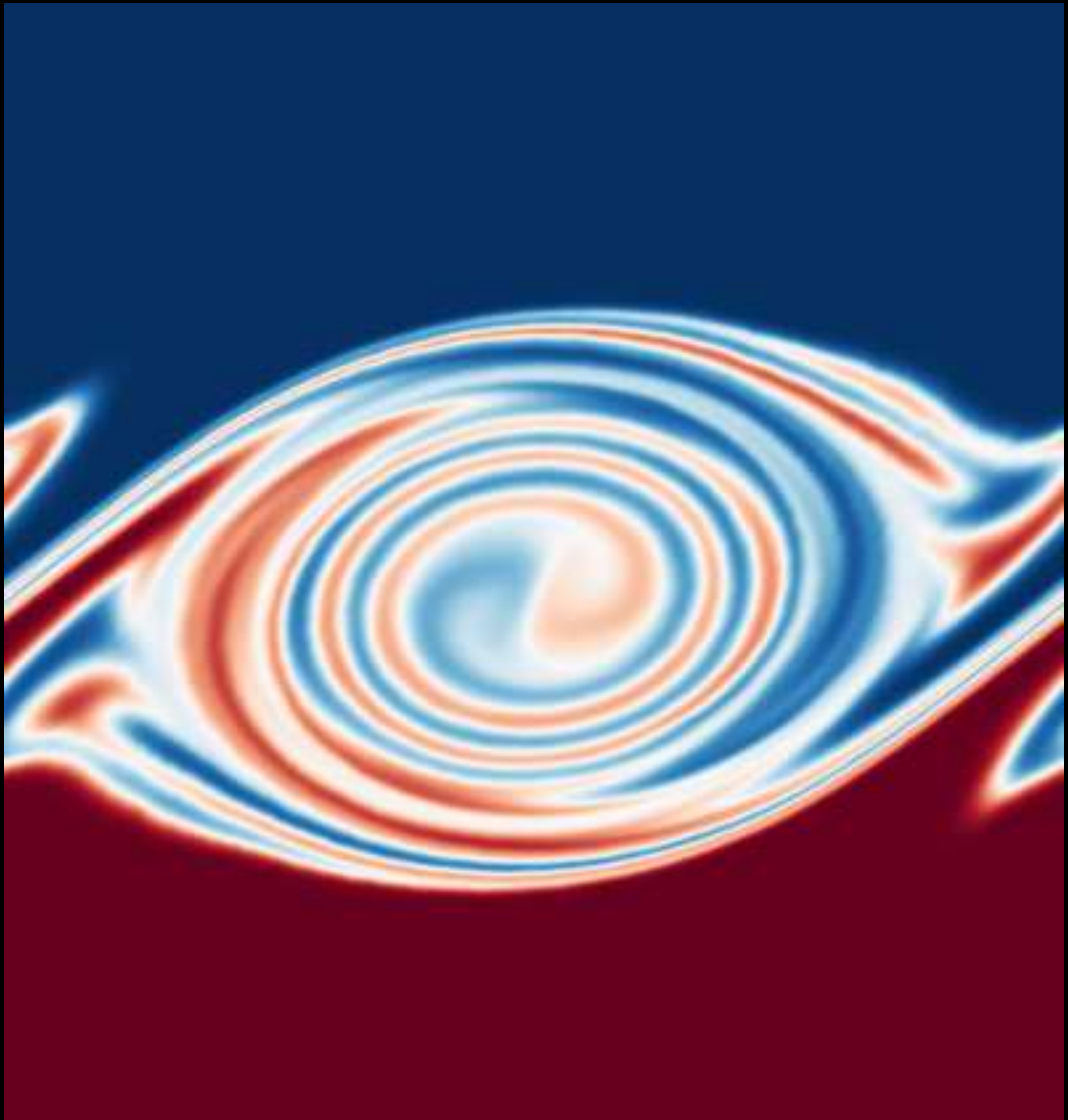
Stratified KH Test of Lecoanet et al (2016)



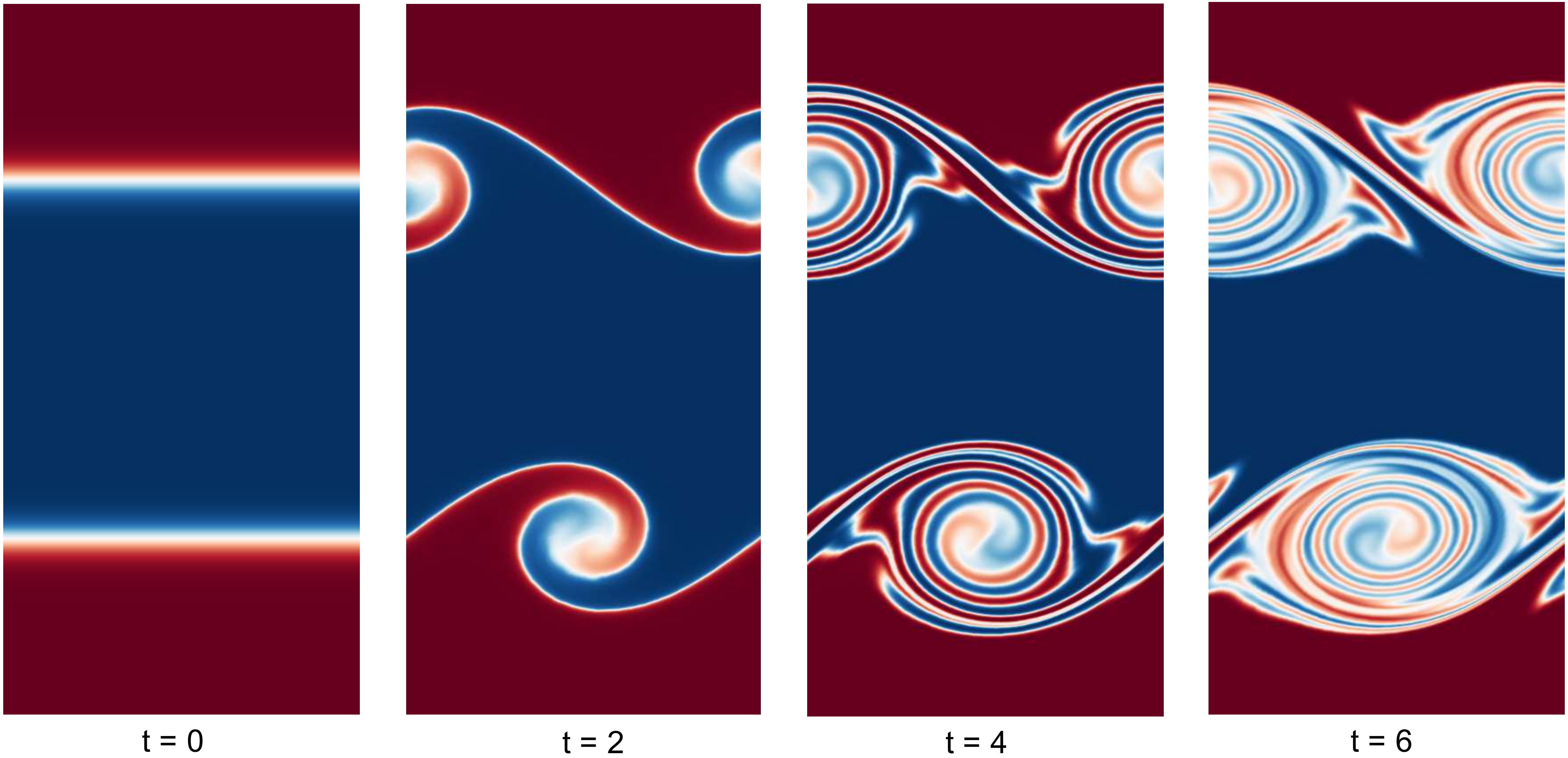
SPH Simulations

- I am using the Re=10^5 unstratified (uniform density) KH test ($\mathrm{Re} = \frac{L\Delta u}{\nu}$)
- Comparison to n_x = 2048 Dedalus calculation (spectral code)

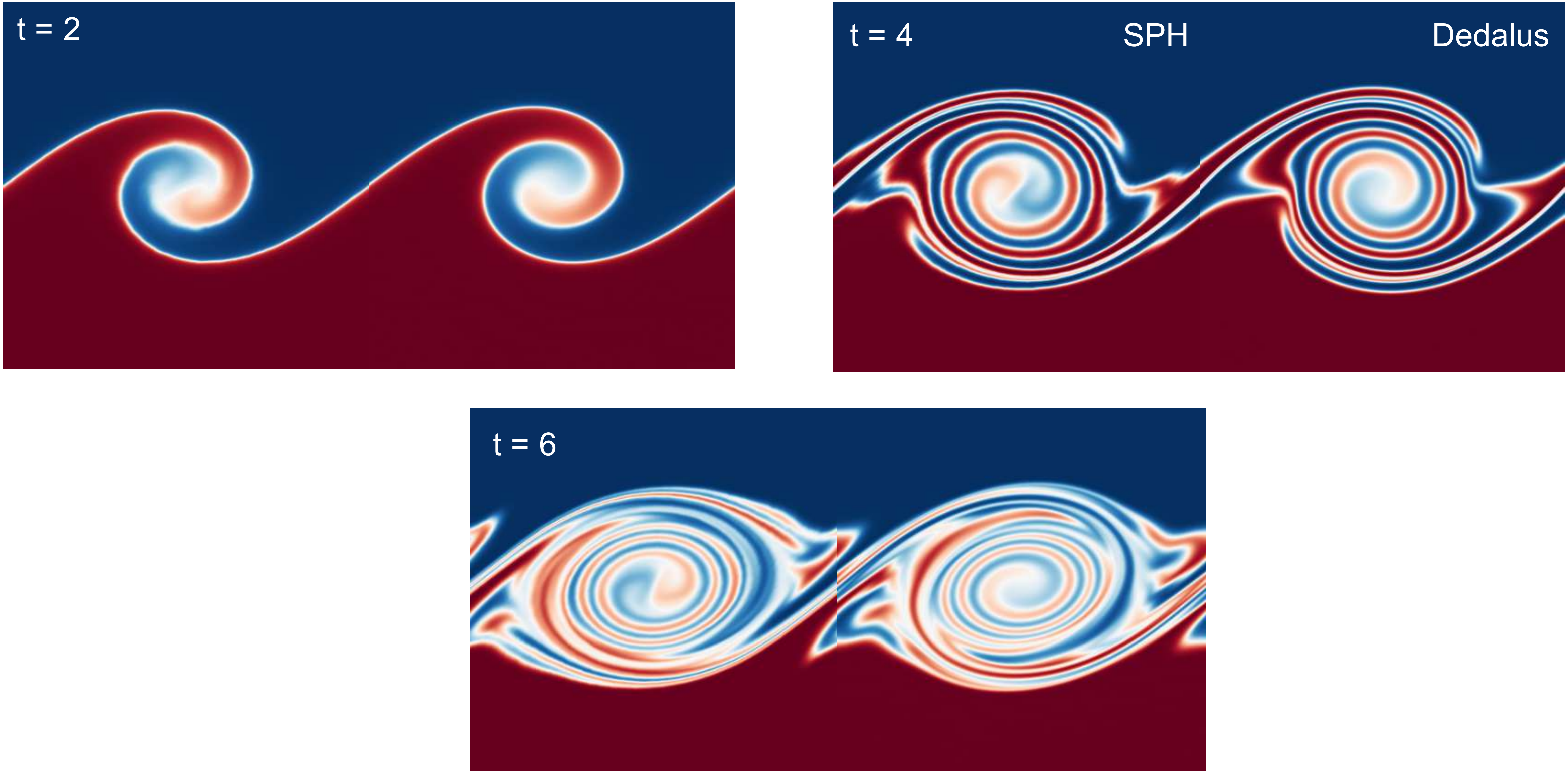
- Goal: obtain convergence of SPH results towards reference solution
- Resolution: n_x = 256, 512, 1024, 2048 particles (~8 million)
- Dissipation Implementation: direct second derivative style for Navier-Stokes viscosity, thermal conduction, and colour diffusion (efficiency, consistency)



SPH results (n_x = 1024 particles)



SPH results (n_x = 1024 particles)



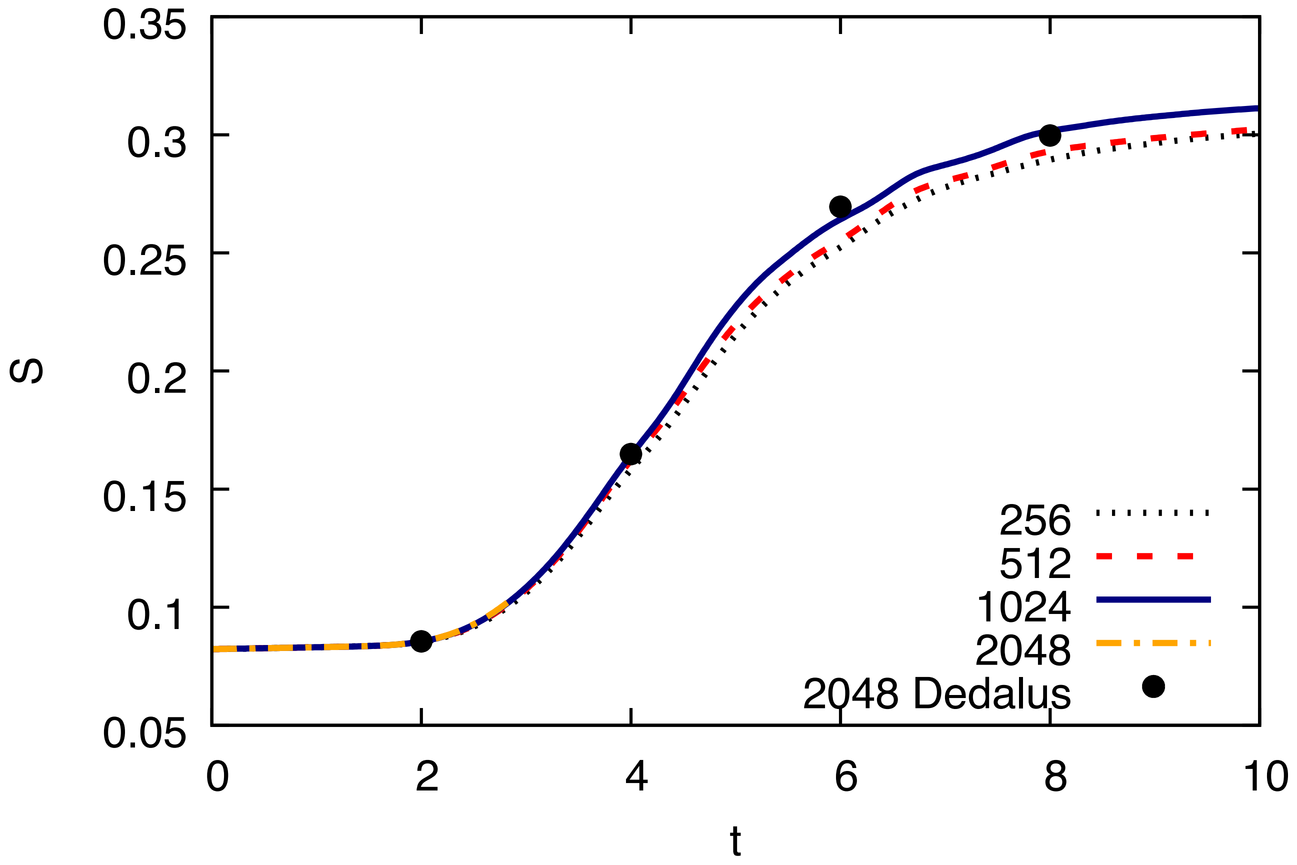
Colour Entropy

- Define entropy for colour

$$s \equiv -c \ln c$$

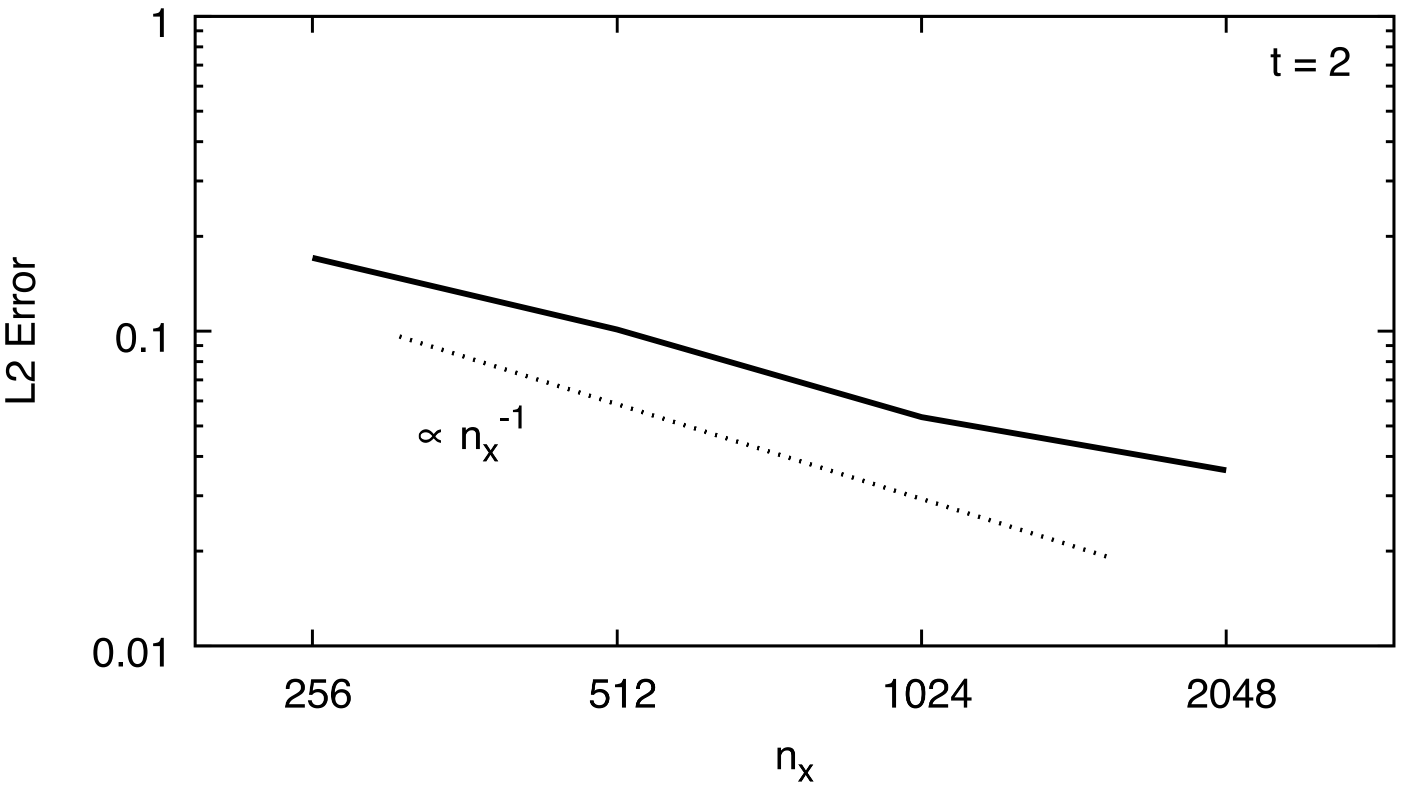
- and total colour entropy

$$S \equiv \int \rho \, s \, \mathrm{d}V$$

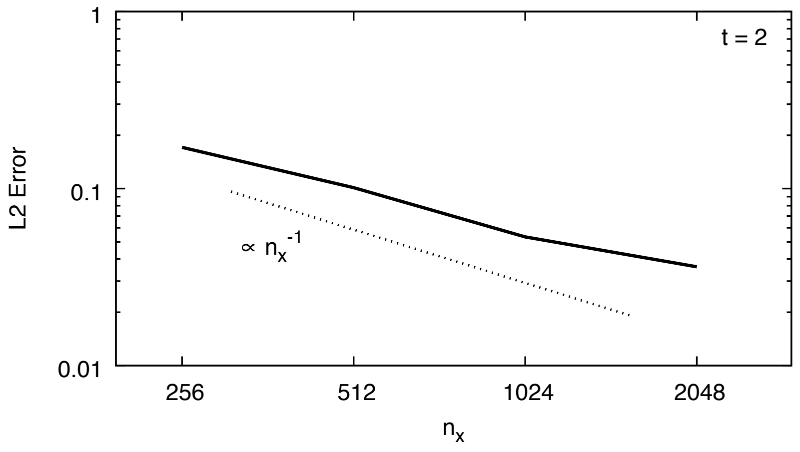


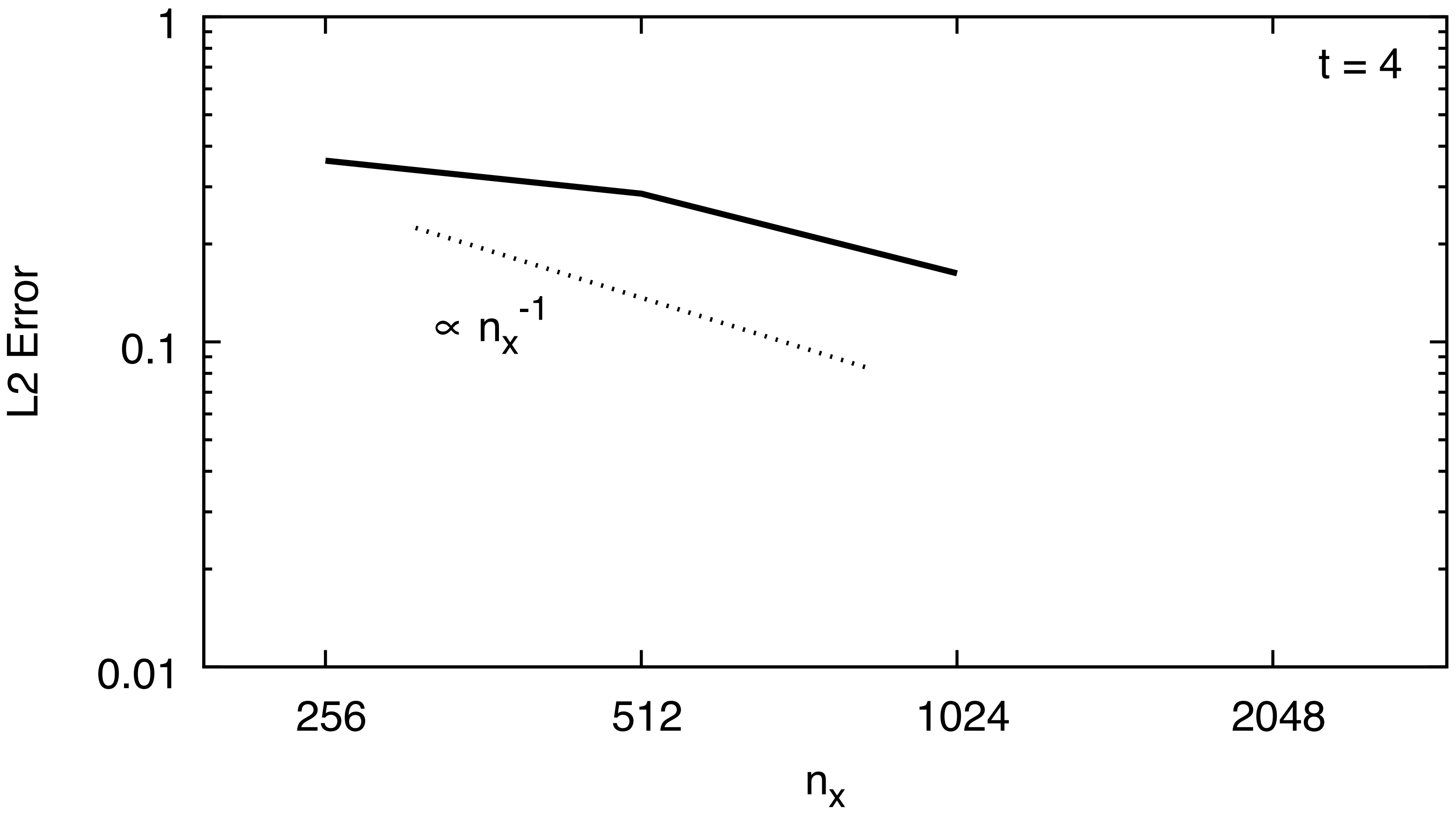
- Results are converging towards reference solution
- Numerical dissipation (artificial viscosity) still relevant up till n_x = 1024 or 2048, so don't except convergence yet

L2 error Convergence (t = 2)

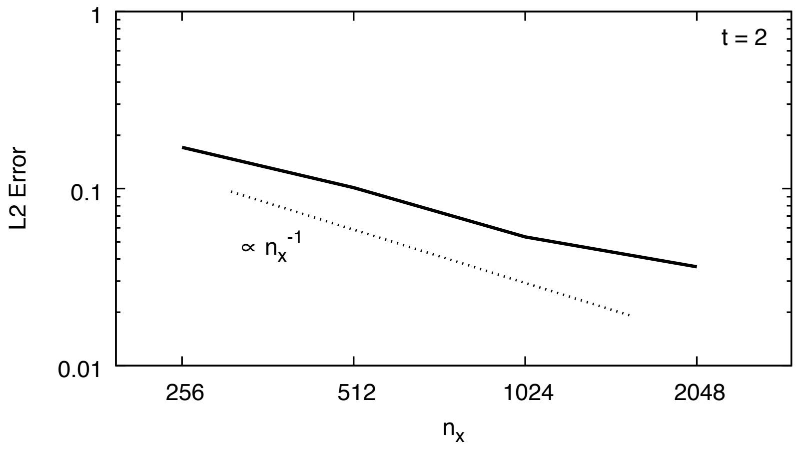


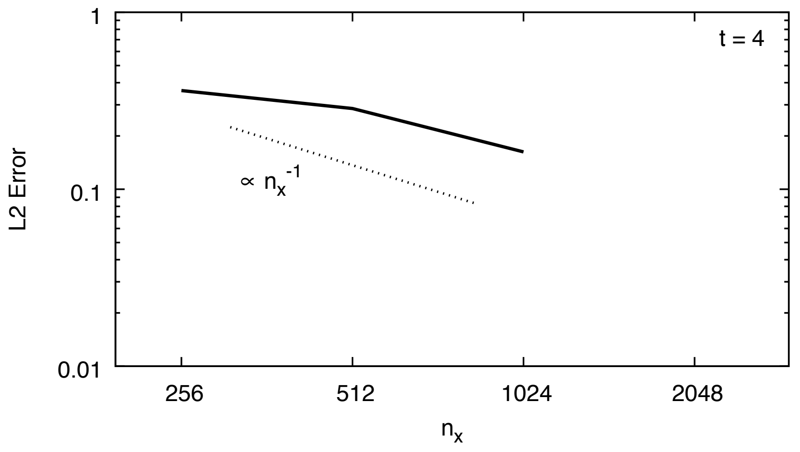
L2 error Convergence (t = 4)

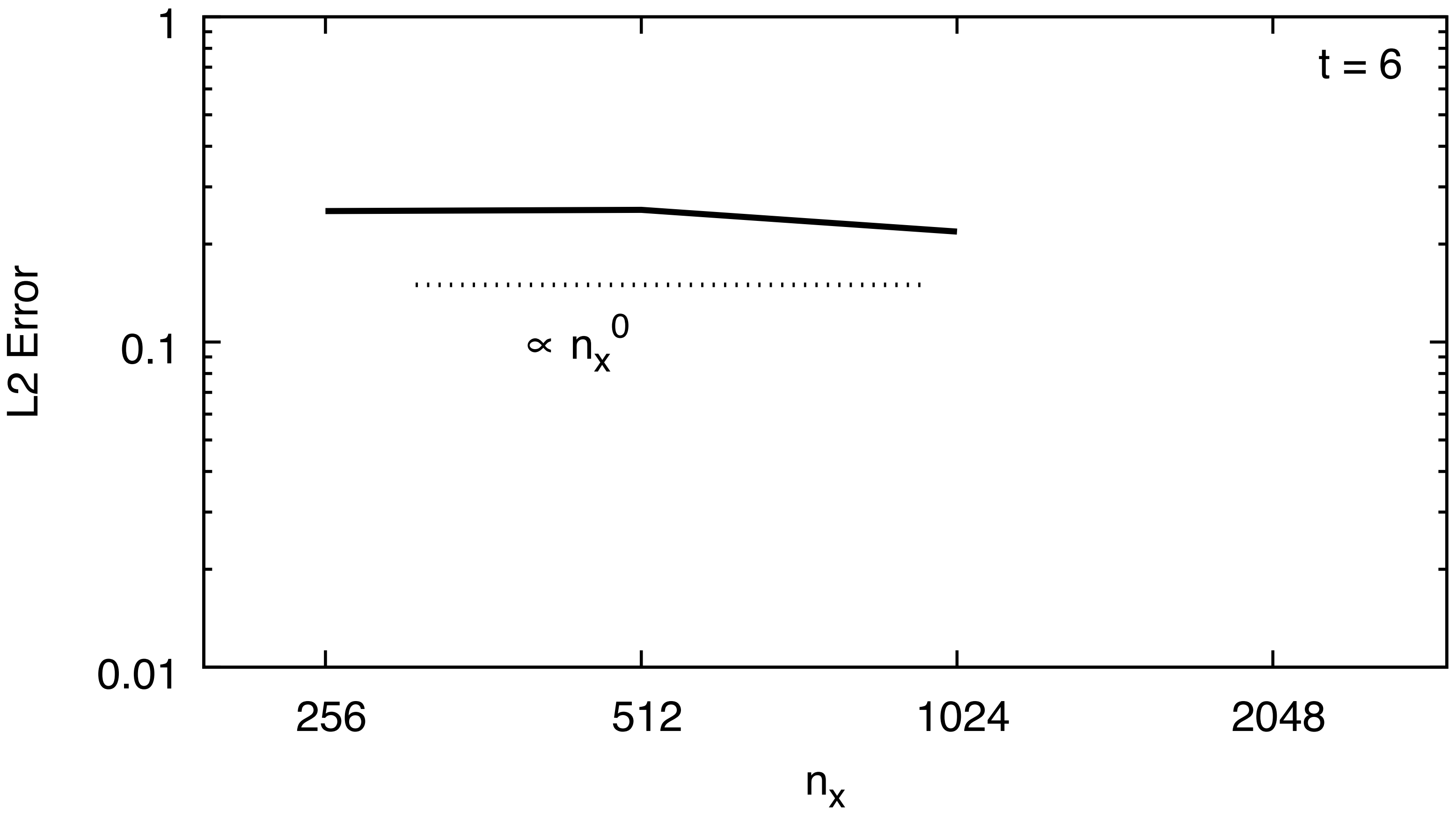




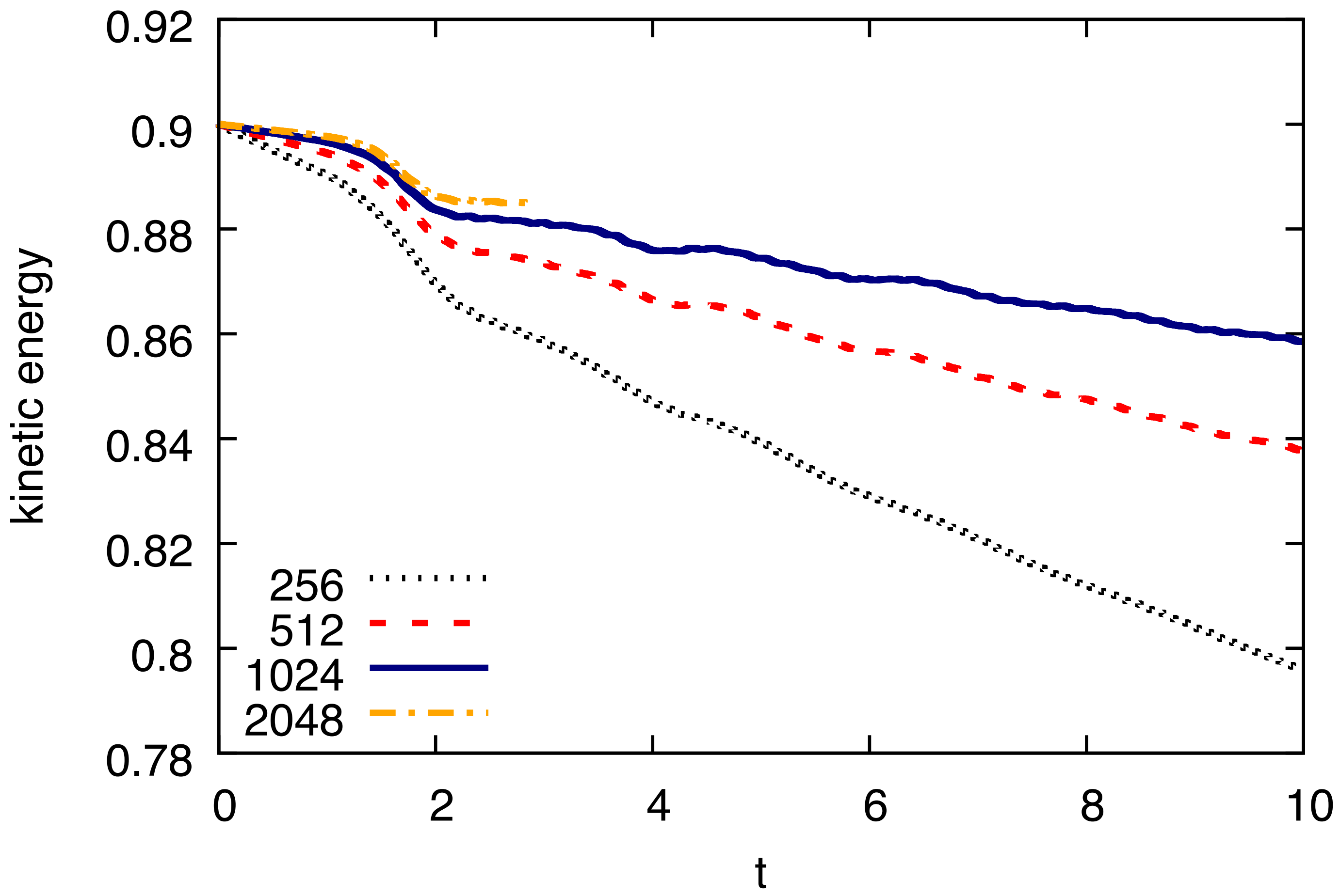
L2 error Convergence (t = 6)





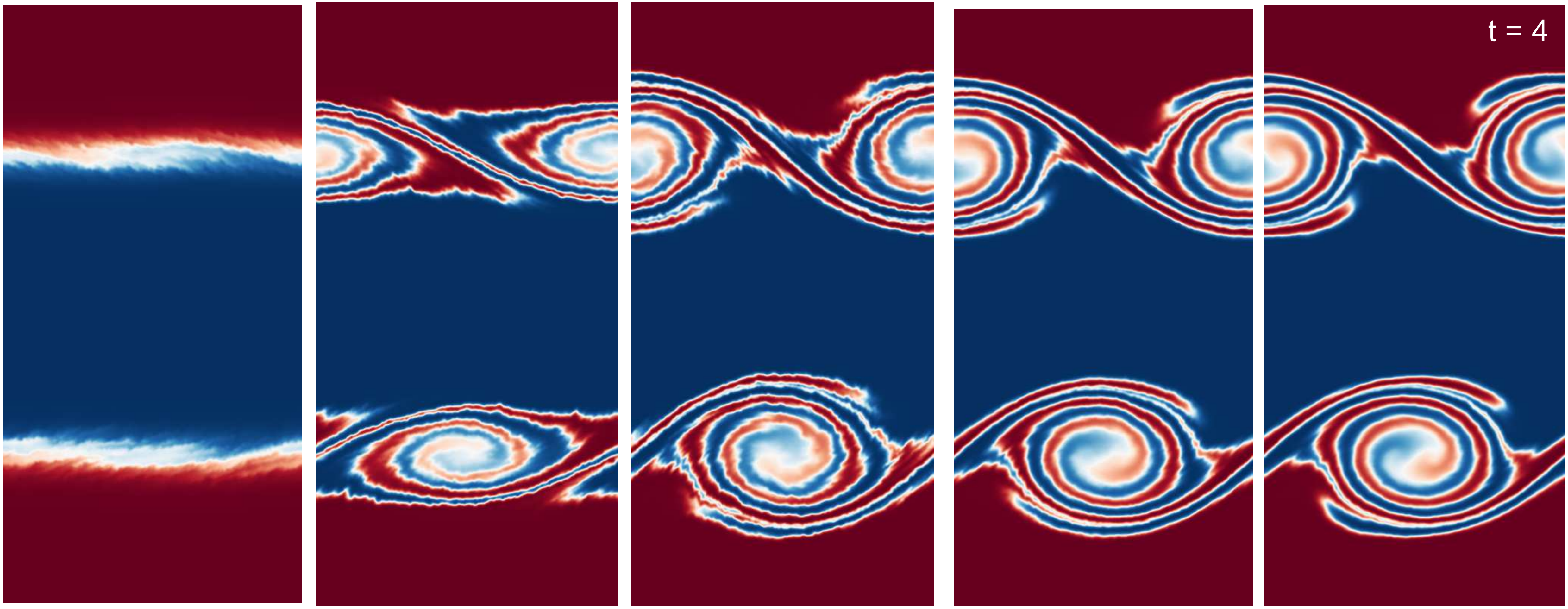


Kinetic Energy



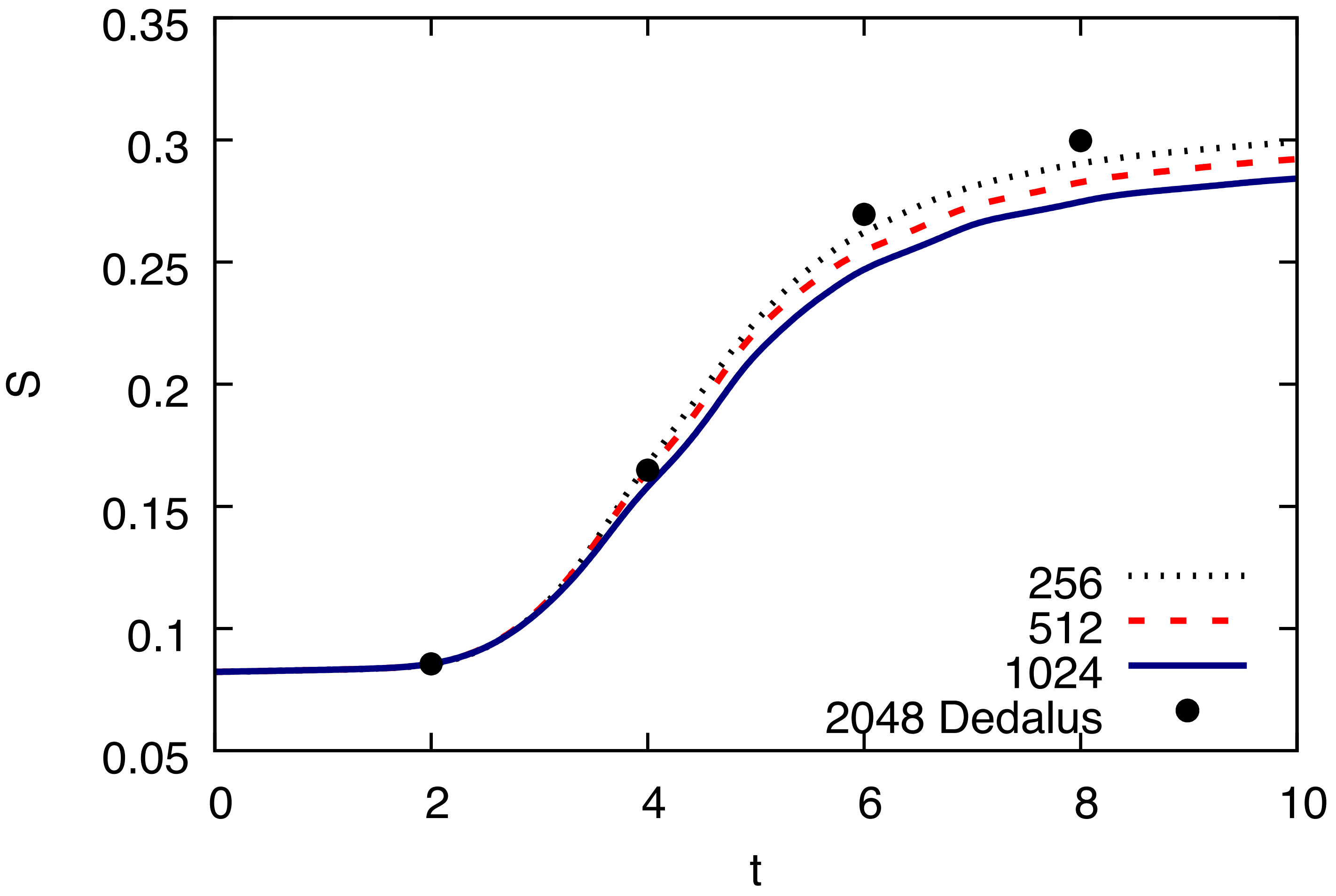
- Dissipation rate of kinetic energy not yet converged!
- Expected from analytic translation of artificial viscosity to physical dissipation

Quality of Smoothing Kernel Matters



Cubic Spline | Quartic | Quintic | Sextic | Heptic

Colour Entropy for Quintic Spline



Conclusions

- SPH can activate the Kelvin-Helmholtz instability!
  (*that is, SPH can do hydrodynamics – not a surprise to anyone in this room*)
- May need to use n_x = 4096 to achieve formal convergence
  (*32 million particles – I hope not!*)
- Currently running octic and nonic splines (R = 5h!) to check kernel bias convergence.

- It may not be as difficult (resolution requirement, kernel bias) to activate KH as found here for other conditions (i.e., Reynolds number).
- Not shown, but Wendland family of kernels demonstrate same behaviour.

- My belief is that SPH will converge to the agreed solution.