Kinematics and dynamics of gravitationally unstable discs with PHANTOM

PHANTOM and MCFOST users workshop 2023
Monash University

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SG and GI in accretion discs

Self gravity

The gravitational attraction of the gas itself contributes to the total gravitational potential

Main consequences:

- Super-Keplerian rotation curve (**Benedetta's** talk)

- Thickness of the disc

Gravitational instability

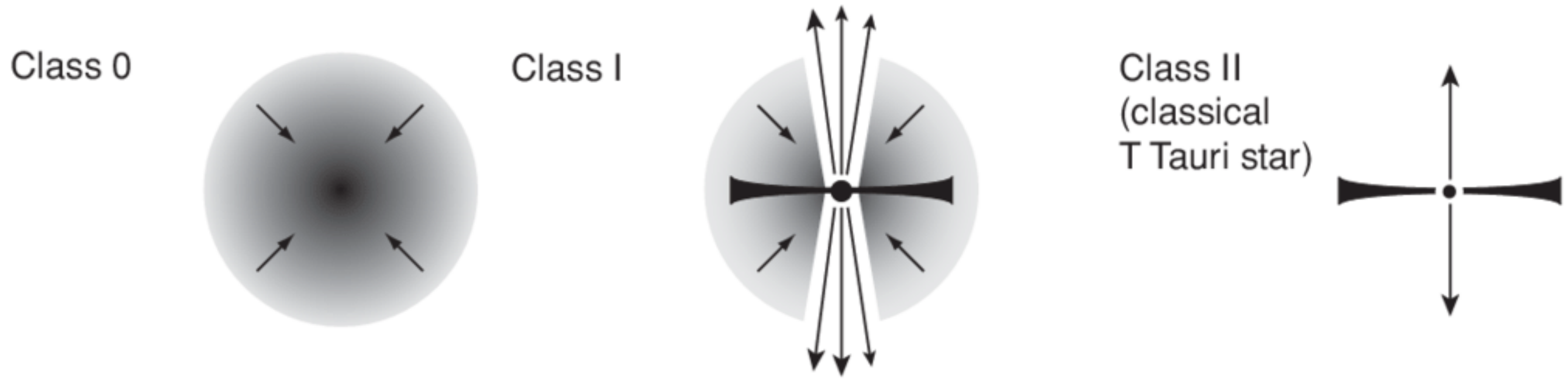
Linear hydrodynamical instability that can arise in self gravitating systems

Main consequences:

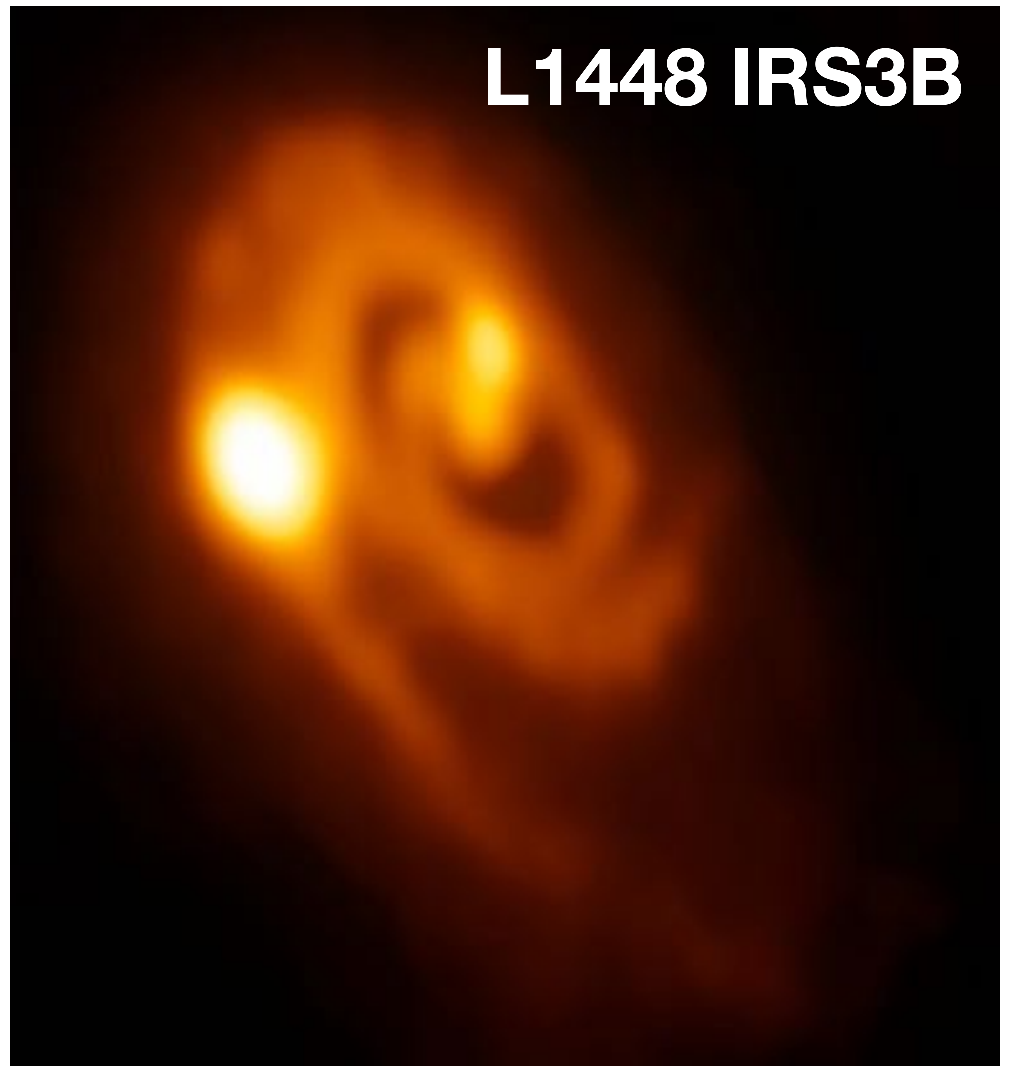
- Spiral density waves (in density and velocity)

- Transport of angular momentum and fragmentation

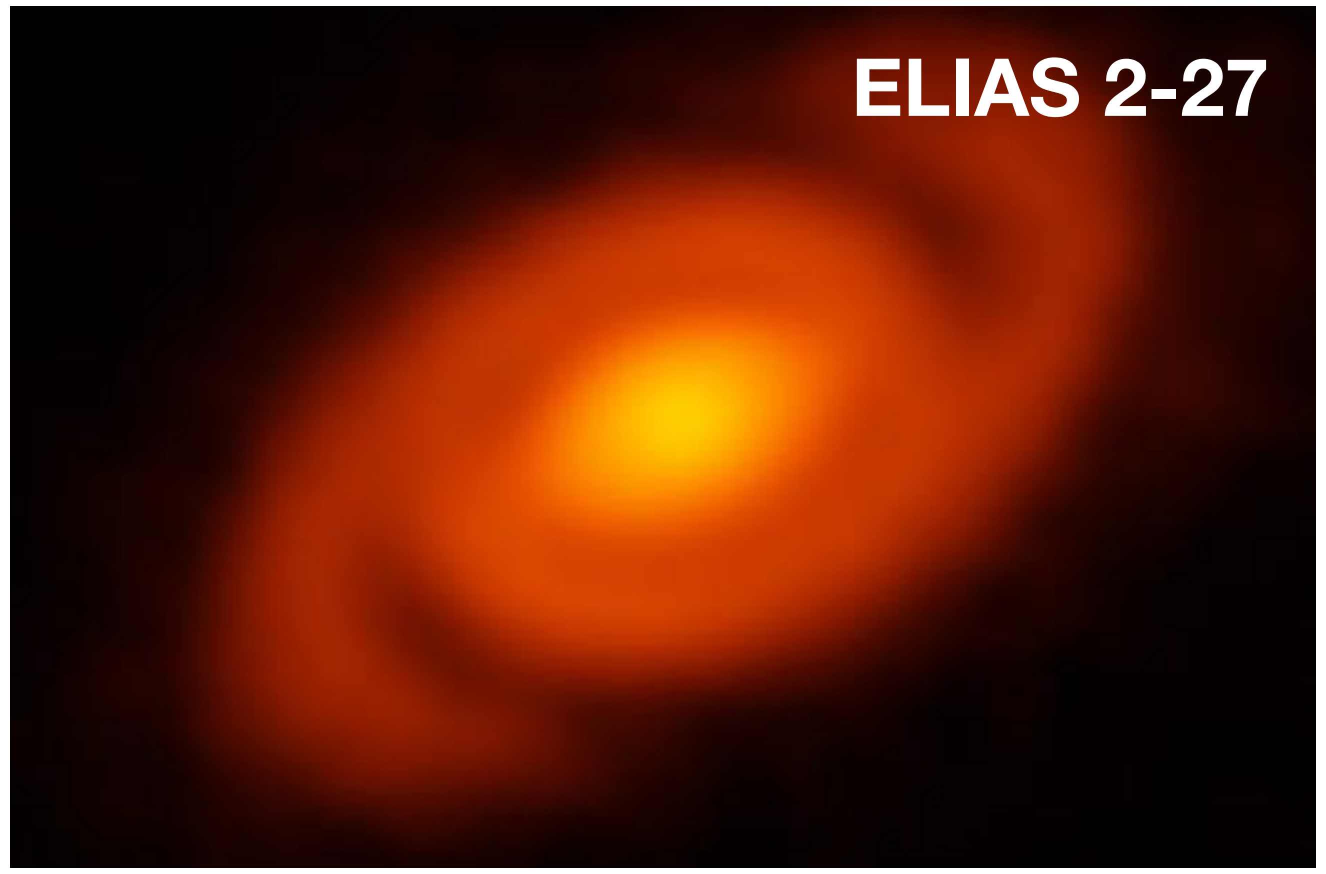
Why GI?



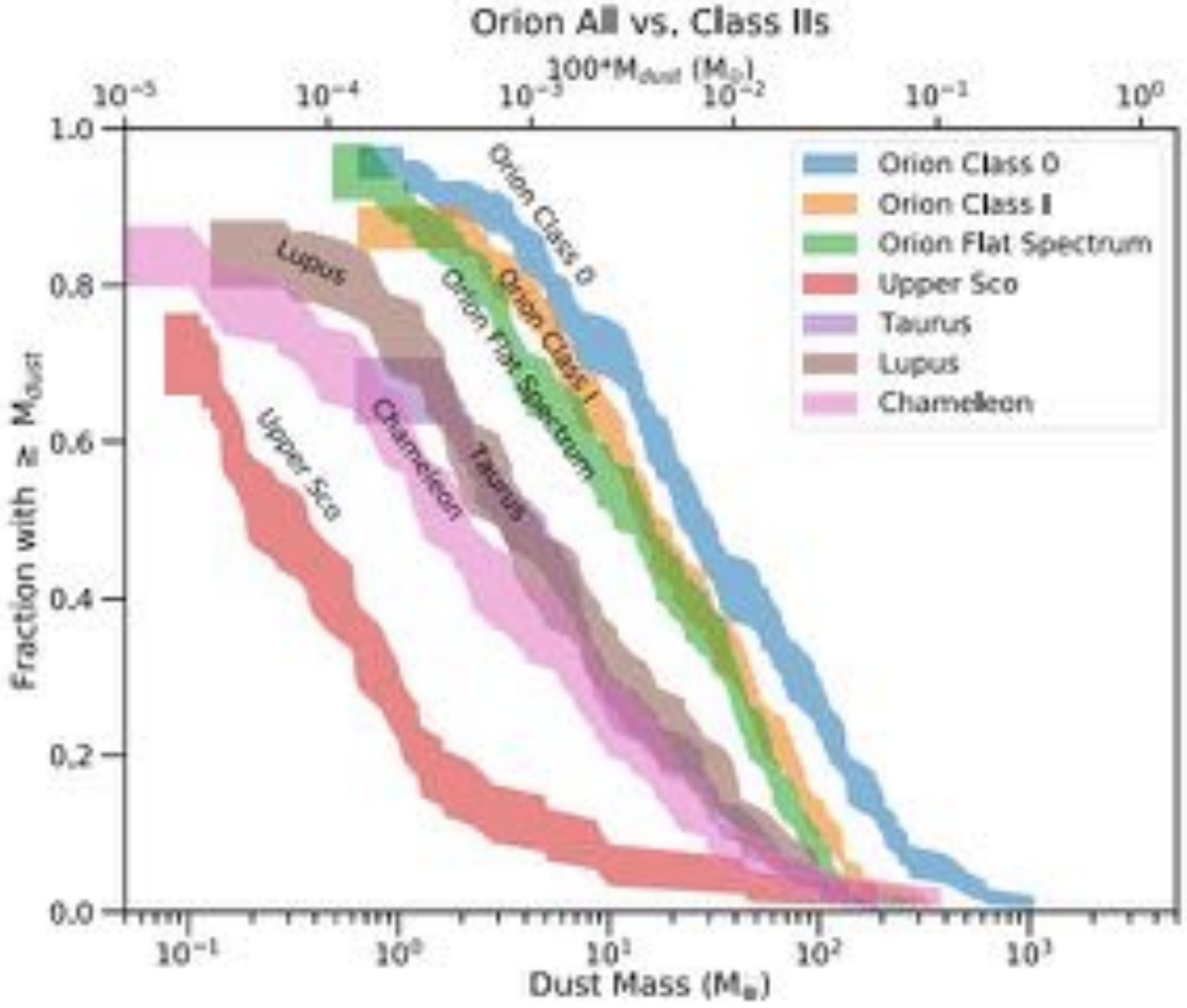
M_d Decreasing



Tobin+2017



Pérez+ 2016



VANDAM Survey of Orion protostars, Tobin 2020

Important to understand the outcome of GI in order to understand how planet formation works

Gravitational instability: linear theory

Dispersion relation for one fluid component thin disc:

$$(\omega - m\Omega)^2 = c^2k^2 - 2\pi G\Sigma |k| + \kappa^2$$

Doppler-shifted perturbation frequency

Sound speed stabilising

Surface density destabilising

Epicyclic frequency stabilising

$$Q = \frac{c\kappa}{\pi G\Sigma}$$

$\frac{\text{Stabilising}}{\text{Destabilising}}$

- $Q > 1$ stability $(\omega^2 > 0)$
- $Q < 1$ instability $(\omega^2 < 0)$

Lin & Shu 1964

Toomre 1964

Beyond linear theory

Saturation:
Spiral structure heats the disc with shocks balancing the cooling.
→ angular momentum transport

Fragmentation:
Spiral structure does not balance the cooling. Exp growth of the perturbation
→ fragmentation of the disc

Beyond linear theory

Saturation:

Spiral structure heats the disc with ... ing the cooling.

→ angular momentum t...

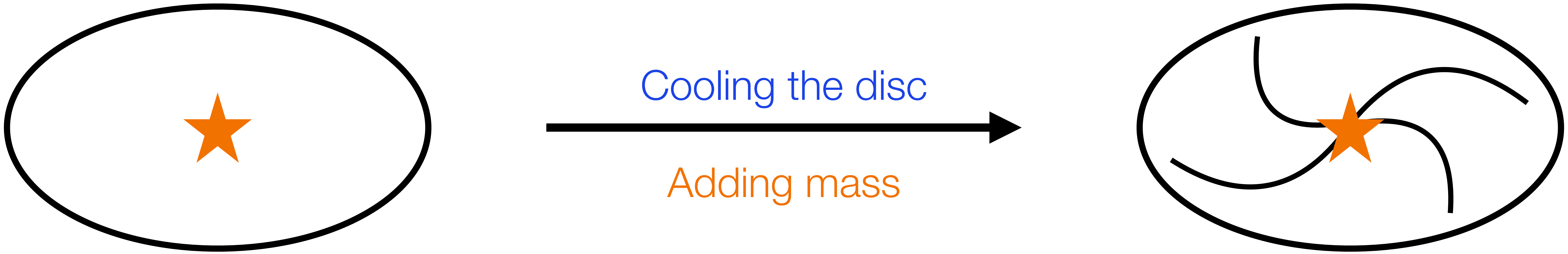
NEGATIVE FEEDBACK

Fragmentation:

Spiral structure does not balance the cooling. Exp growth of the perturbation

→ fragmentation of the disc

Beyond linear theory



$Q >> 1$ $\qquad$ $Q = 1$

Saturation:
Spiral structure heats the disc with… NEGATIVE FEEDBACK …ing the cooling.
→ angular momentum t…

Fragmentation:
Spiral structure does not bal… POSITIVE FEEDBACK …p growth of the perturbation
→ fragmentation of the d…

GI : main characteristics

β cooling

thermodynamics

$$\beta_{cool} = \Omega t_{cool}$$

$\beta_{cool} > \beta_{cr} \rightarrow$ Not fragmenting

Thermal saturation

$$\delta\Sigma/\Sigma = \chi\beta^{-1/2}$$

$\beta_{cool} < \beta_{cr} \rightarrow$ Fragmenting

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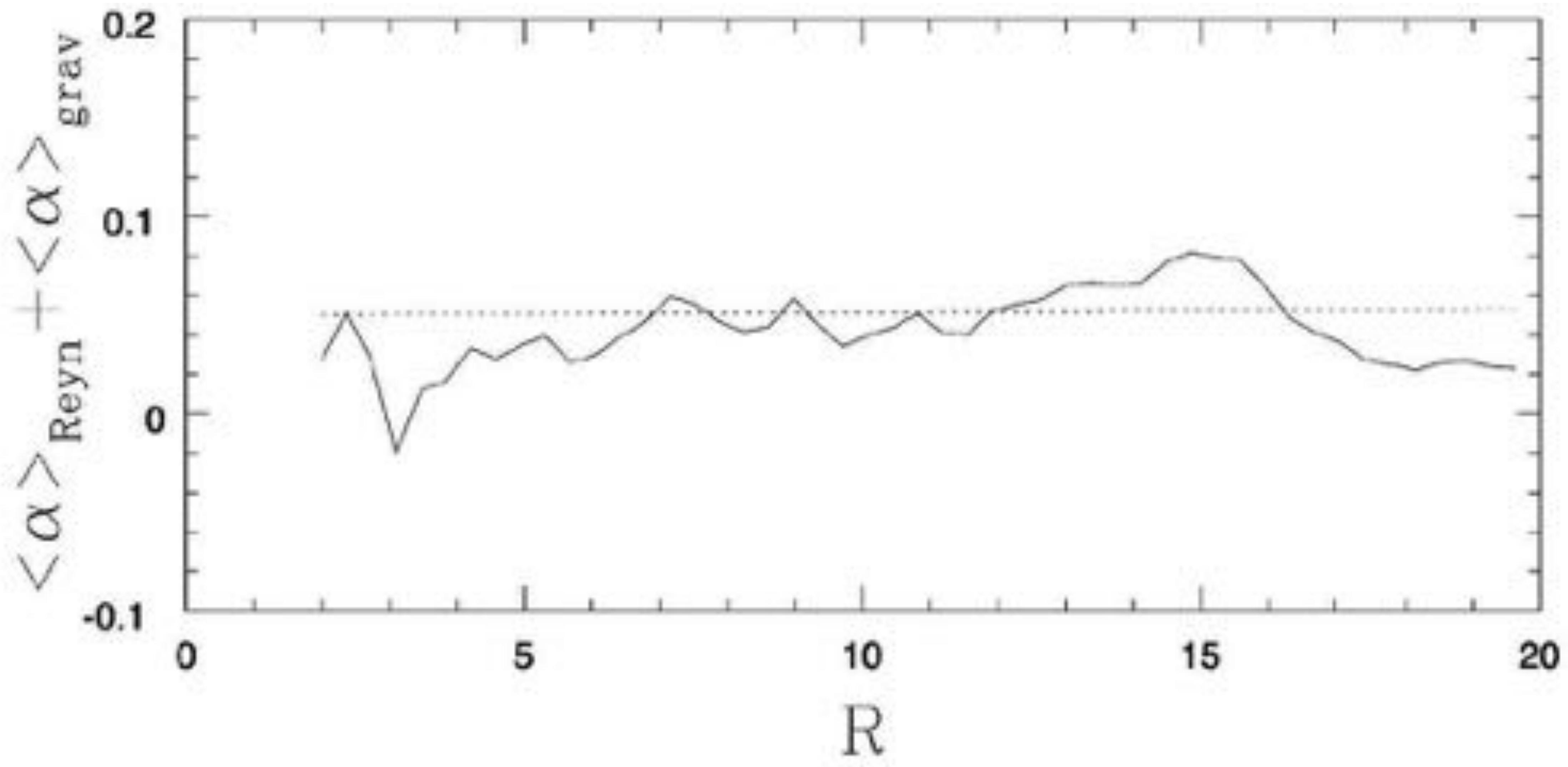
Within $\beta-$cooling prescription, the angular momentum transport induced by GI is

$$\alpha_{GI} = \frac{4}{9\gamma(\gamma - 1)\beta}$$



Lodato 2008

GI : main characteristics

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$\beta_{cool} < \beta_{cr} \rightarrow$ Fragmenting

	β_{cr}	Method
Gammie 2001	$\beta_{cr} = 3$	2D local simulations
Rice et al. 2005	$\beta_{cr} \simeq 6$	3D SPH simulations
Meru & Bate 2012	$\beta_{cr} > 15$	3D SPH & 2D grid Non convergence
Deng et al. 2018	$\beta_{cr} = 3$	3D SPH & 3d MFM (grid) Convergence

GI : main characteristics

β cooling

thermodynamics

Determines the strength of spiral perturbation and the angular momentum transport

Disc to star mass ratio

global behaviour - morphology

$$\frac{M_d}{M_\star} = Q\frac{H}{r}$$

$$\frac{M_d}{M_\star} \simeq \frac{\tan \boxed{\alpha_p}}{\boxed{m}}$$

Winding angle

Az. wavenumber

Massive discs show fewer spiral arms and more open spirals

GI : main characteristics

β cooling

thermodynamics

Determines the strength of spiral perturbation and the angular momentum transport

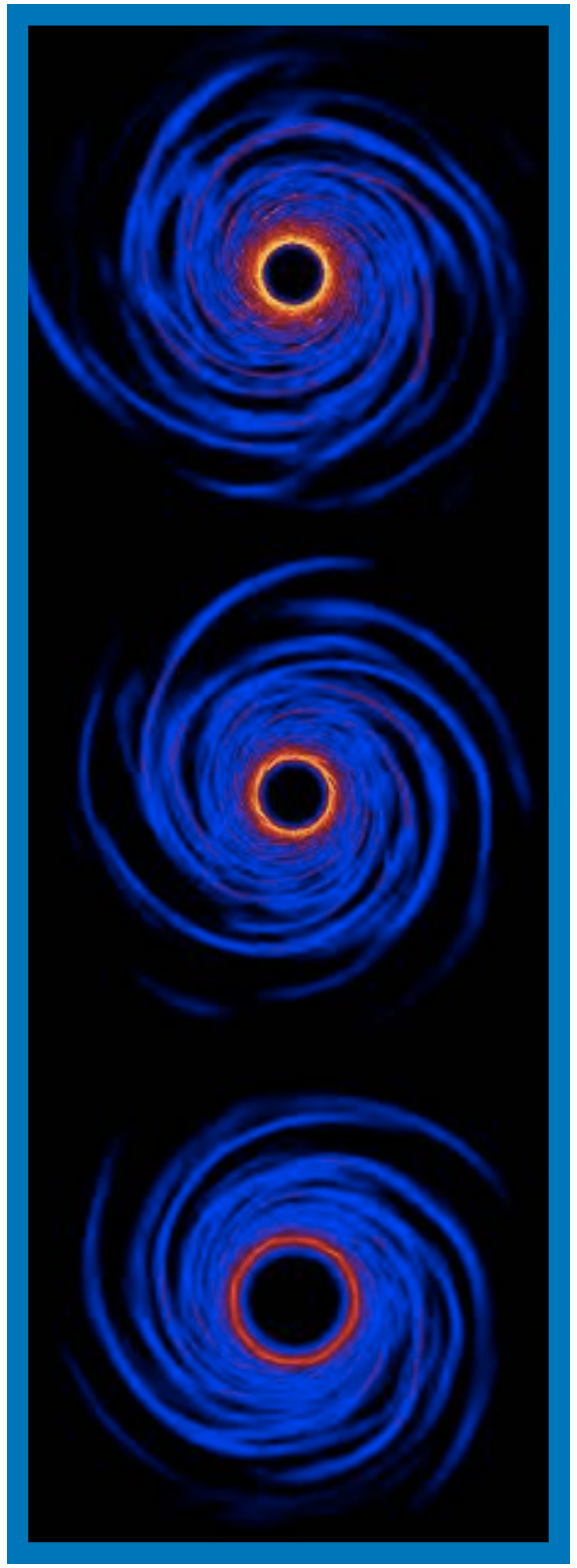
Disc to star mass ratio

global behaviour - morphology

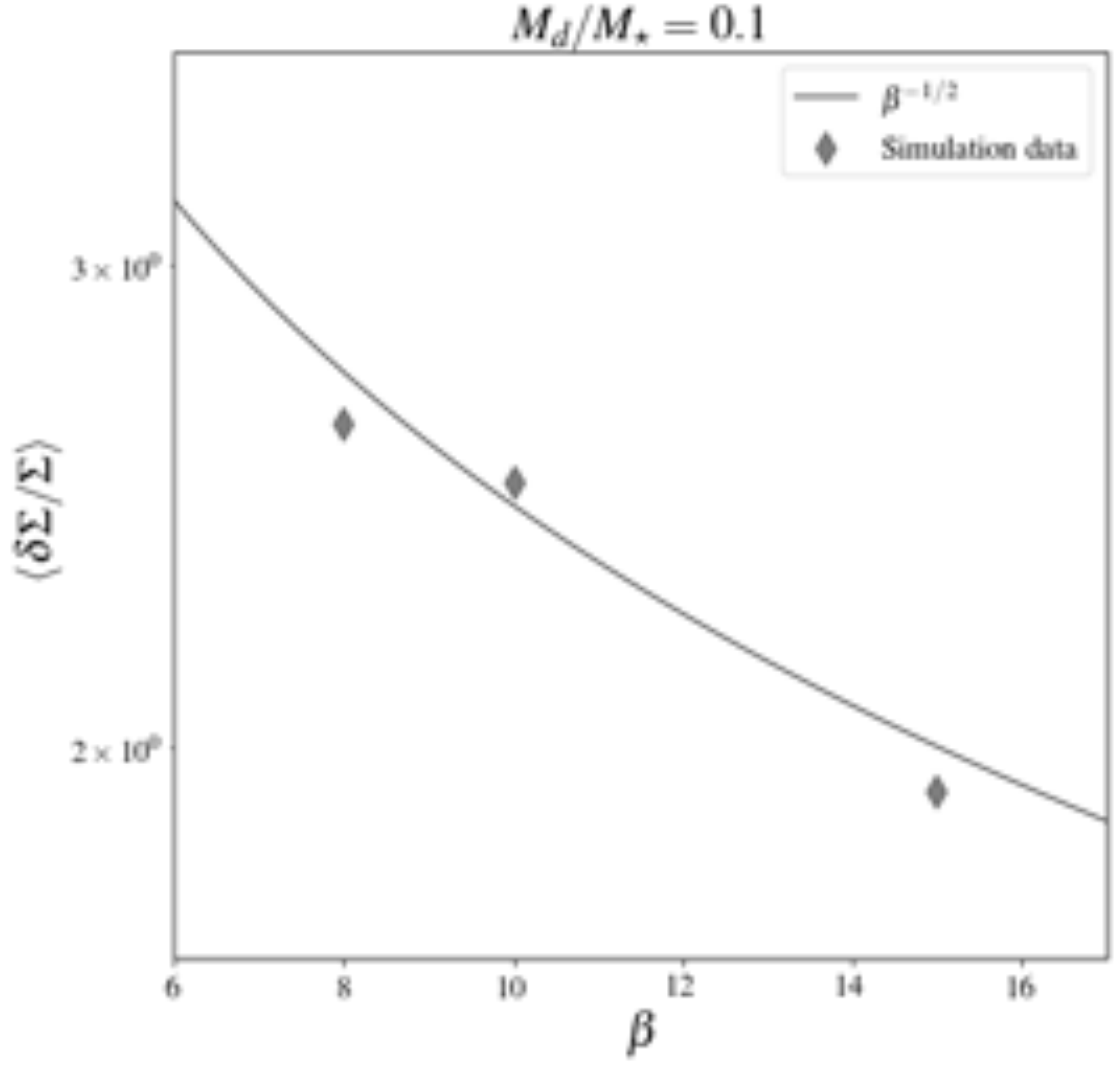
Determines the morphology of the spiral

Cooling factor

$\delta\Sigma/\Sigma \propto \beta^{-1/2}$



β_{cool}



Cooling factor

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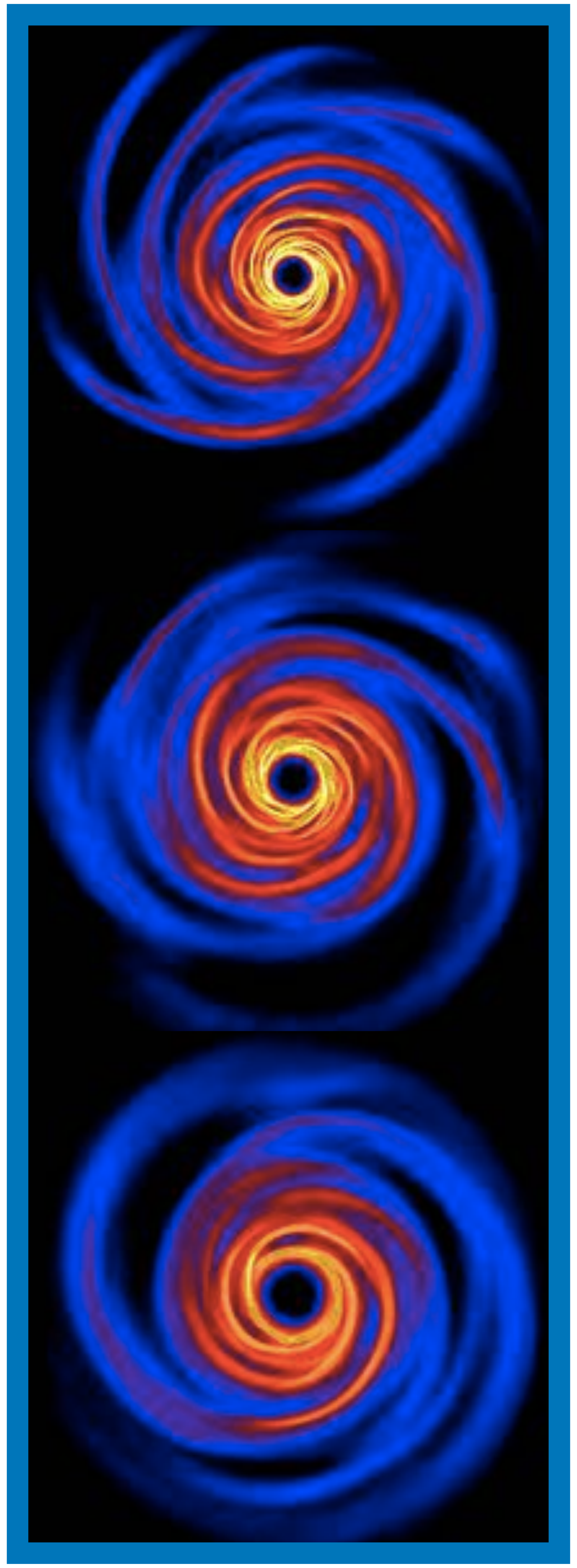
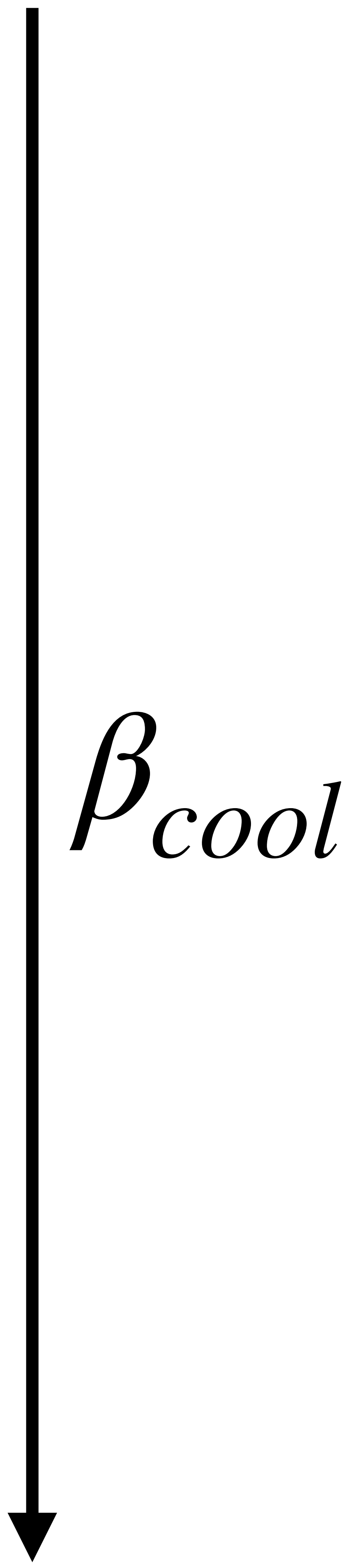
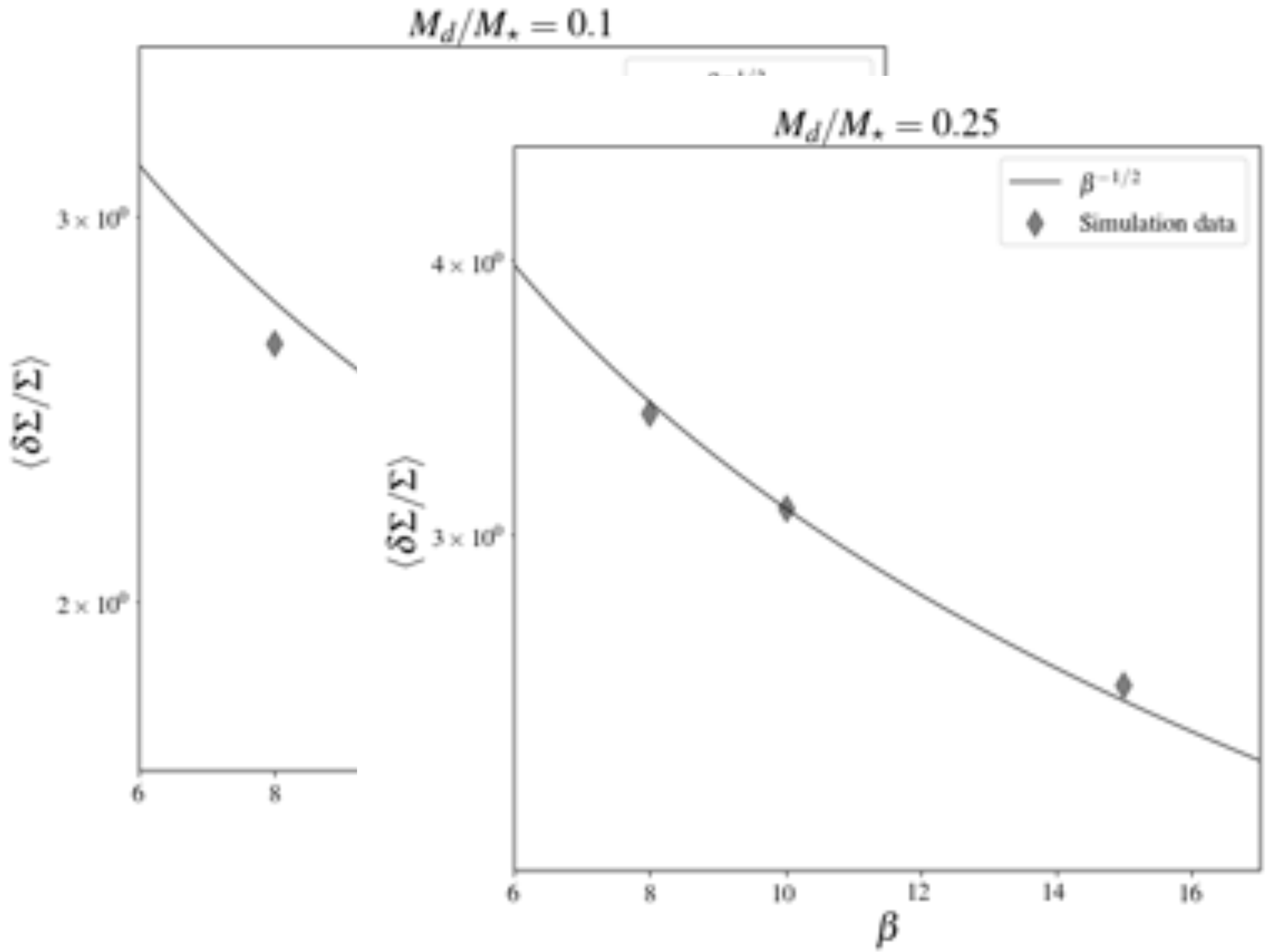
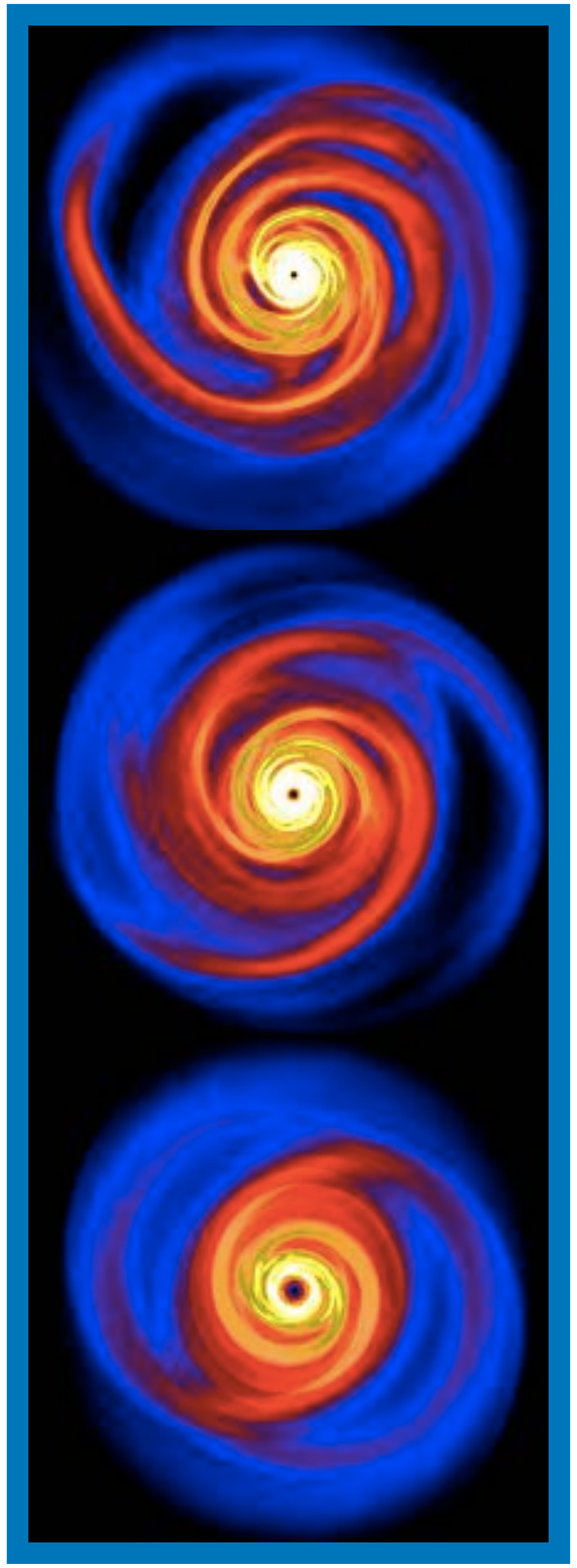


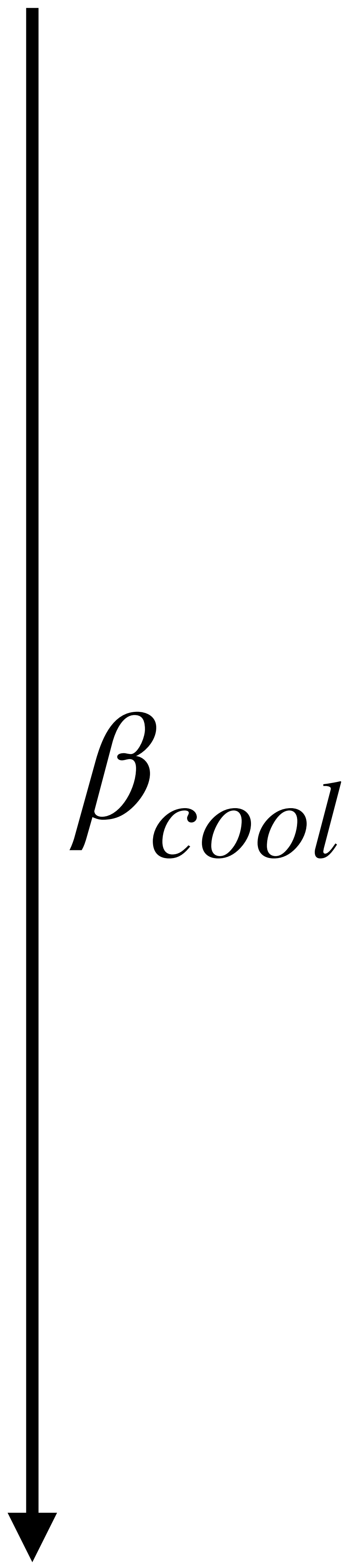


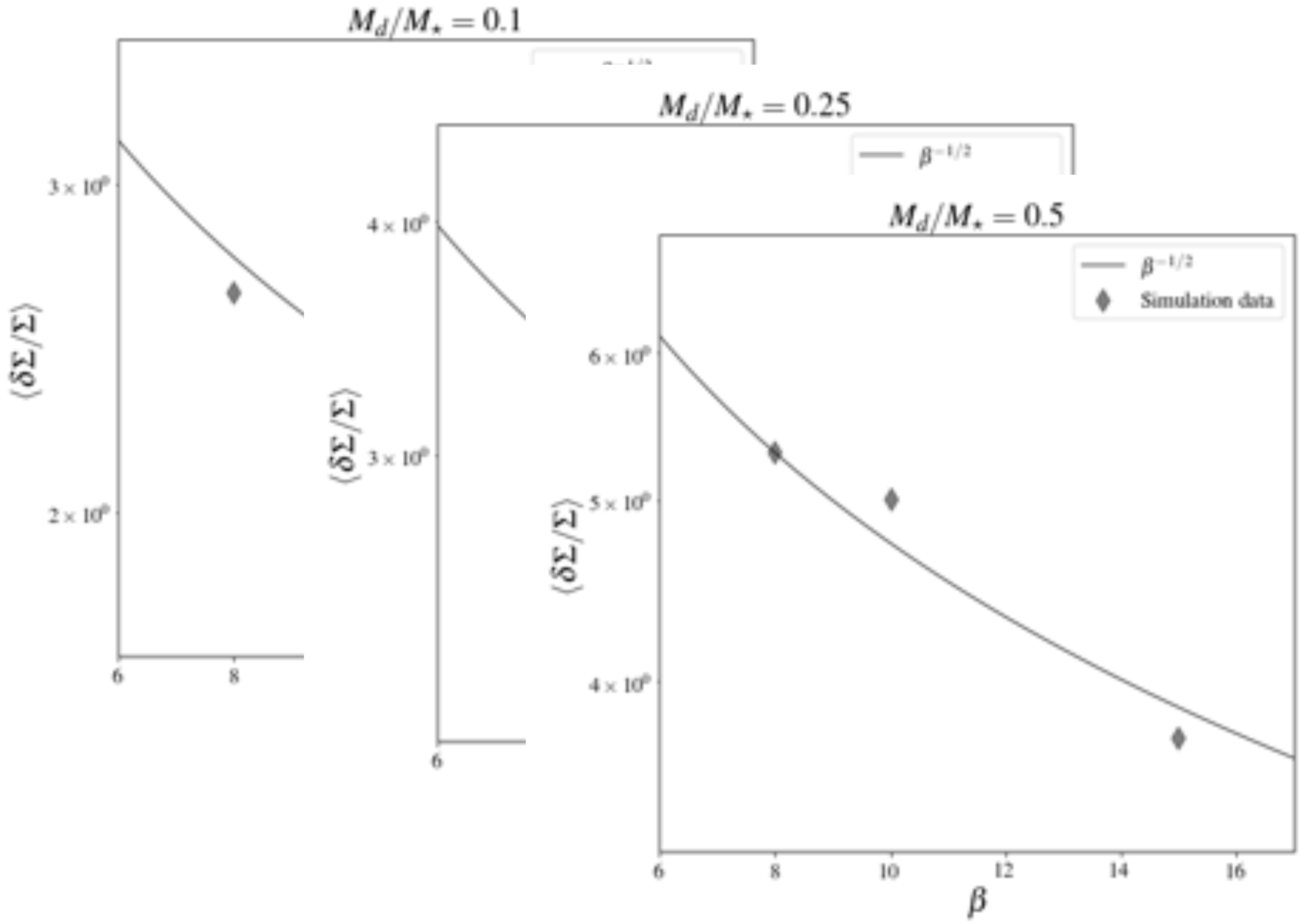


Cooling factor

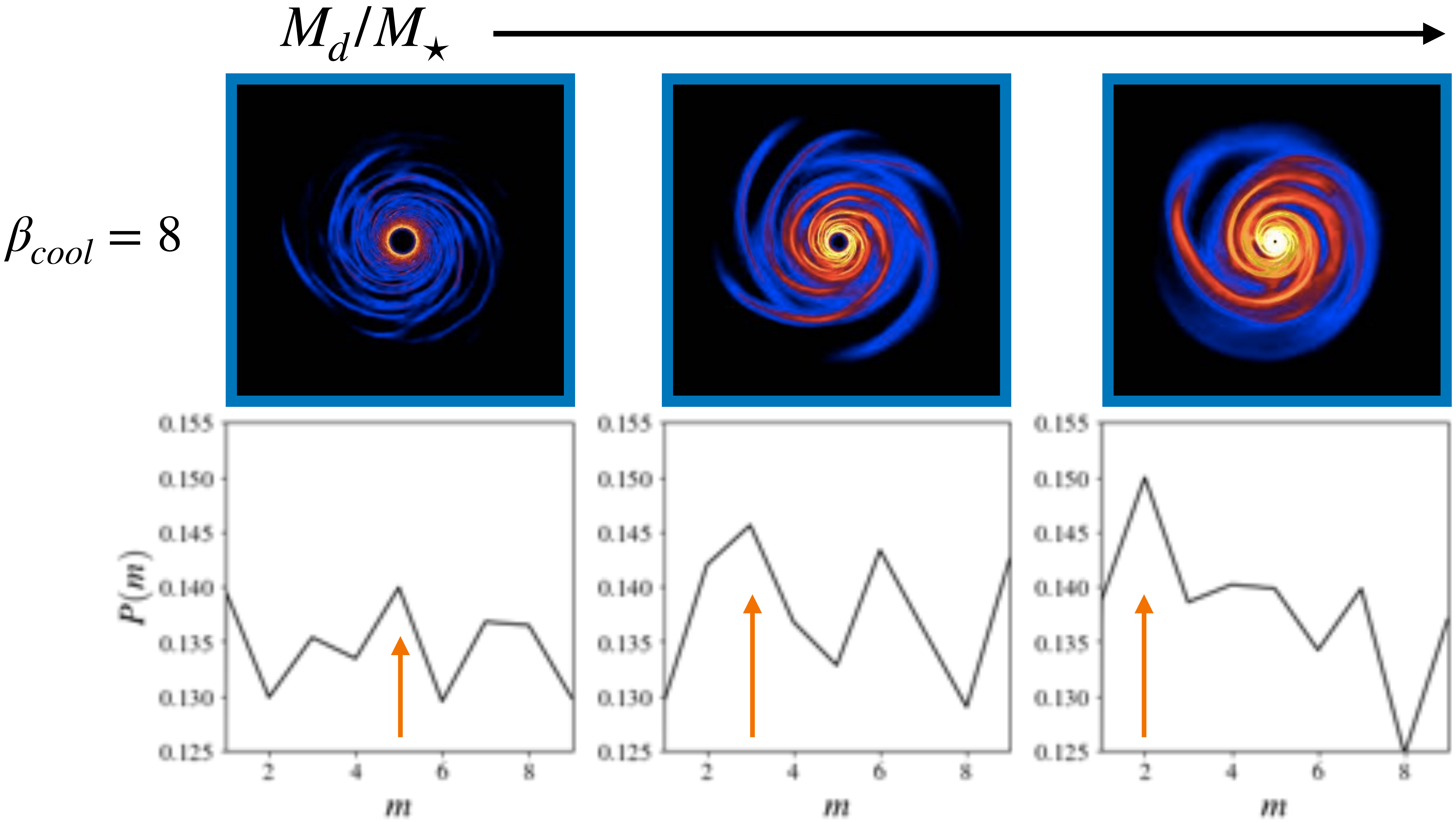
$$\delta\Sigma/\Sigma \propto \beta^{-1/2}$$







Disc to star mass ratio



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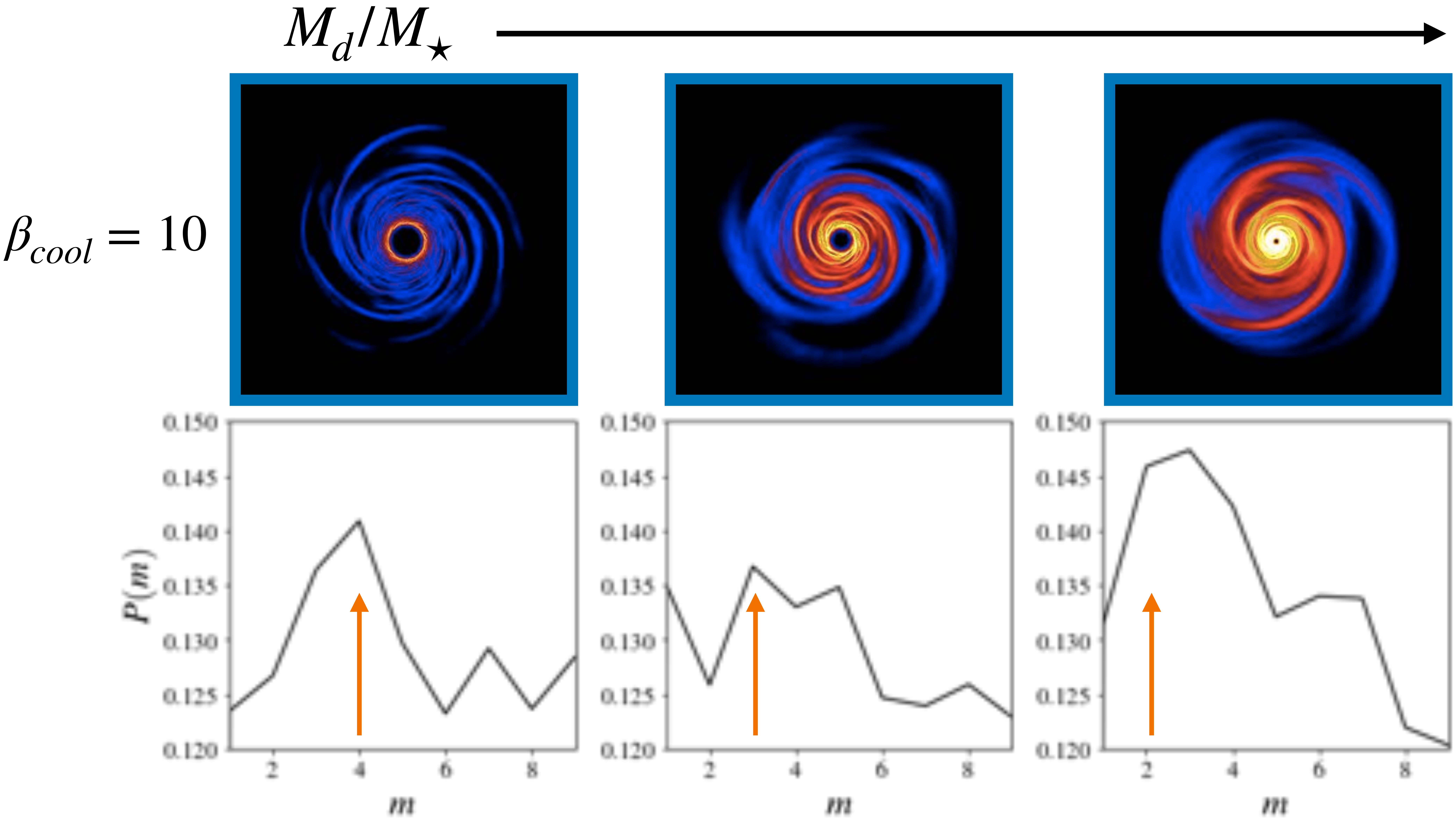
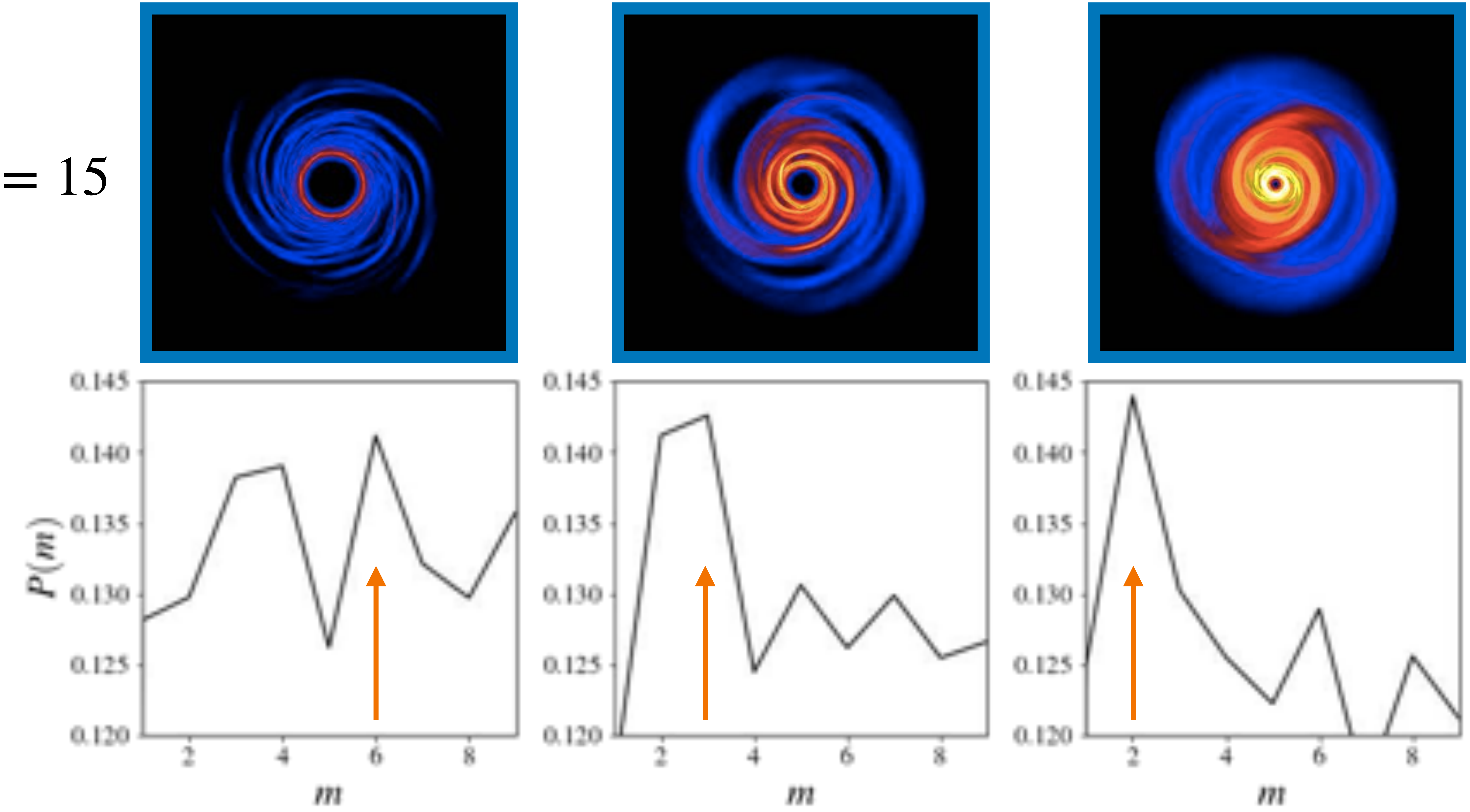


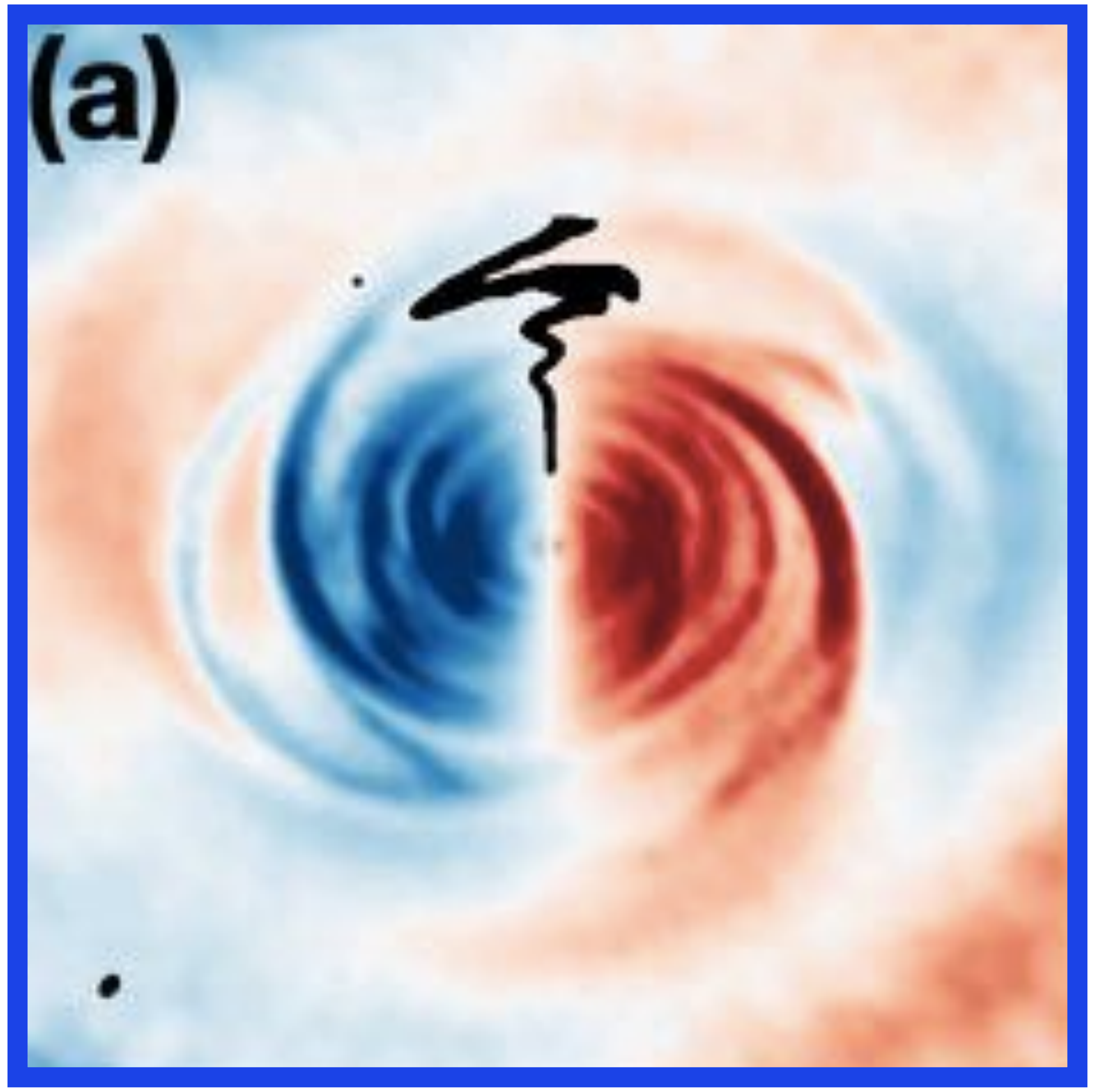
Disc to star mass ratio

$M_d/M_\star$ →

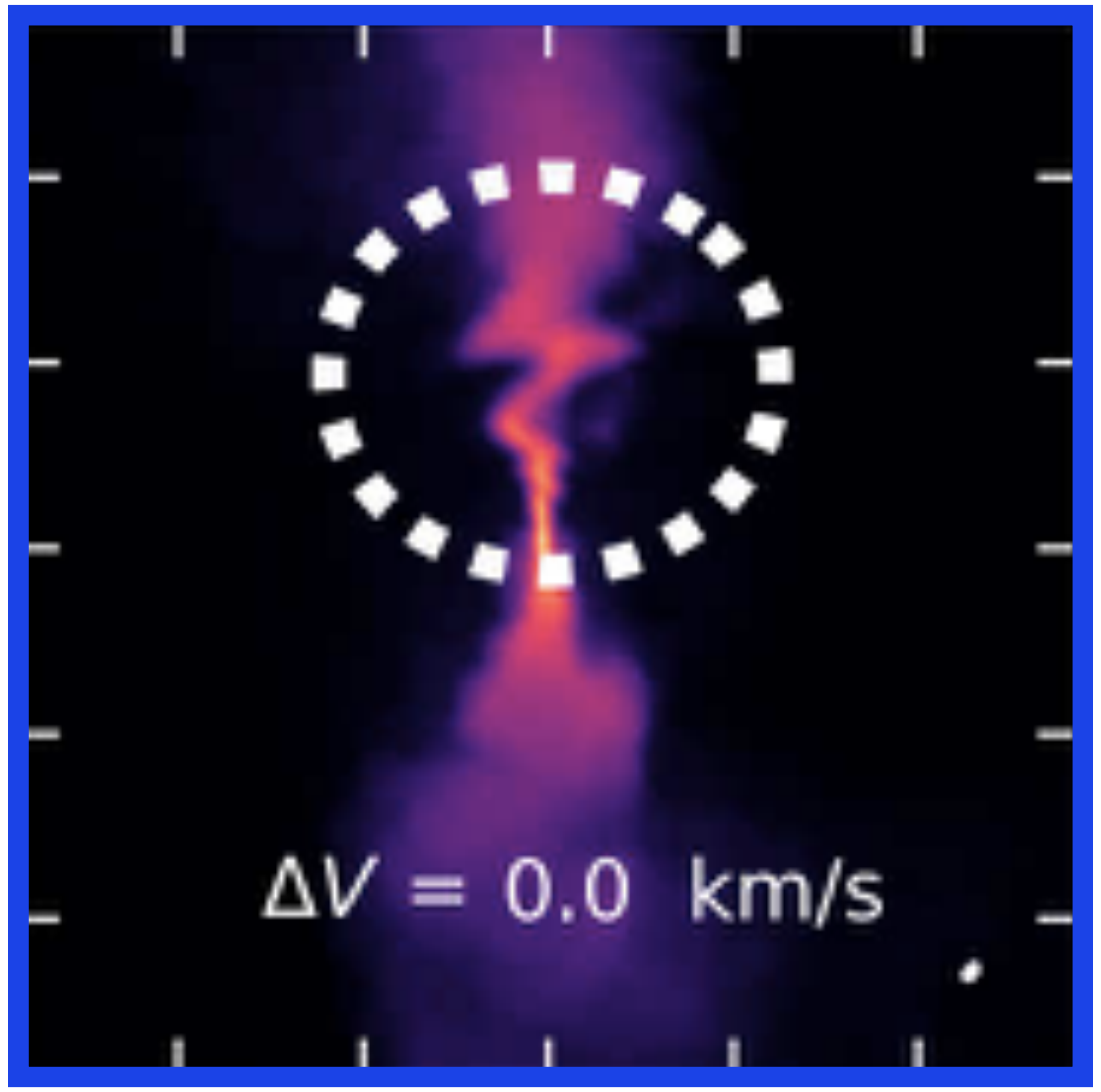
$\beta_{cool} = 15$



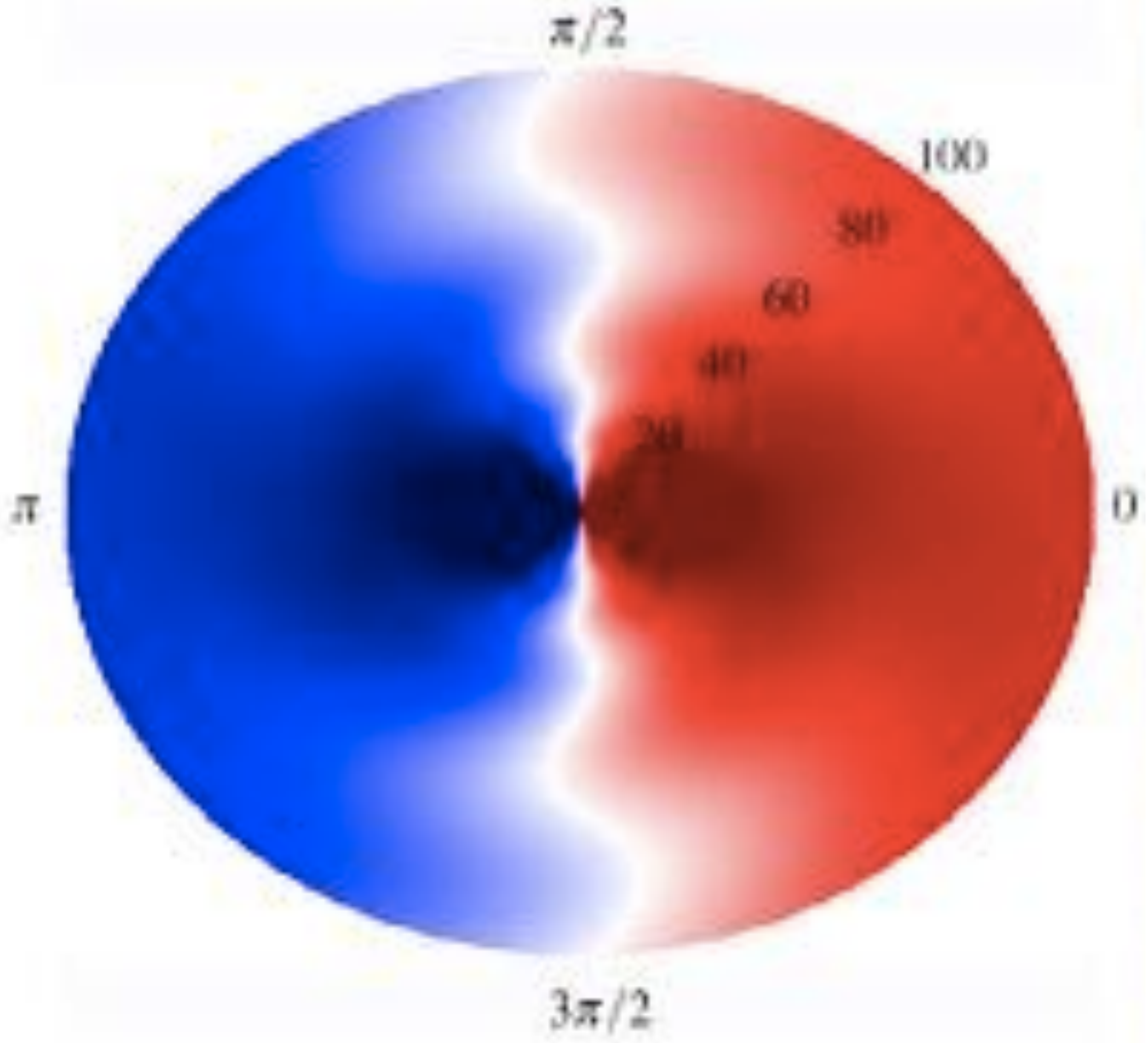
Gas kinematics in GI discs



Terry+ 2021



Hall+ 2020



Longarini+ 2021

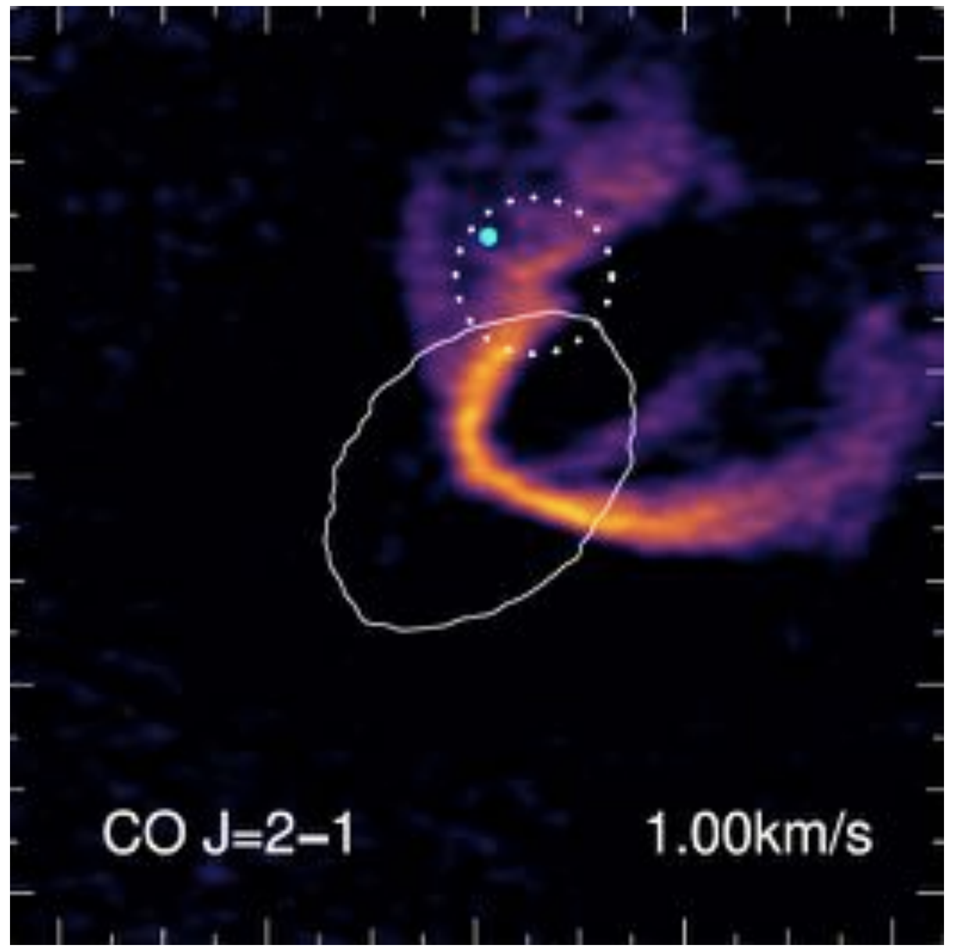
Protoplanetary discs kinematics

- Doppler shift of molecular emission (^{12}CO, ^{13}CO, $C^{18}O$, …) → local gas velocity

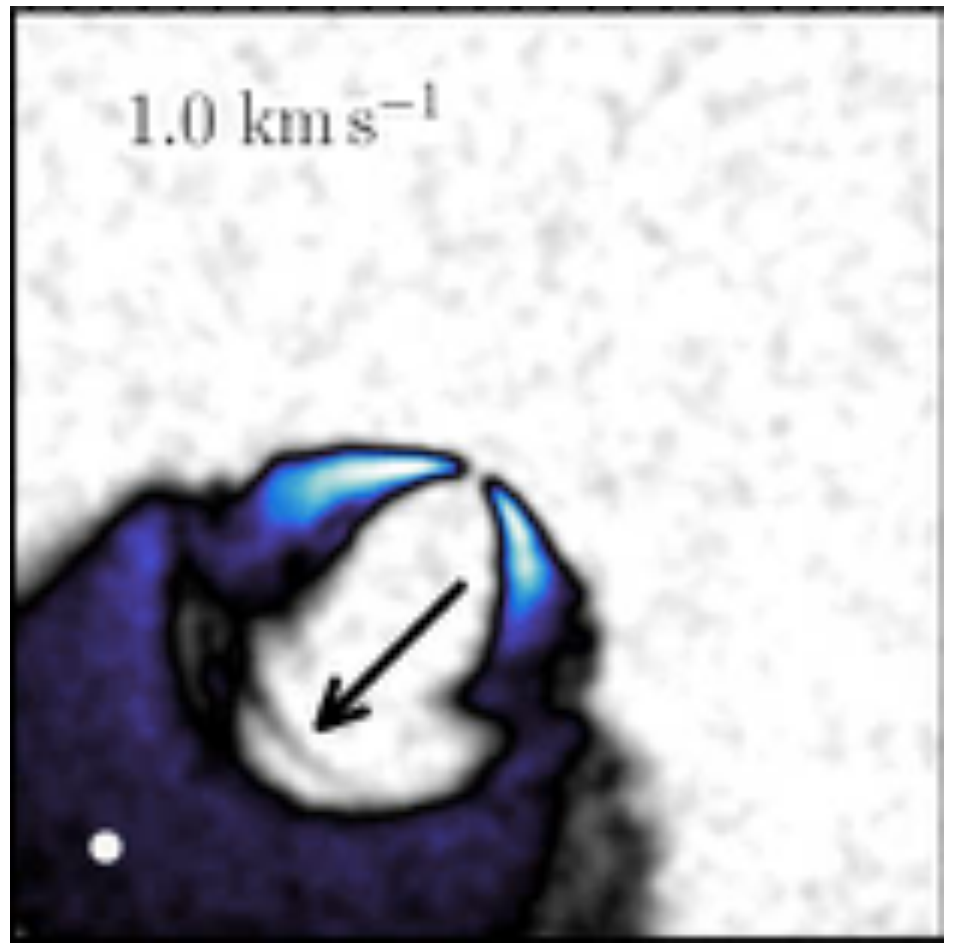
- Because of projection effects, we measure

$$v_{obs} = v_r \sin\phi \sin i + v_\phi \cos\phi \sin i + v_z \cos i$$

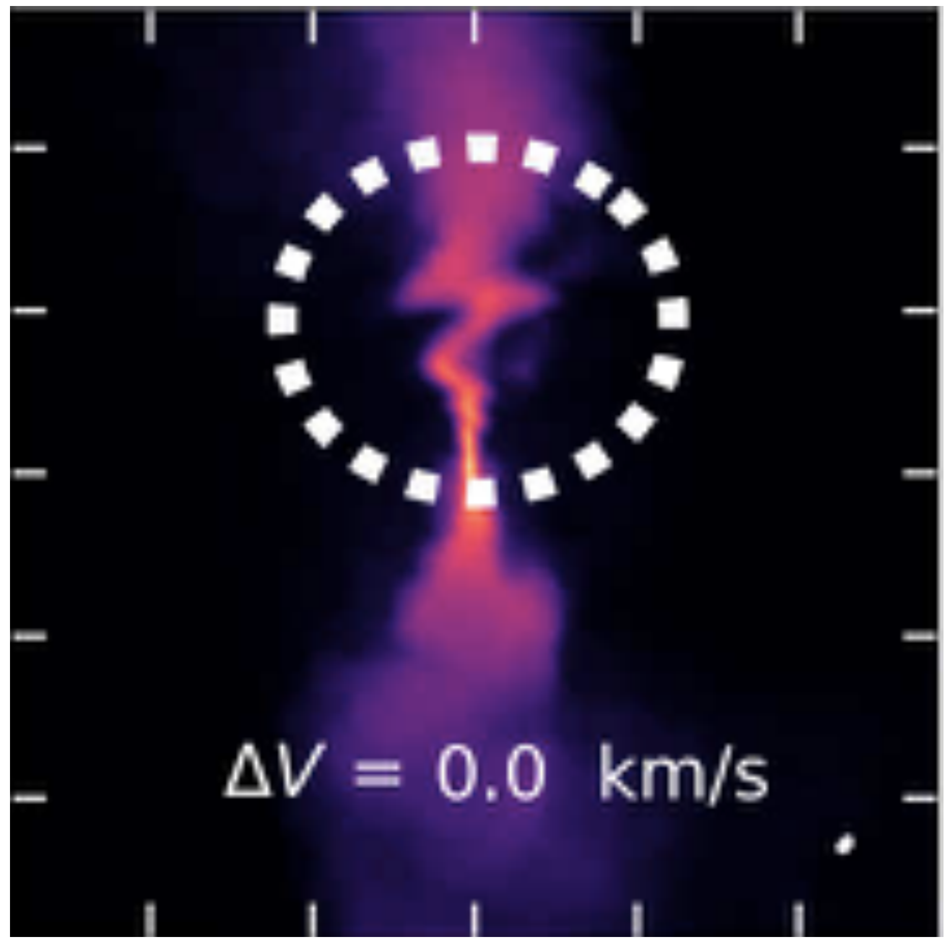
- We observe several deviations from Keplerian rotation that give insights about processes in discs



Pinte+ 2018

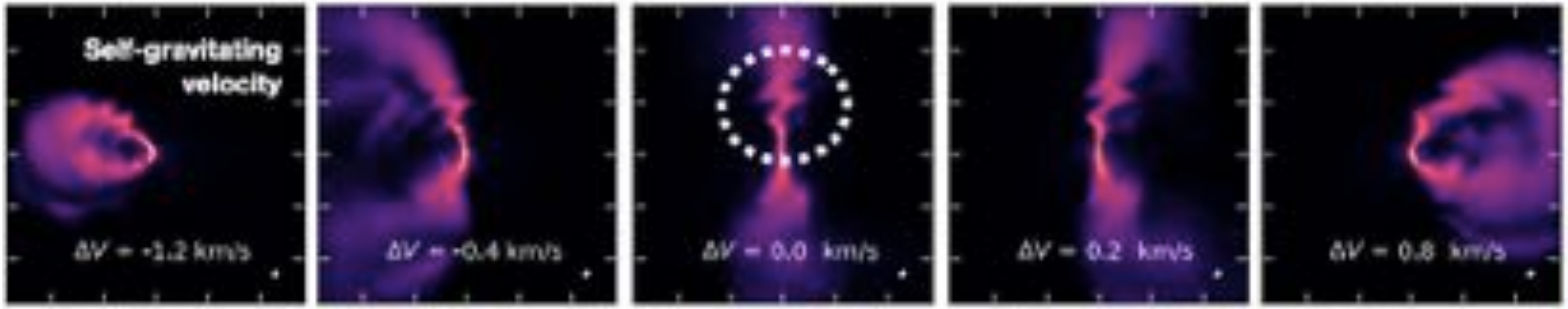


Bae+ 2021



Hall+ 2020

Kinematic deviations induced by GI



- Hall 2020 found that GI has clear kinematic signatures in molecular line observations (PHANTOM + MCFOST) called wiggles
- The nature of the signatures is global and their shape is determined by the spiral characteristics → opening angle, number of arms, amplitude

Analytical "GI Wiggle"

- 1st order perturbations to the fluid equations and we solve for the velocity

$$v_{r1} = \frac{i}{\Delta}\left[(\omega - m\Omega)\partial_r(\Phi_1 + h_1) - \frac{2m\Omega}{r}(\Phi_1 + h_1)\right], \quad v_{\phi 1} = -\frac{1}{\Delta}\left[2B\partial_r(\Phi_1 + h_1) + \frac{m(\omega - m\Omega)}{r}(\Phi_1 + h_1)\right]$$

$$\Delta = \kappa^2 - (\omega - m\Omega)^2$$

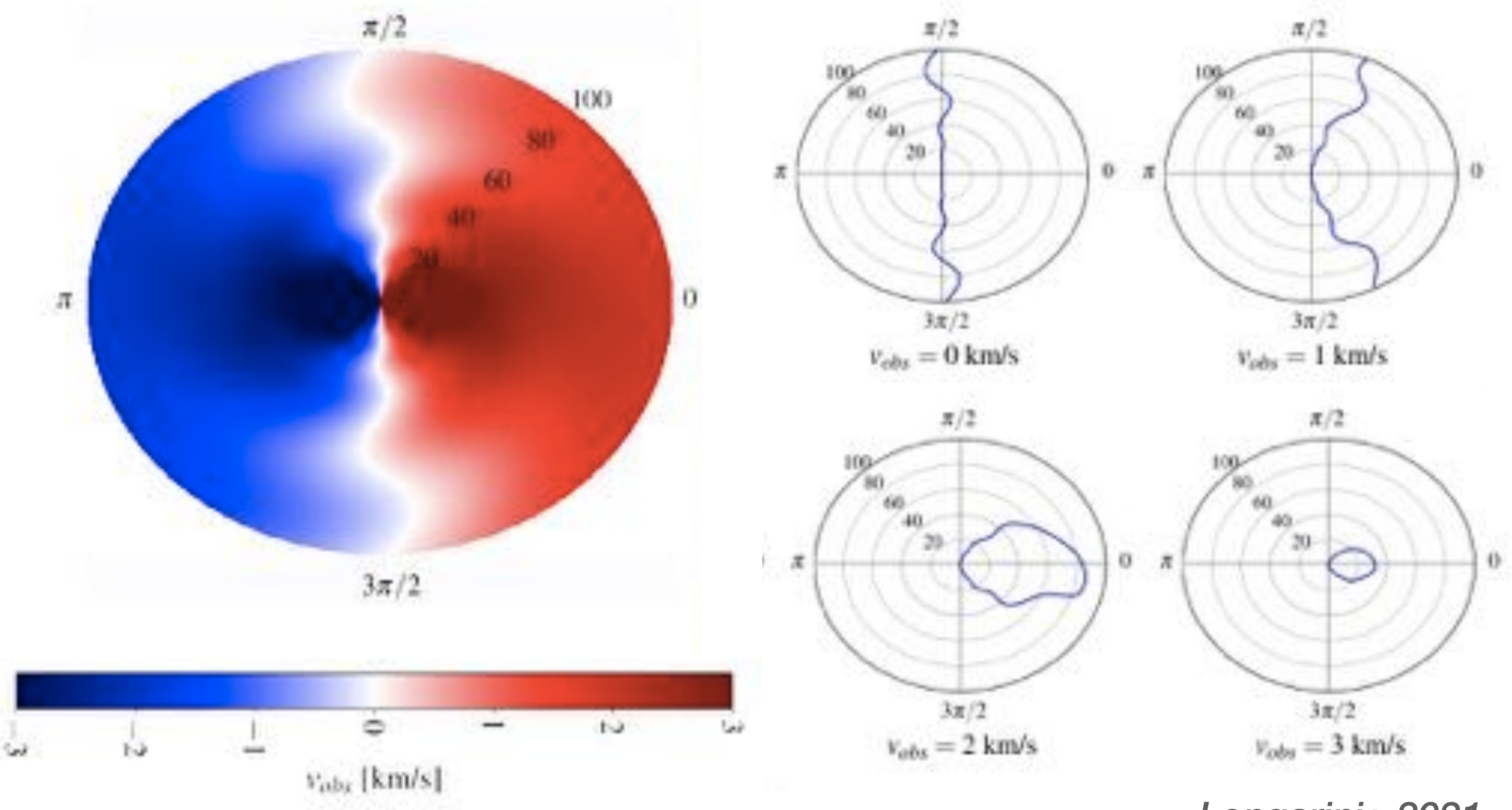
- Hypotheses:
  - Thin disc (r, ϕ)
  - Marginally unstable disc $Q \simeq 1$
  - Thermal saturation $\delta\Sigma \propto \beta_c^{-1/2}$
  - Nearly Keplerian disc $\kappa \simeq \Omega$

$$\delta v_r = 2im\chi\beta_c^{-1/2}\left(\frac{M_d}{M_\star}\right)^2 v_k$$

$$\delta v_\phi = -\frac{i\chi\beta_c^{-1/2}}{2}\left(\frac{M_d}{M_\star}\right) v_k$$

Analytical “GI Wiggle”

$$v_{obs} = v_r \sin\phi \sin i + v_\phi \cos\phi \sin i + v_z \cos i$$



Longarini+ 2021

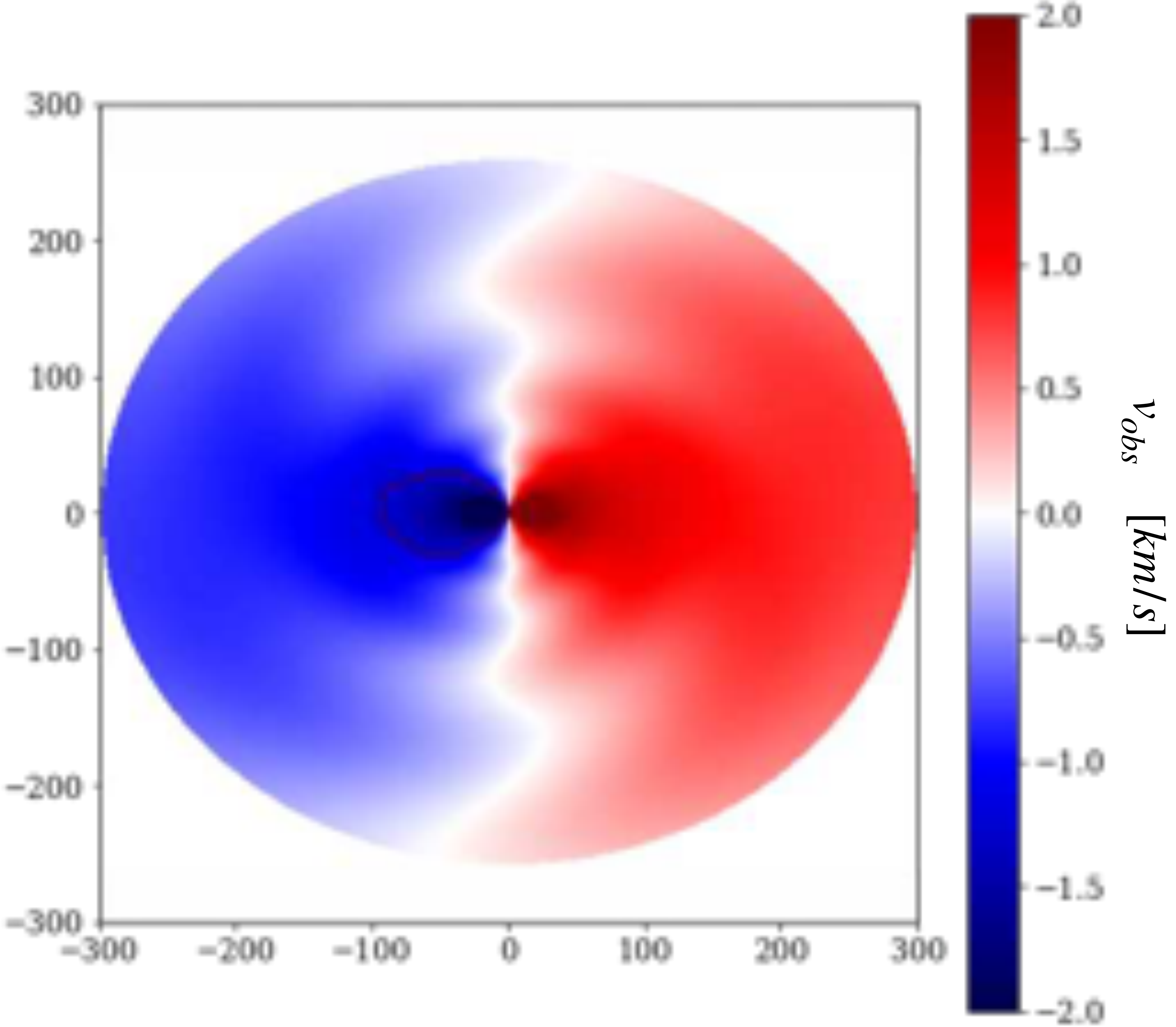
- The perturbation is global rather than localised
- The central channel is not a straight line

$M_\star = 0.5 M_\odot$

$M_d = 0.1 M_\odot$

$\beta = 7$

$i = 30^\circ$



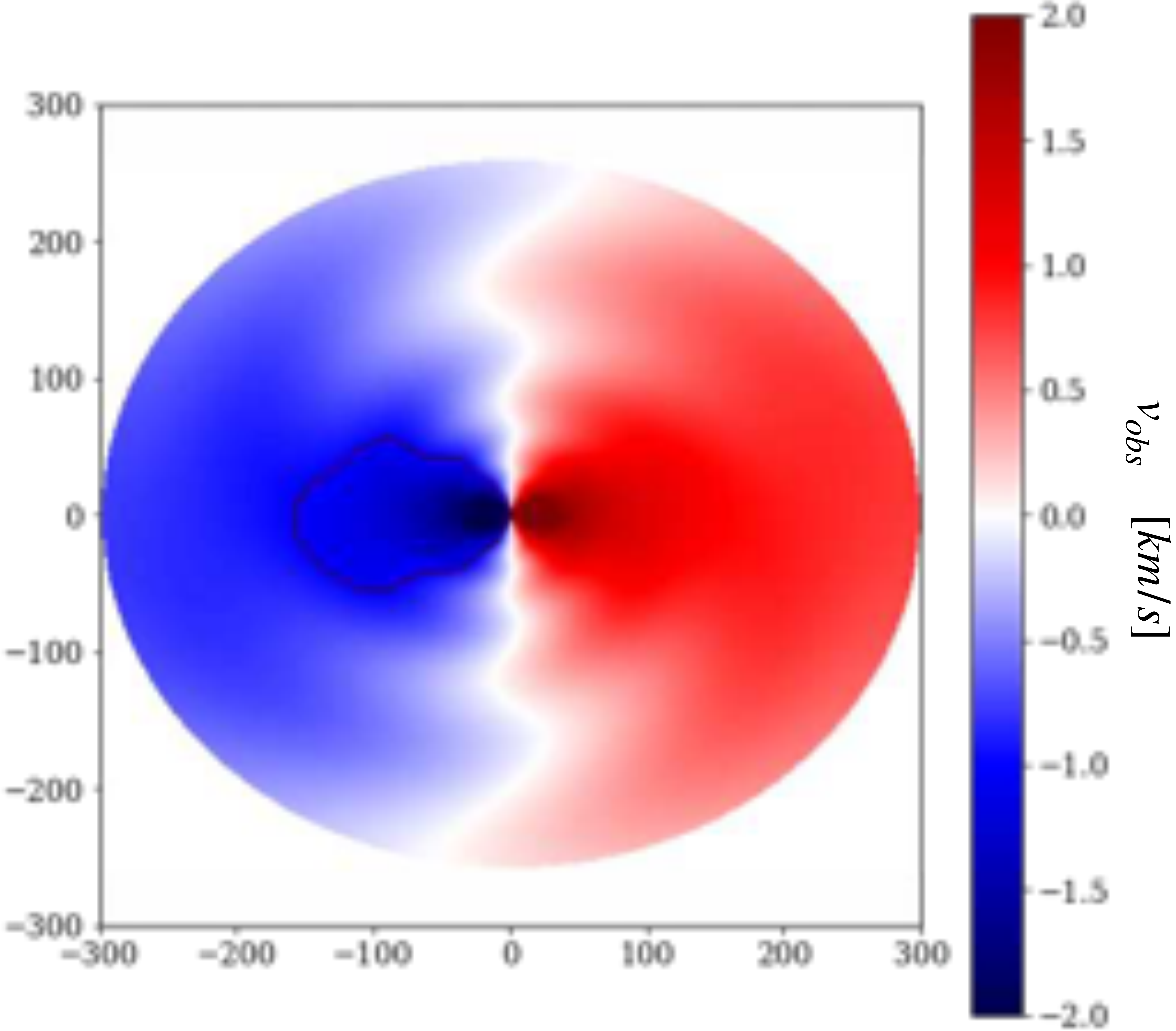
$v_{obs} = -1.25 km/s$

$M_\star = 0.5 M_\odot$

$M_d = 0.1 M_\odot$

$\beta = 7$

$i = 30^\circ$



$v_{obs} = -1 km/s$

$M_\star = 0.5M_\odot$
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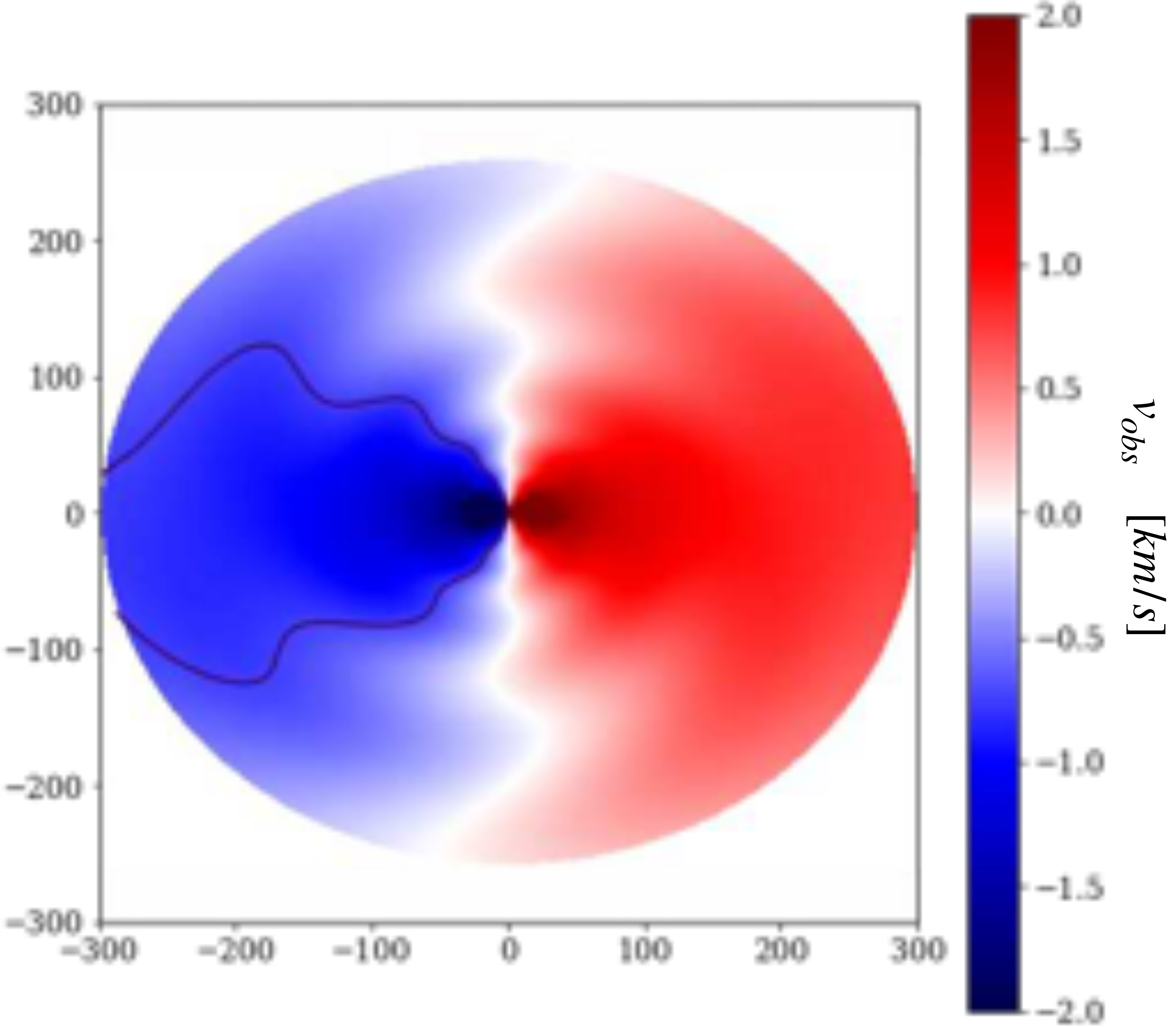
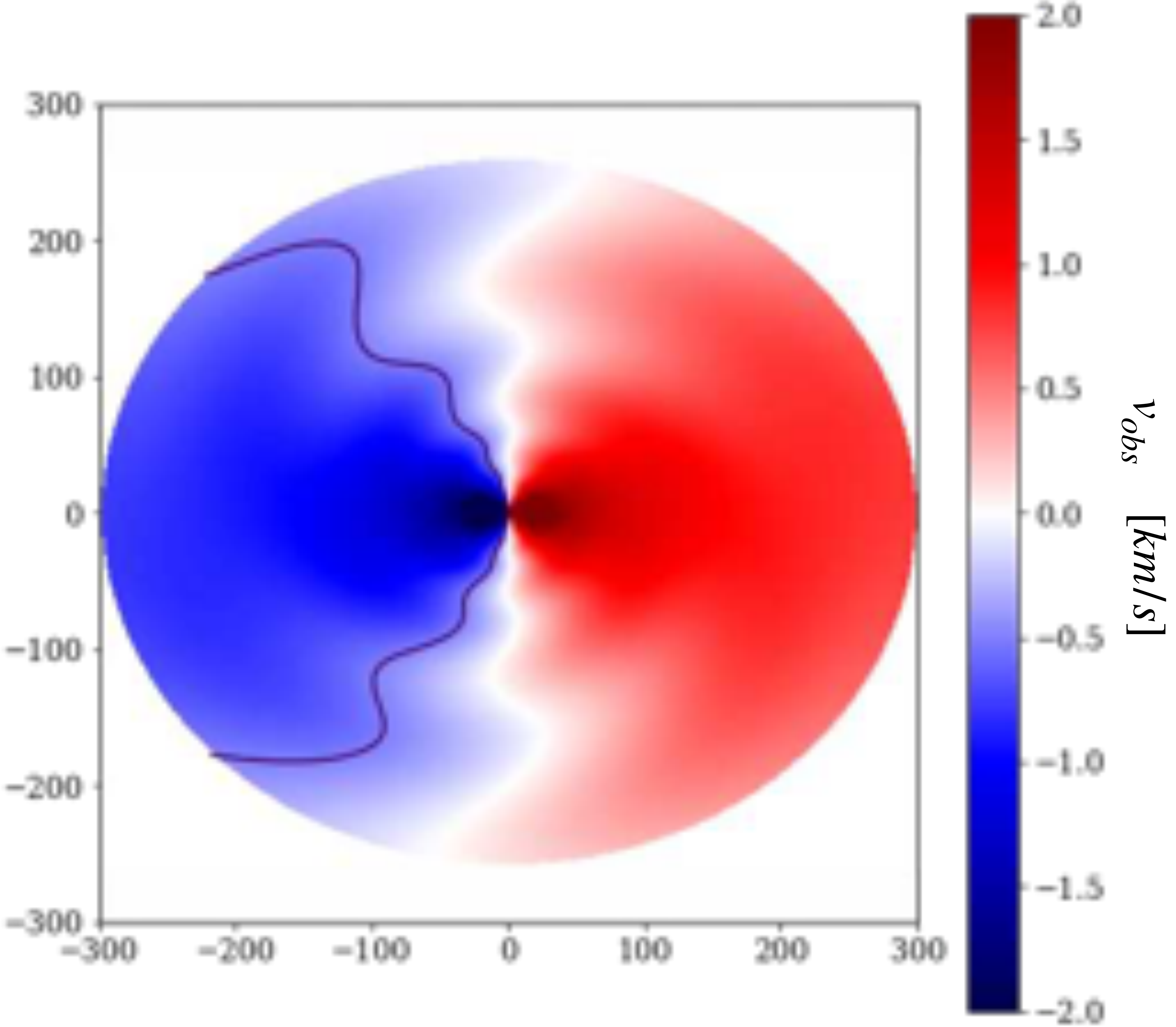
$\beta = 7$
$i = 30^\circ$



$v_{obs} = -0.75 km/s$

$M_\star = 0.5 M_\odot$
$M_d = 0.1 M_\odot$
$\beta = 7$
$i = 30^\circ$



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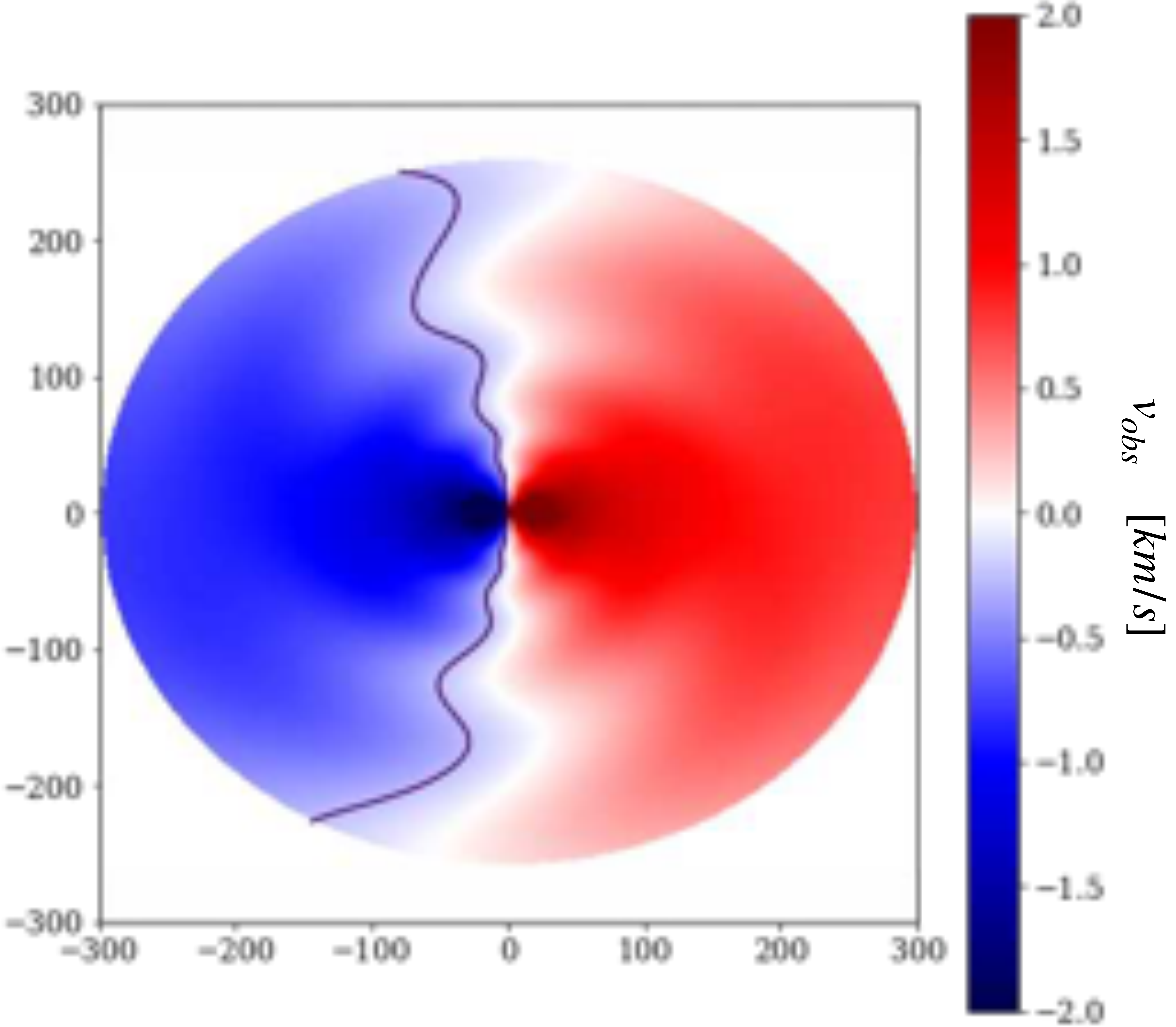

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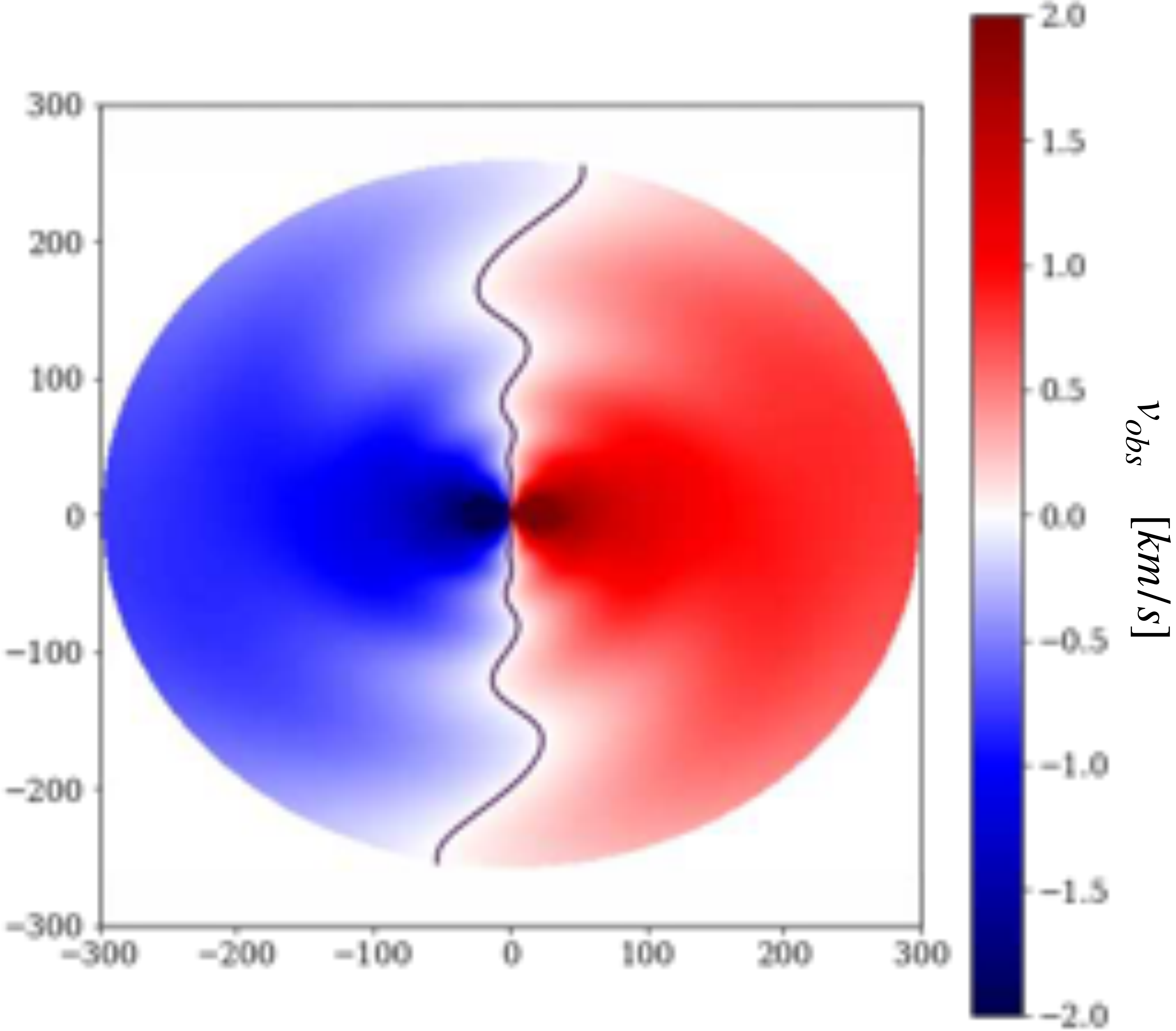
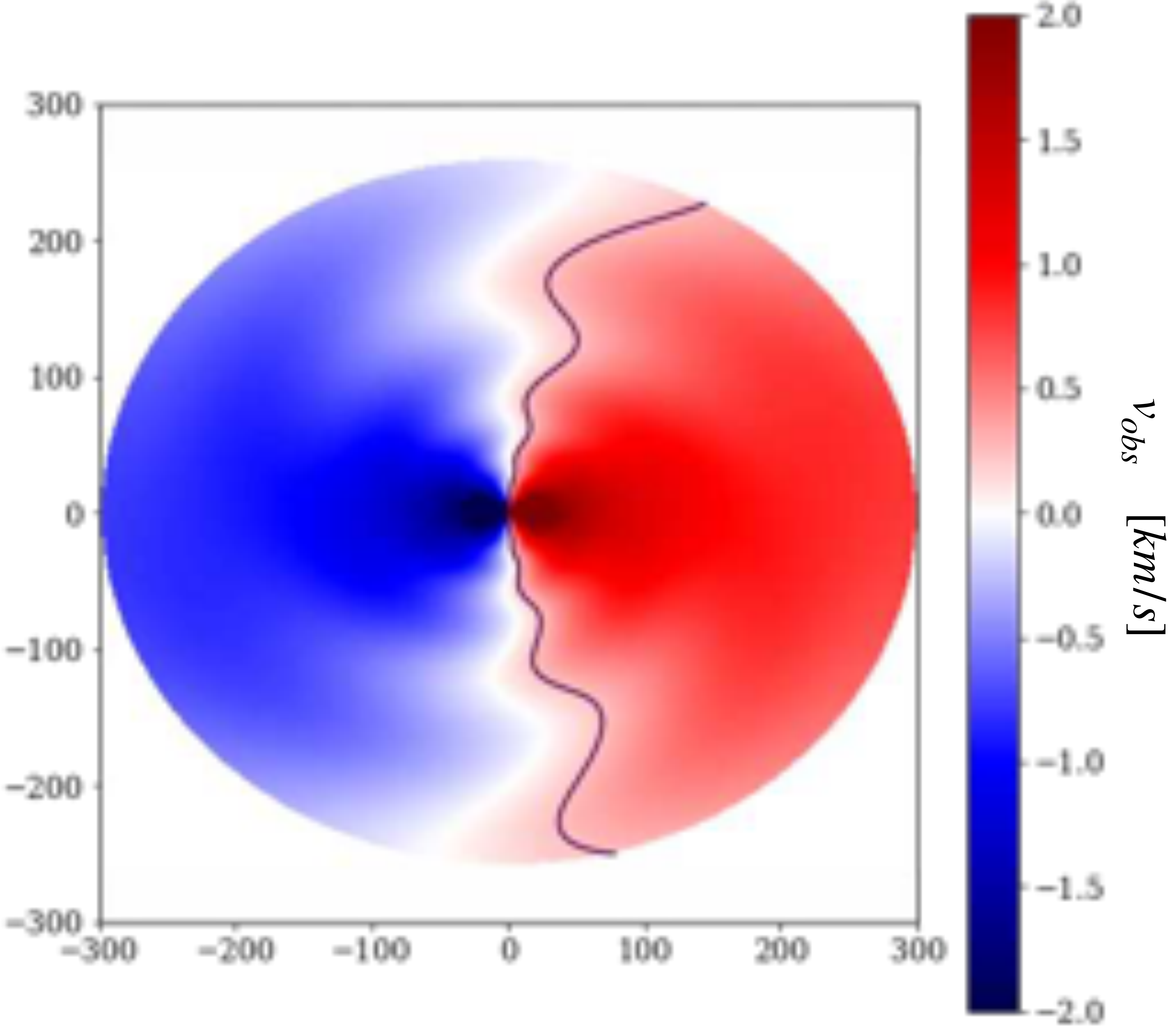


$v_{obs} = 0 km/s$

$M_\star = 0.5 M_\odot$
$M_d = 0.1 M_\odot$
$\beta = 7$
$i = 30^\circ$



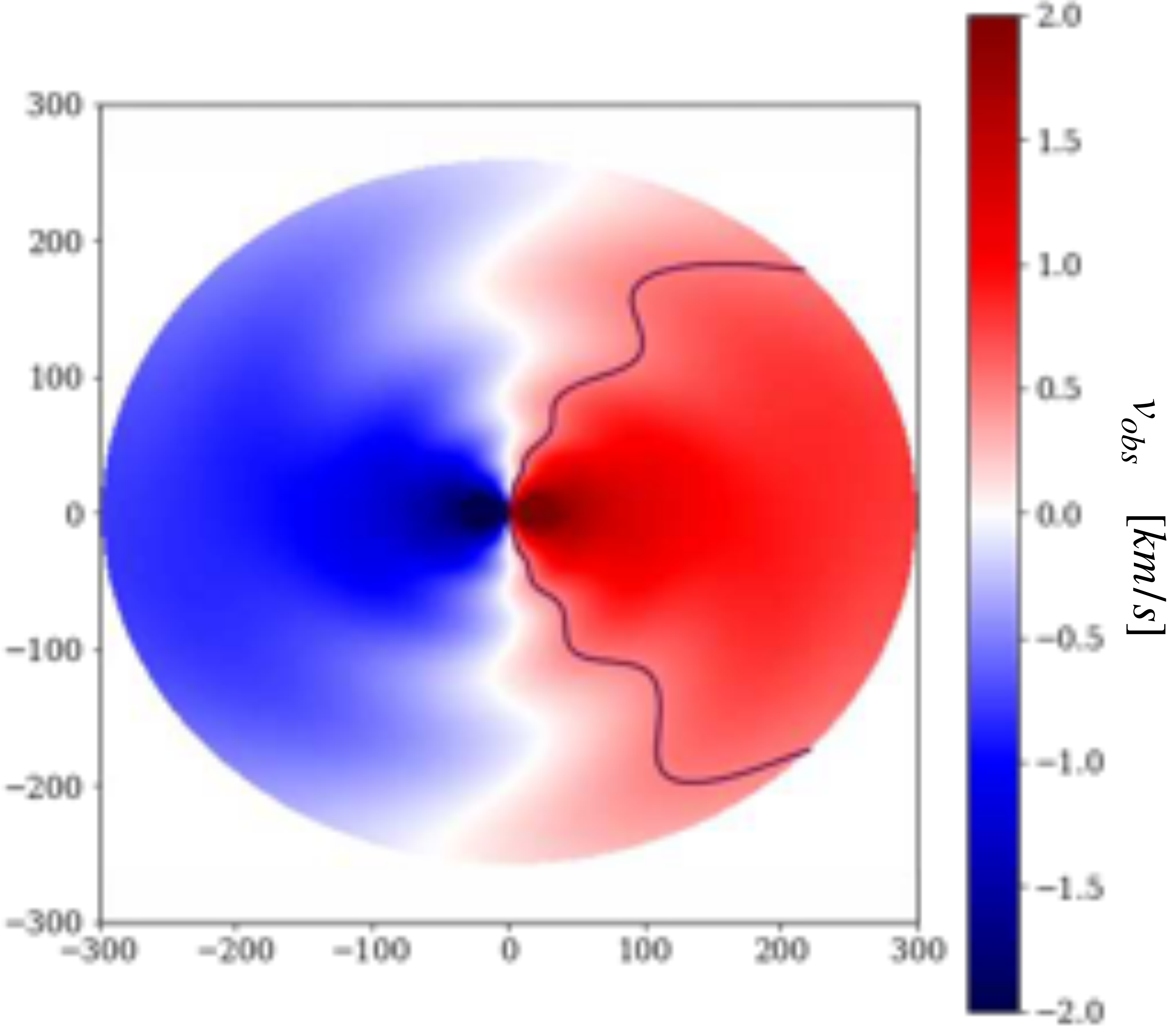
$v_{obs} = 0.25 km/s$

$M_\star = 0.5 M_\odot$

$M_d = 0.1 M_\odot$

$\beta = 7$

$i = 30^\circ$



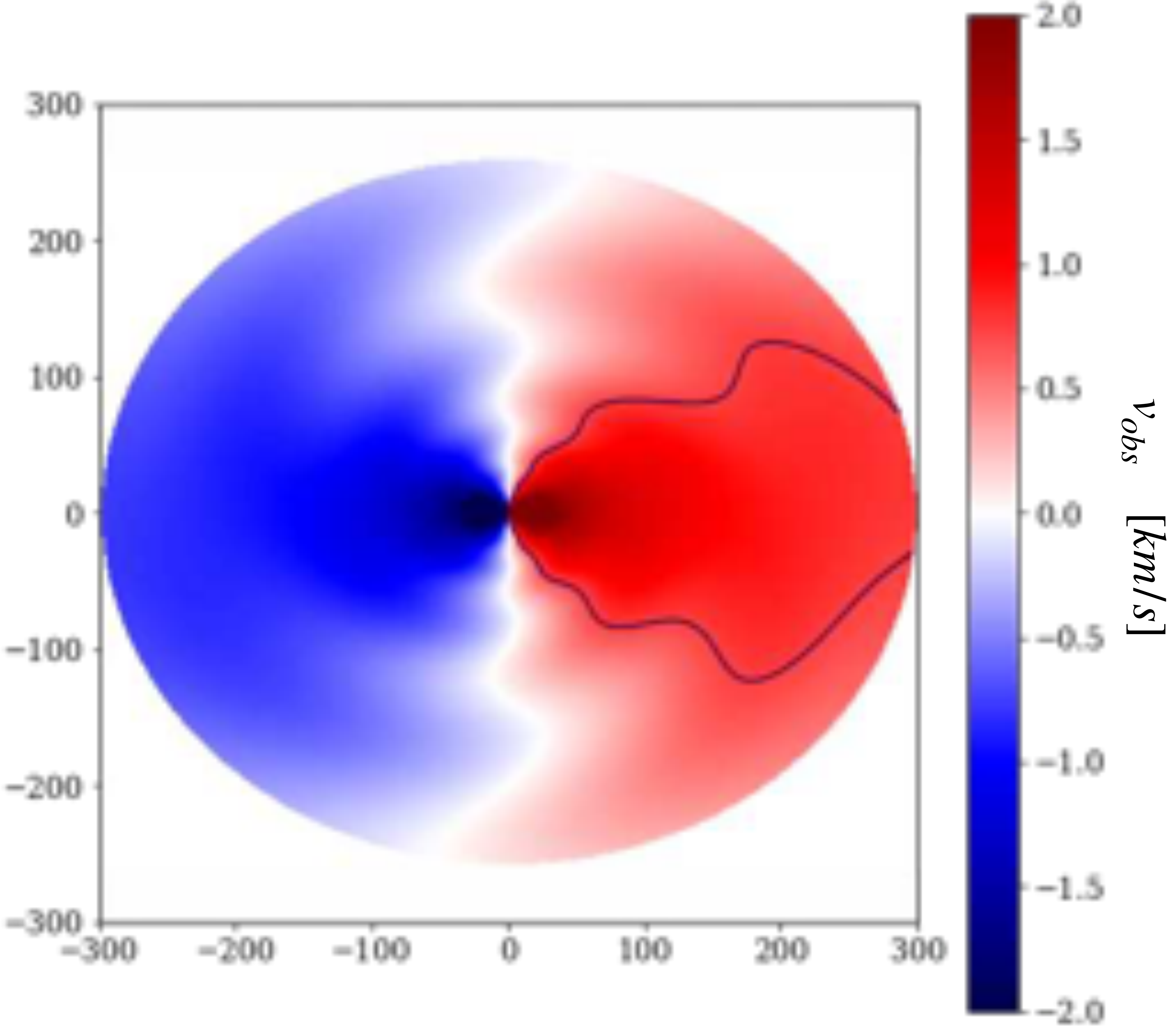
$$v_{obs} = 0.5 km/s$$

$M_\star = 0.5 M_\odot$

$M_d = 0.1 M_\odot$

$\beta = 7$

$i = 30^\circ$



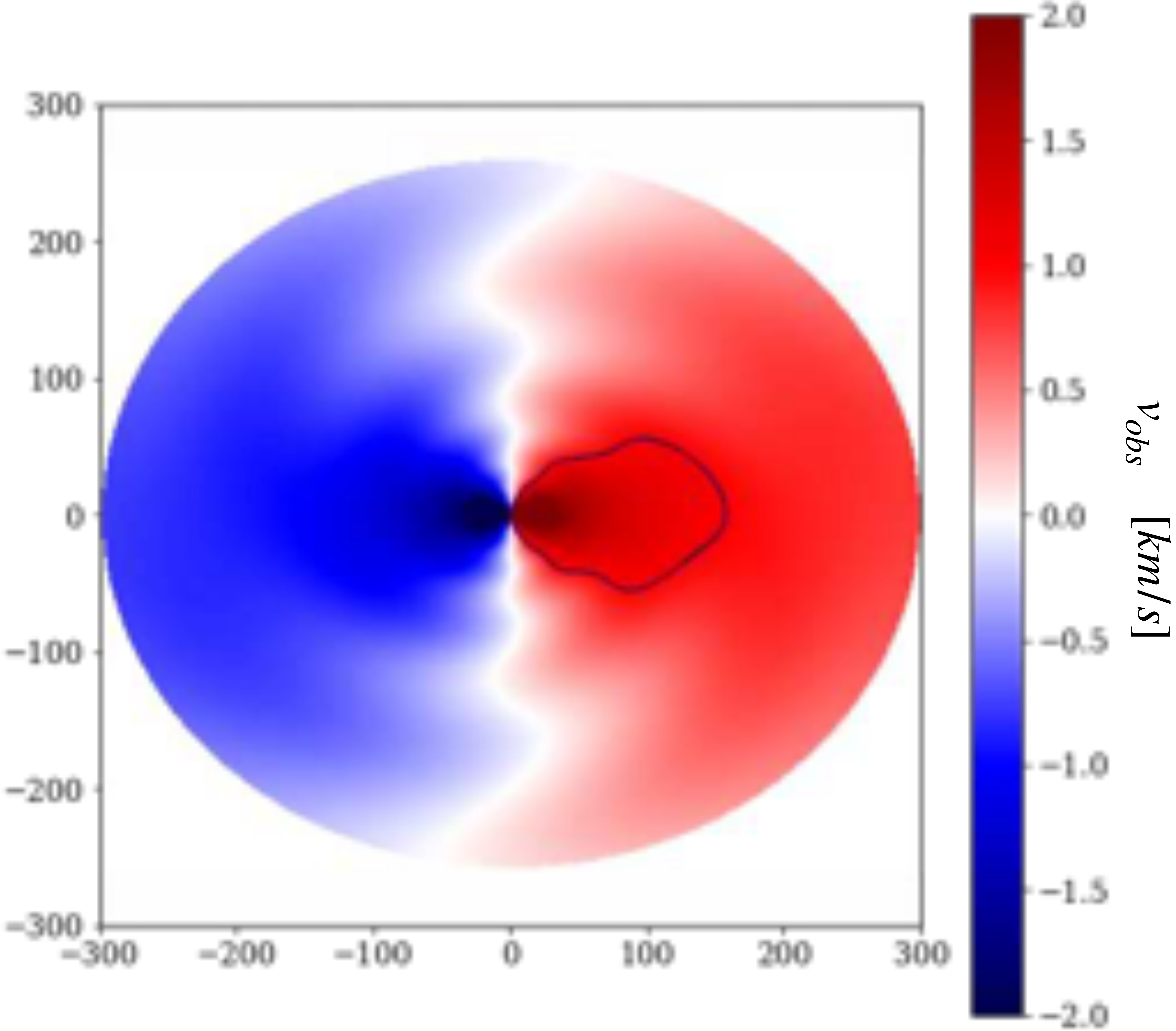
$v_{obs} = 0.75 km/s$

$M_\star = 0.5 M_\odot$

$M_d = 0.1 M_\odot$

$\beta = 7$

$i = 30^\circ$



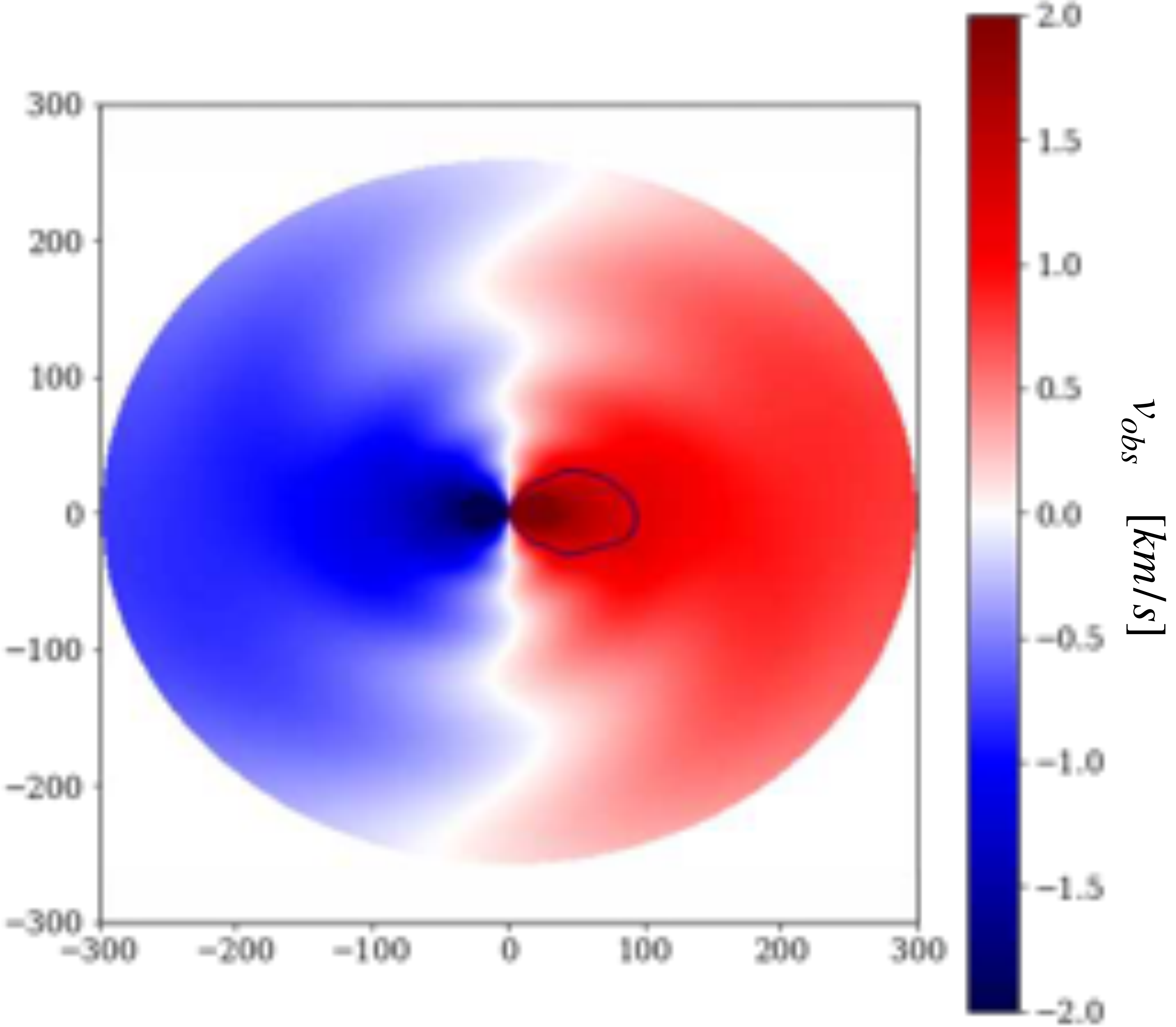
$v_{obs} = 1 km/s$

$M_\star = 0.5 M_\odot$

$M_d = 0.1 M_\odot$

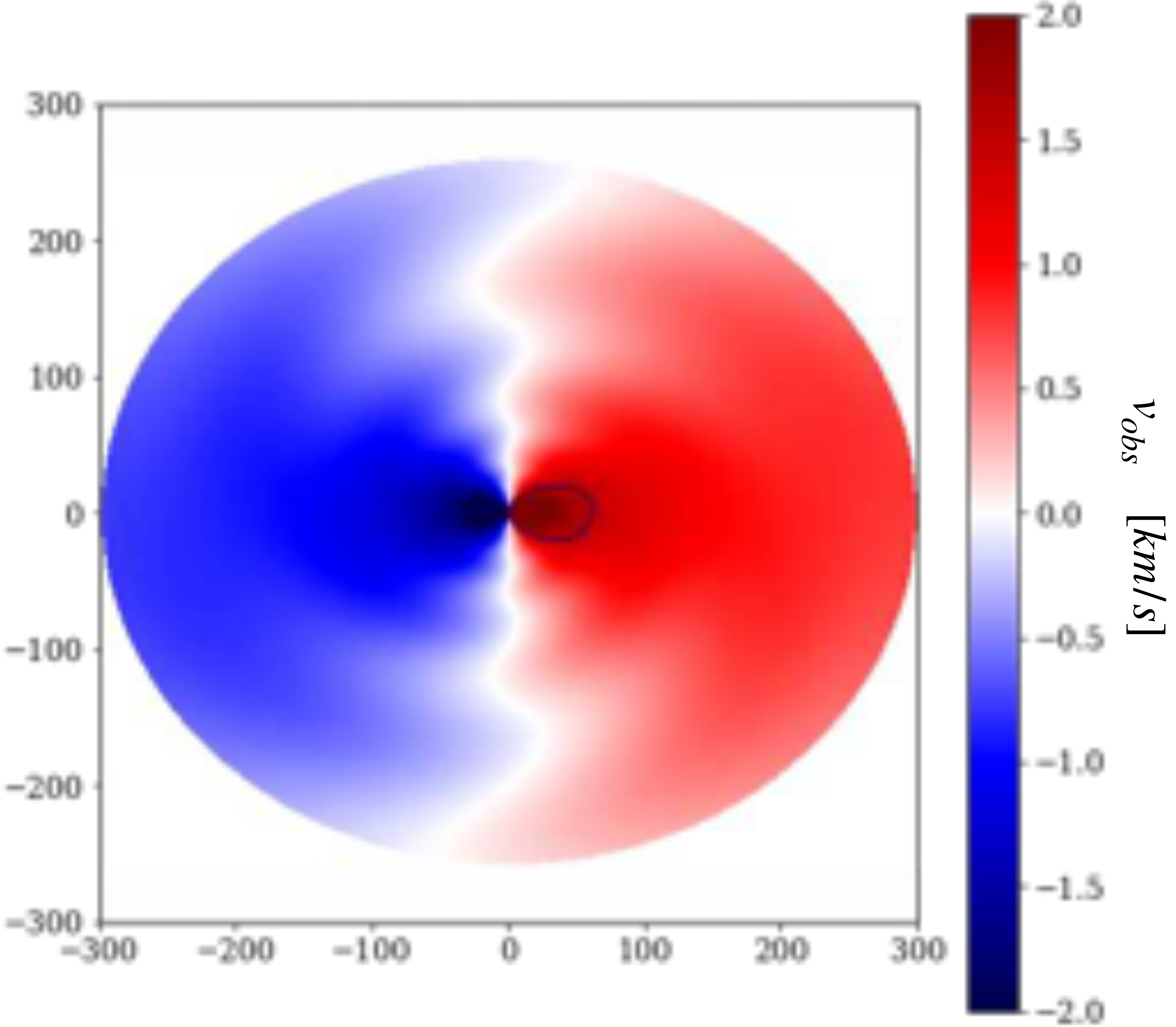
$\beta = 7$

$i = 30^\circ$



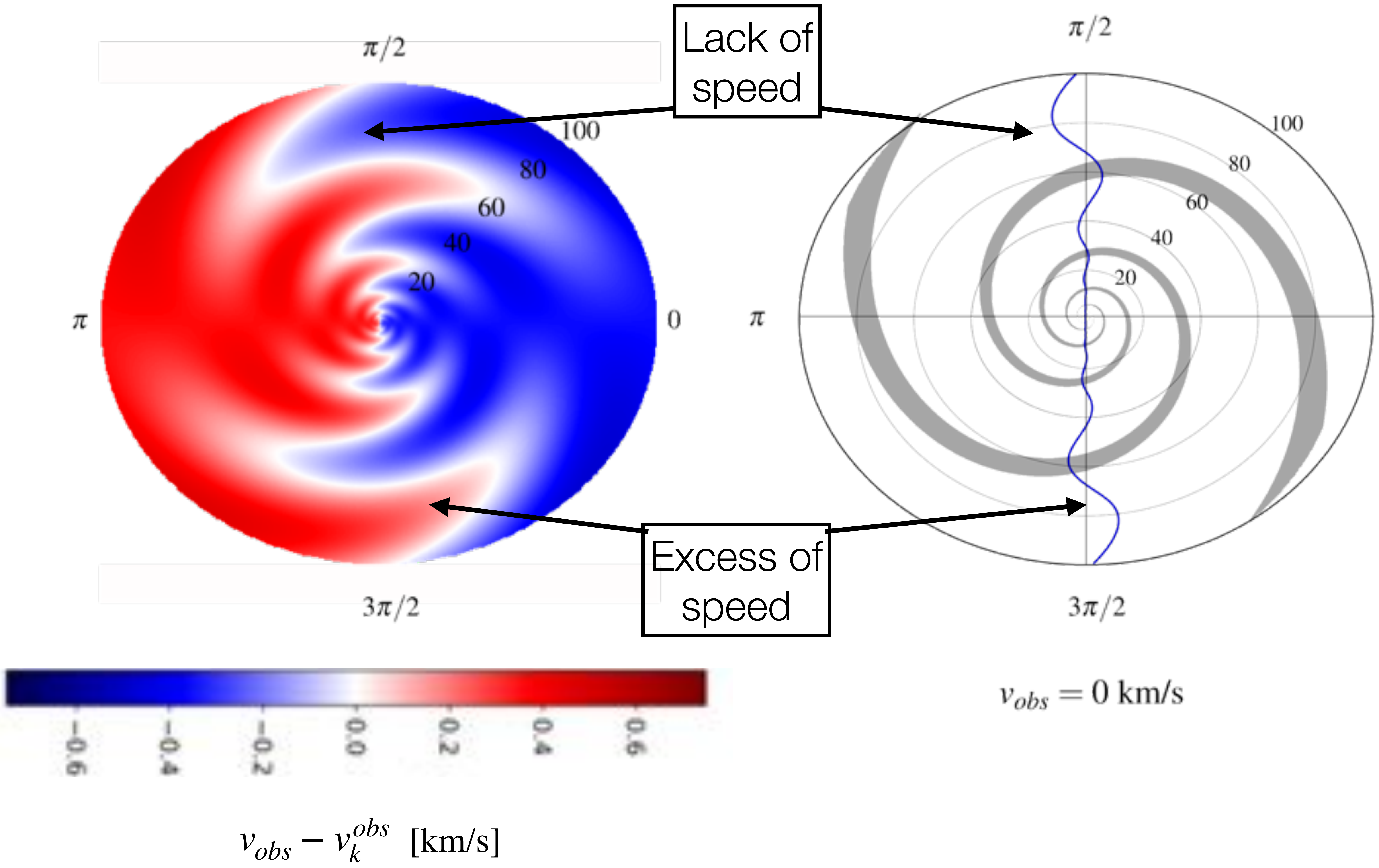
$v_{obs} = 1.25 km/s$

$M_\star = 0.5 M_\odot$
$M_d = 0.1 M_\odot$
$\beta = 7$
$i = 30^\circ$

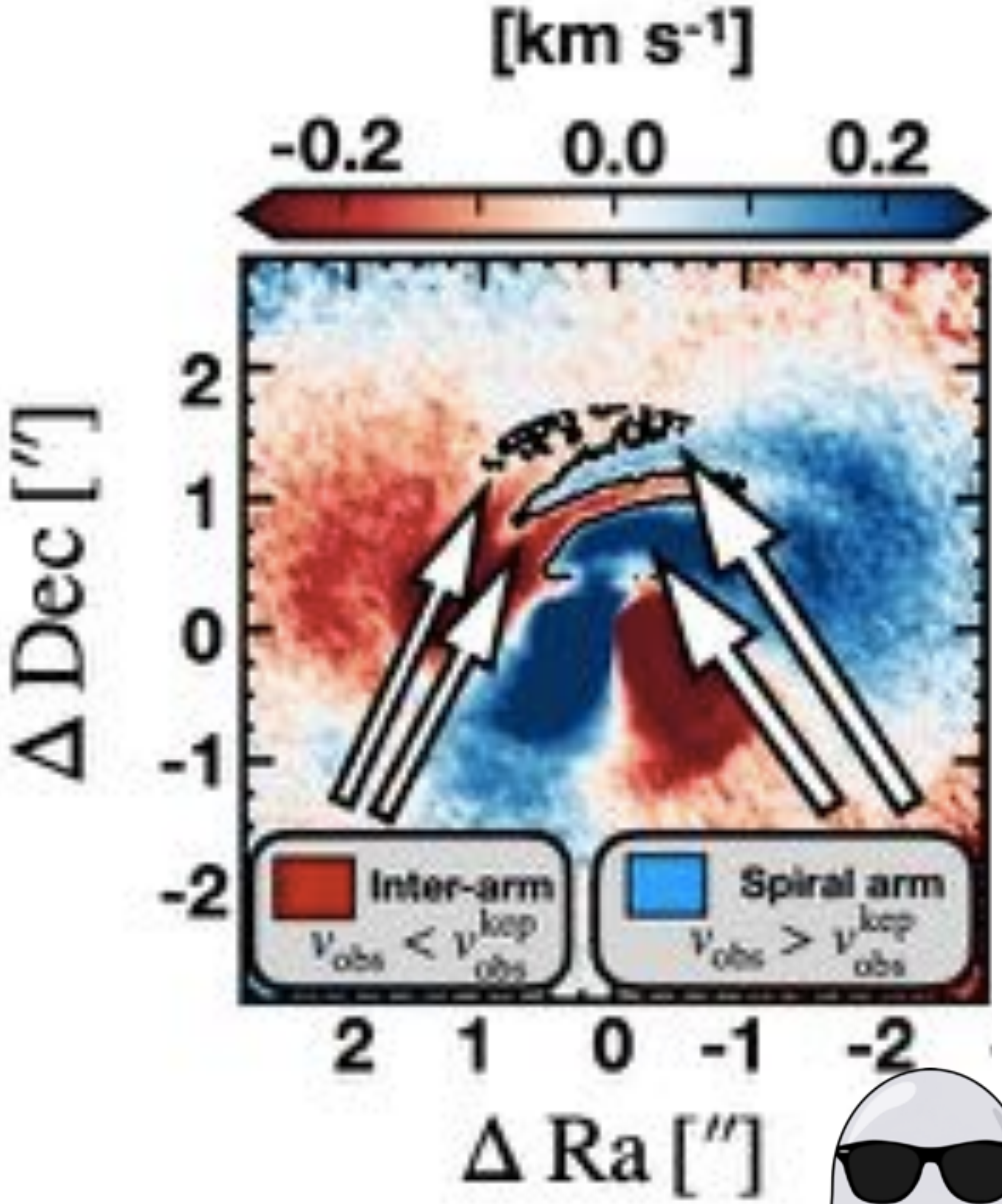


$$v_{obs} = 1.5 km/s$$

Analytical "GI Wiggle"

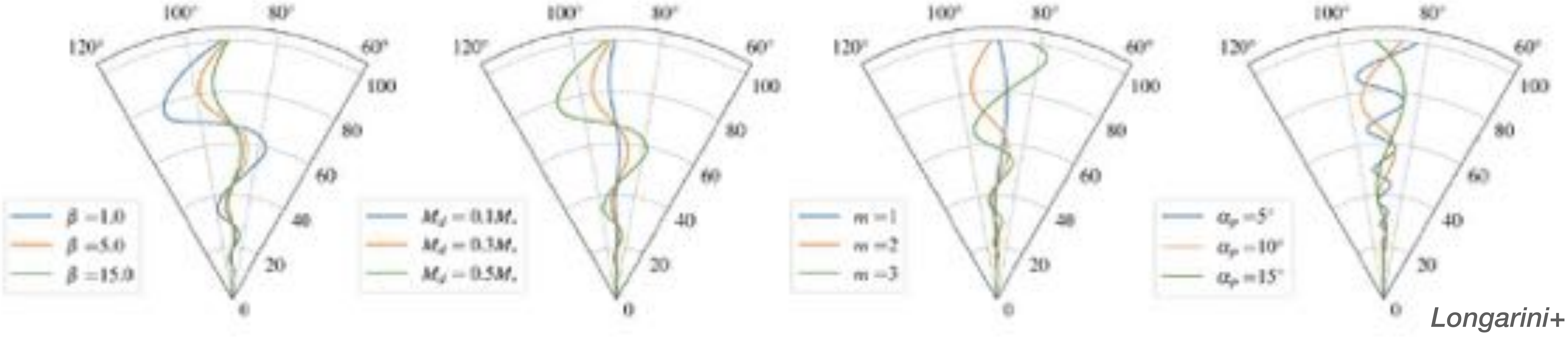


Longarini+ 2021



Hall+ 2020

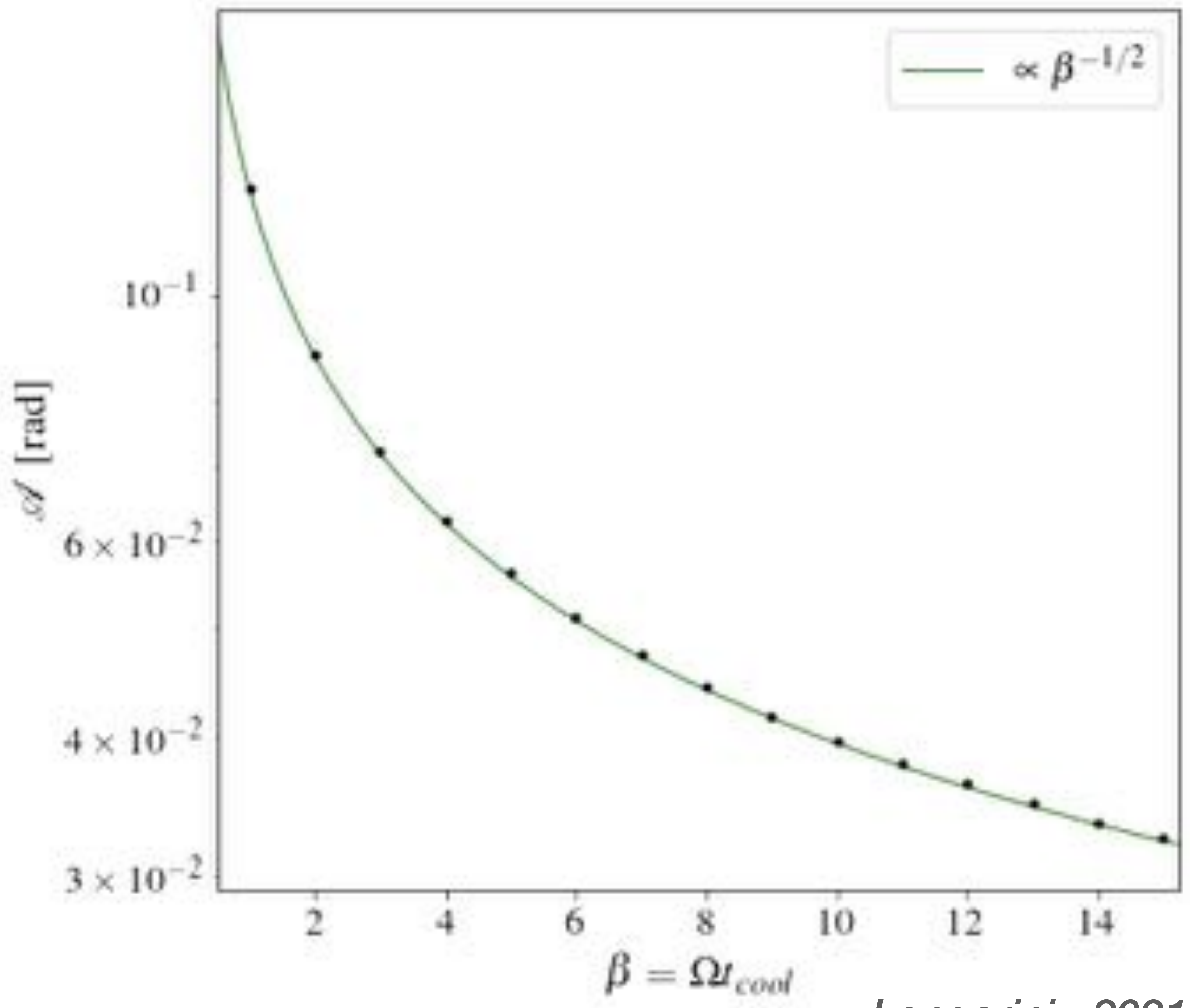
Shape of the perturbation



Longarini+ 2021

- Amplitude decreases with the cooling factor
- Amplitude increases with disc mass *Terry+ 2021*
- Frequency increases with azimuthal wavenumber
- Frequency decreases with opening angle

Constraining the cooling factor

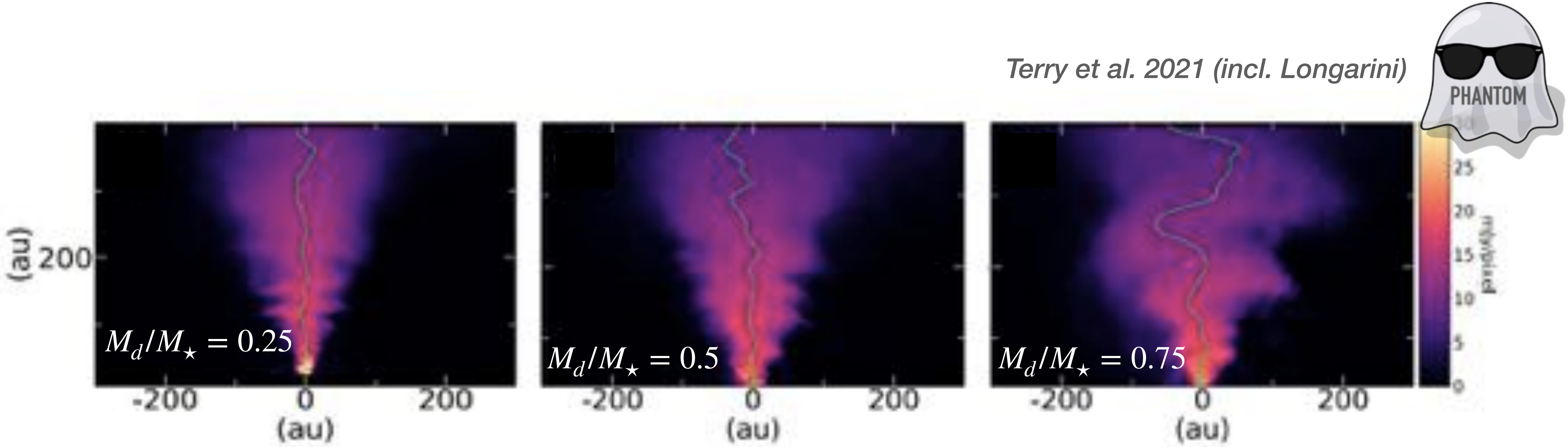


Longarini+ 2021

- Degeneracy between mass and cooling. How to break? Rotation curve! *Veronesi+ 2021*
- Amplitude of the wiggle scaling $\mathscr{A} \propto \beta_c^{-1/2}$
  $\rightarrow \delta v_r, \delta v_\phi \propto \beta_c^{-1/2}$
- Knowing the mass, we can constrain the cooling trough the amplitude

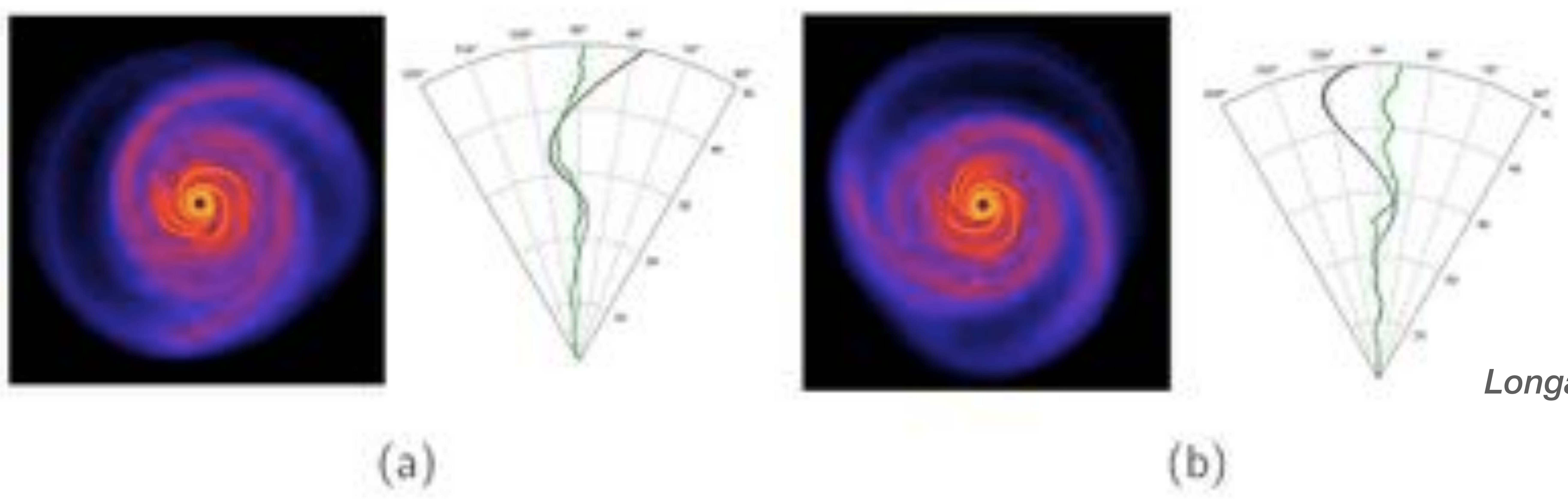
Testing the model

Terry et al. 2021 (incl. Longarini)



- The amplitude of the wiggle increases with the disc mass **as expected**
- Relationship between frequency and # spiral arms + pitch angle **as expected**

Testing the model



Longarini + 2021

- Constraining protoplanetary disc cooling: (disc mass from rotation curve)
  Simulation = 8, Model = 9

- The shape of the wiggle is retrieved → attention to the orientation of the spiral, dissipation due to viscosity

An actual case: Elias 2-27

Longarini, Clarke, Lodato et al in prep

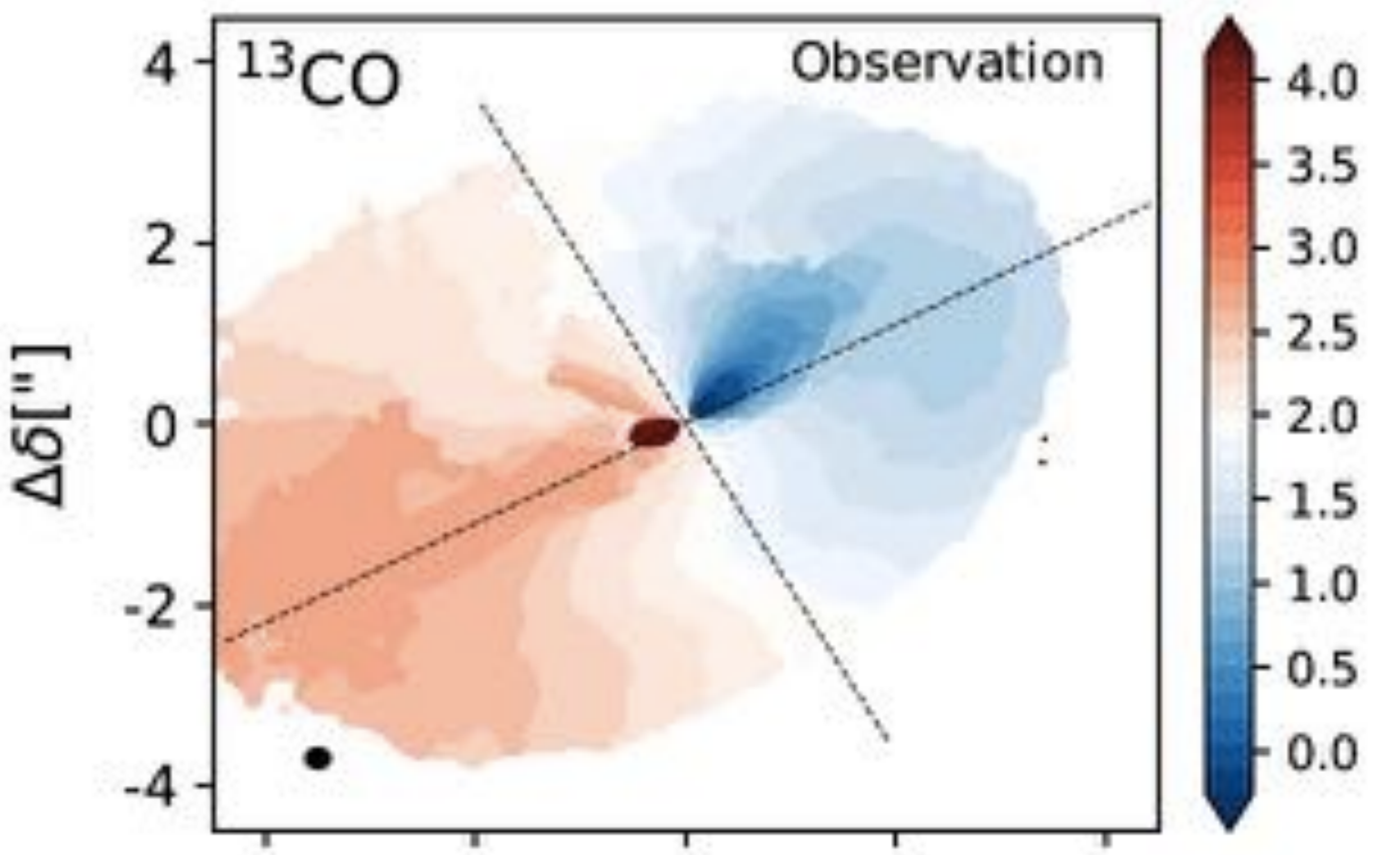
Elias 2-27 is a self gravitating disc:

- $M_\star = 0.4 M_\odot, M_d = 0.08 M_\odot$

  (Veronesi+ 2021)

- It shows deviations from Keplerian motion in velocity field

Constraining cooling - angular momentum transport through kinematics

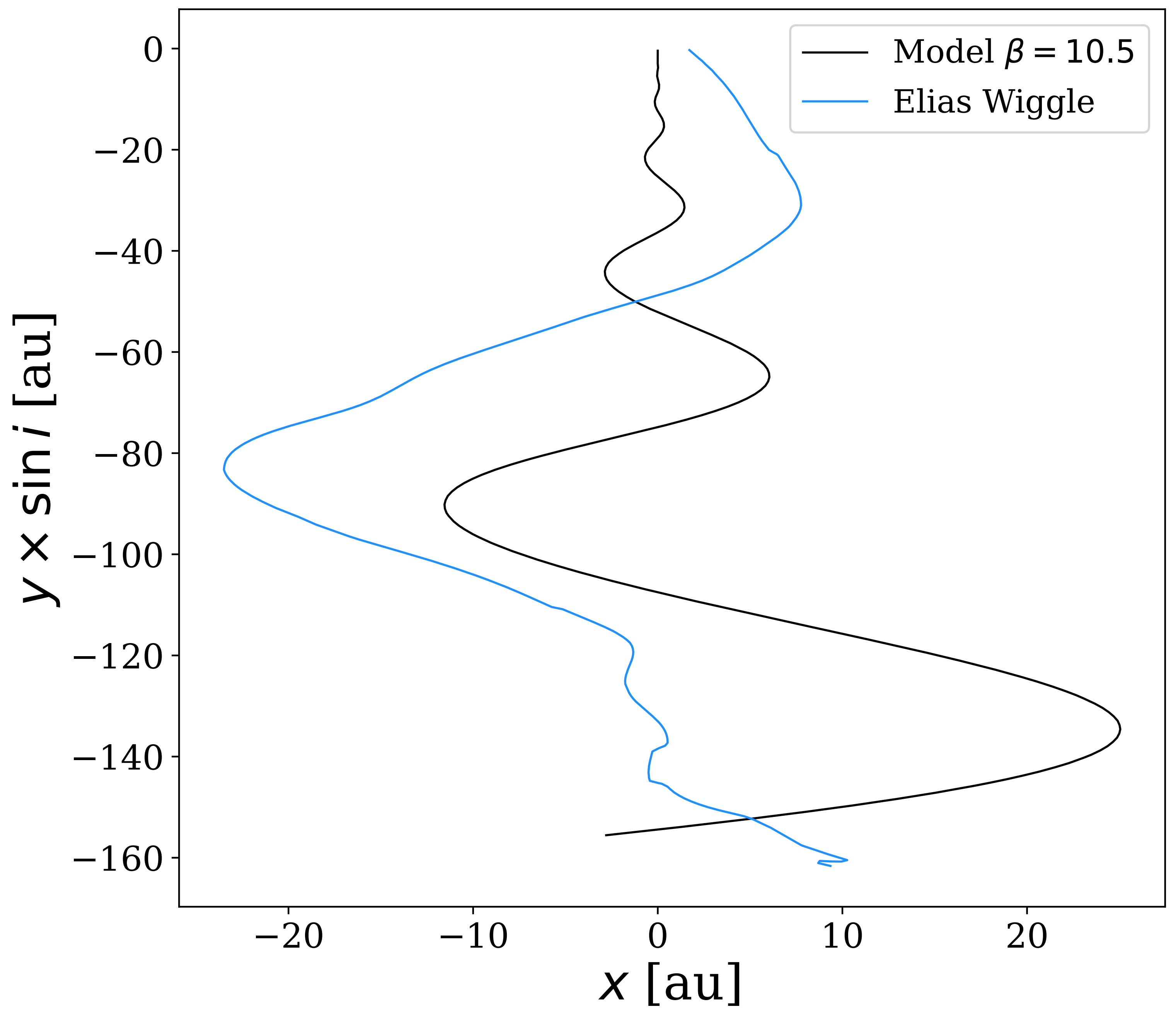


13CO velocity field

(Paneque-Carreño+ 2021)

$$\beta \simeq 10.5$$

$$\alpha_{GI} = 0.038$$



An actual case: Elias 2-27

$\alpha_{GI} = 0.038$ into the self-similar solution

$$\rightarrow \dot{M}_\star = -\frac{3\alpha}{2}\left(\frac{H}{R}\right)^2_{R_c} M_d \Omega_c$$

Measured accretion rate:

$\log \dot{M}_{Elias}[M_\odot/yr] = -7.2 \pm 0.5$

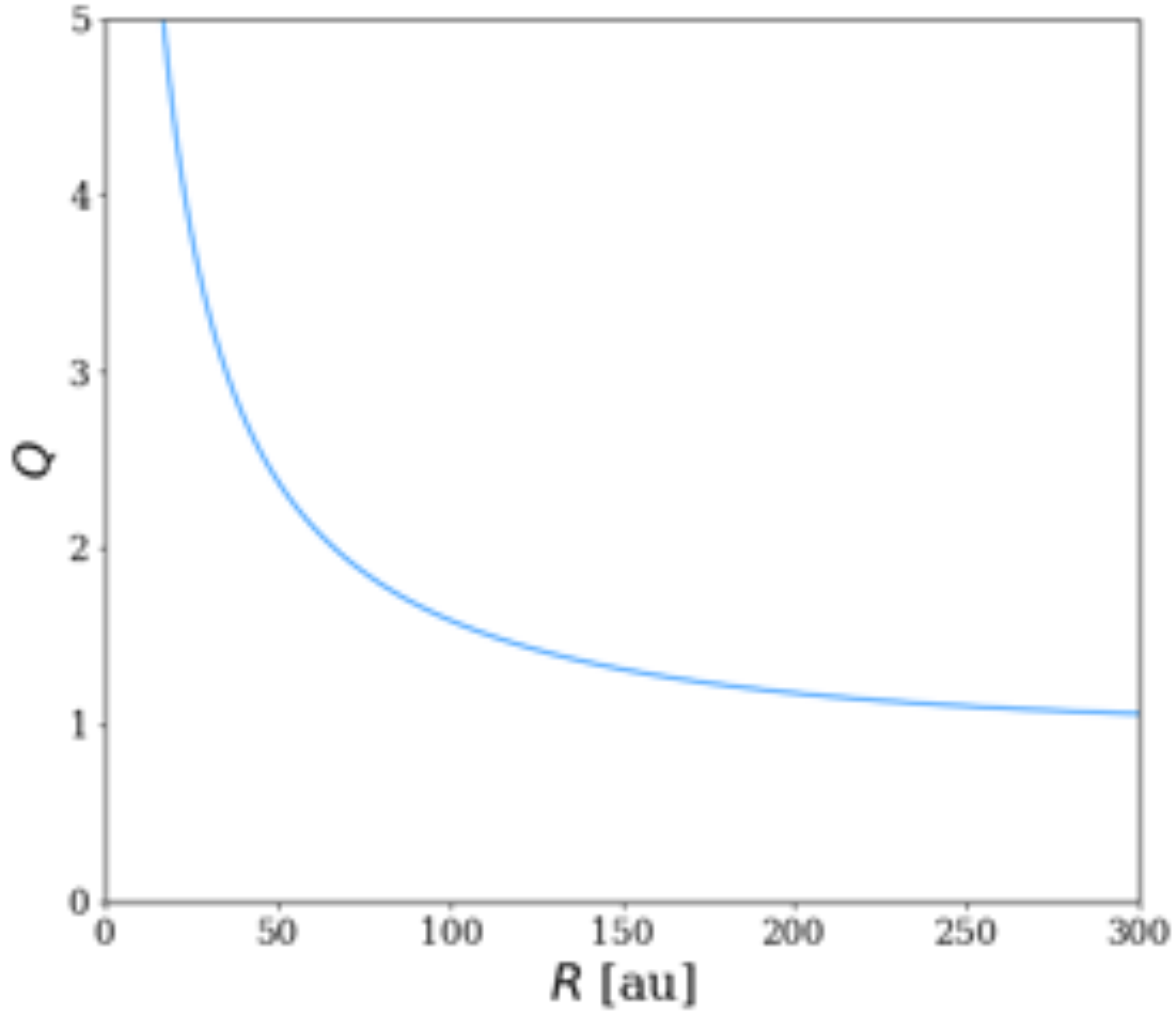
Estimated from gravito-turbulence:

$\log \dot{M}_{GI}[M_\odot/yr] = -6.9 \pm 0.16$

(Error from star and disc masses)

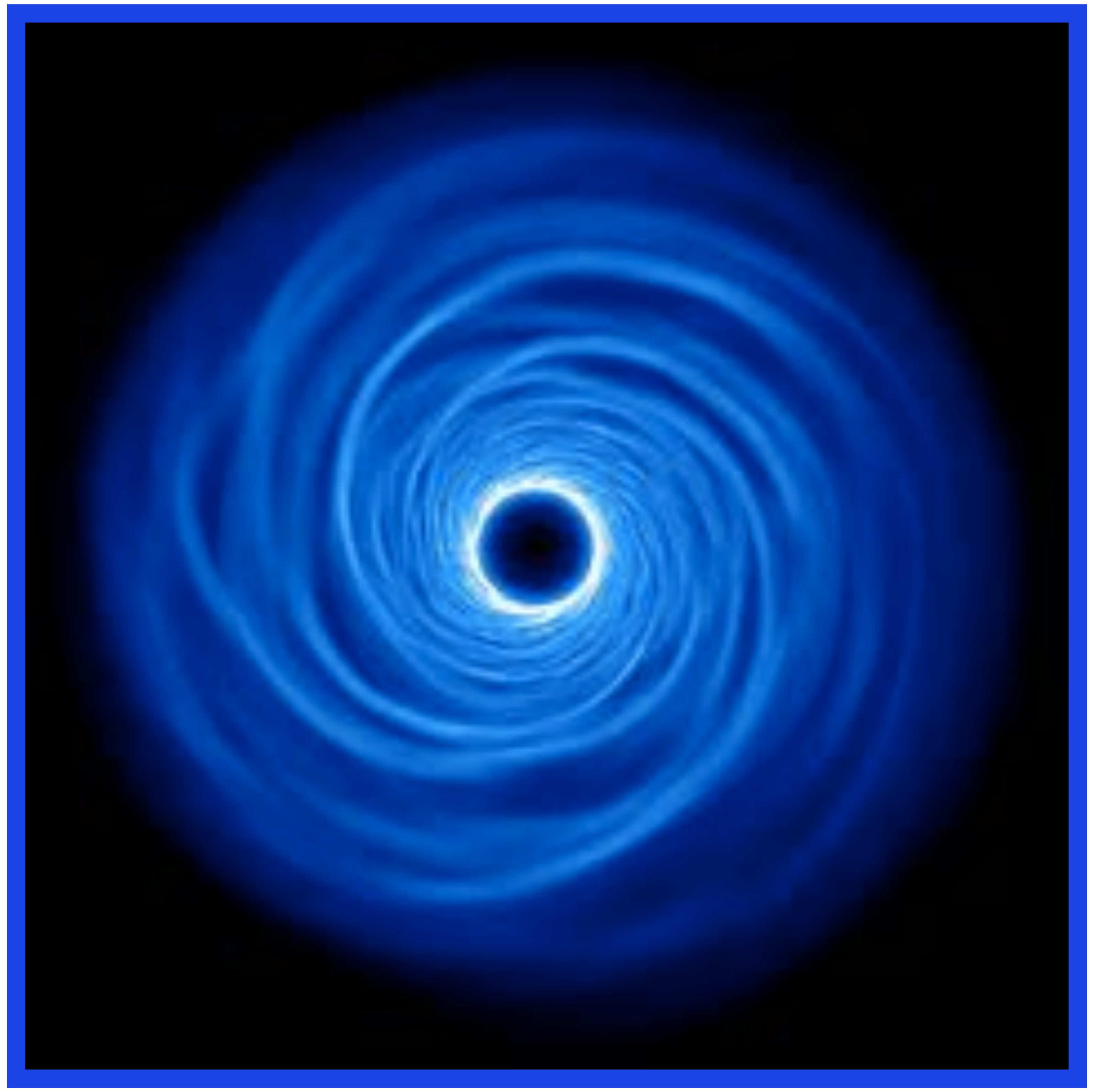
Compatible!

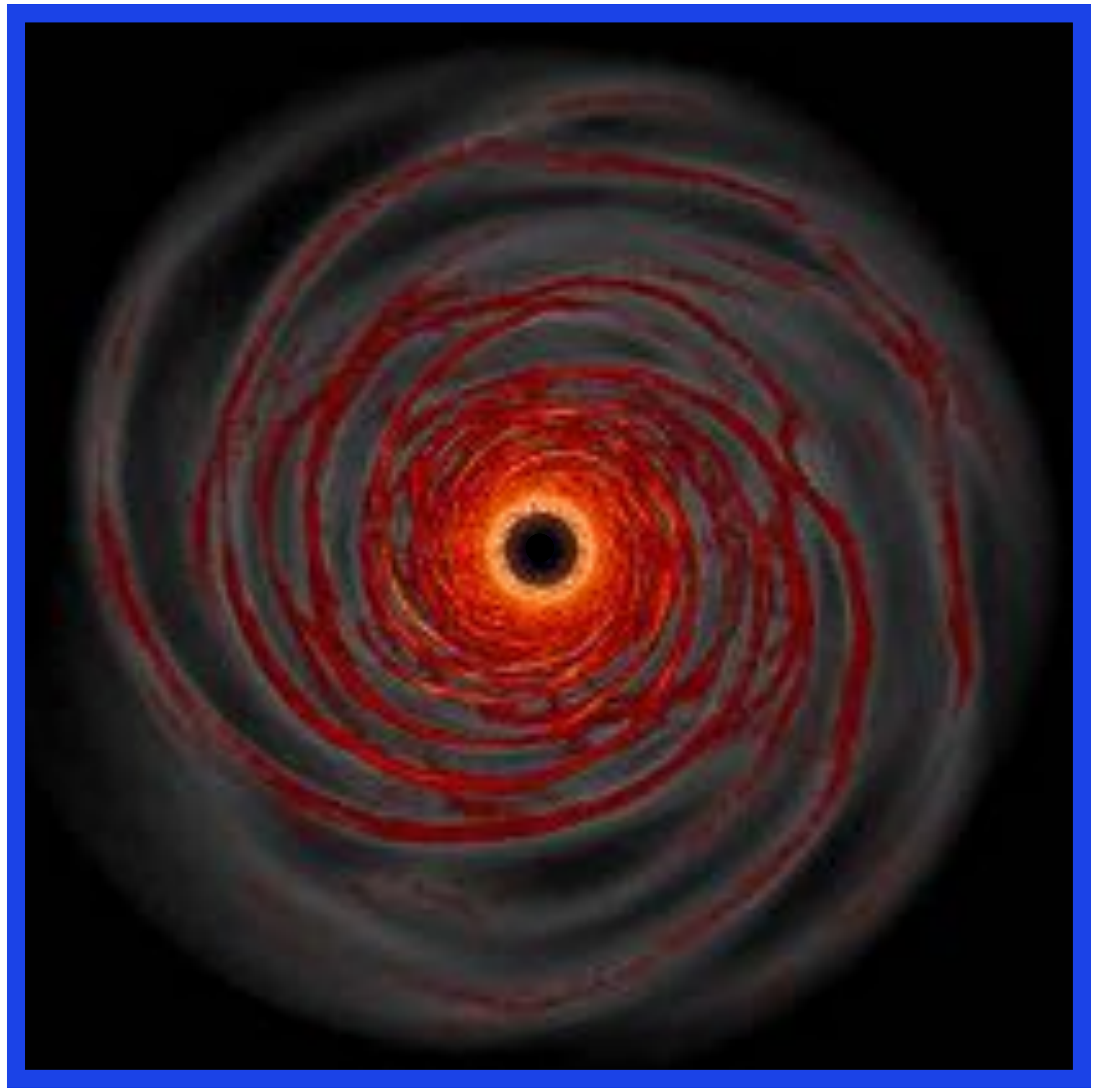
$R_c = 200au$



Longarini, Clarke, Lodato et al in prep

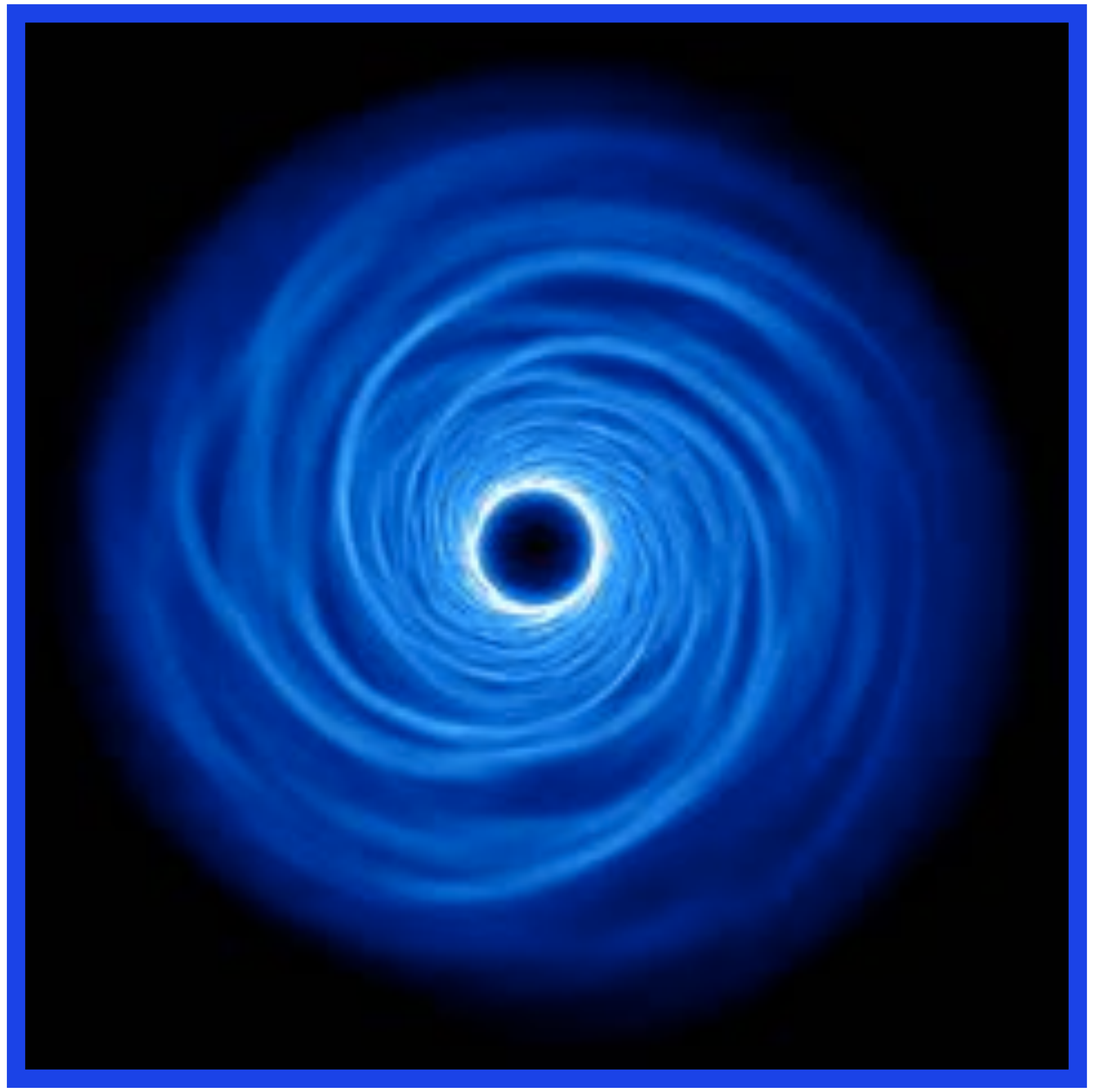
Dust dynamics in GI discs

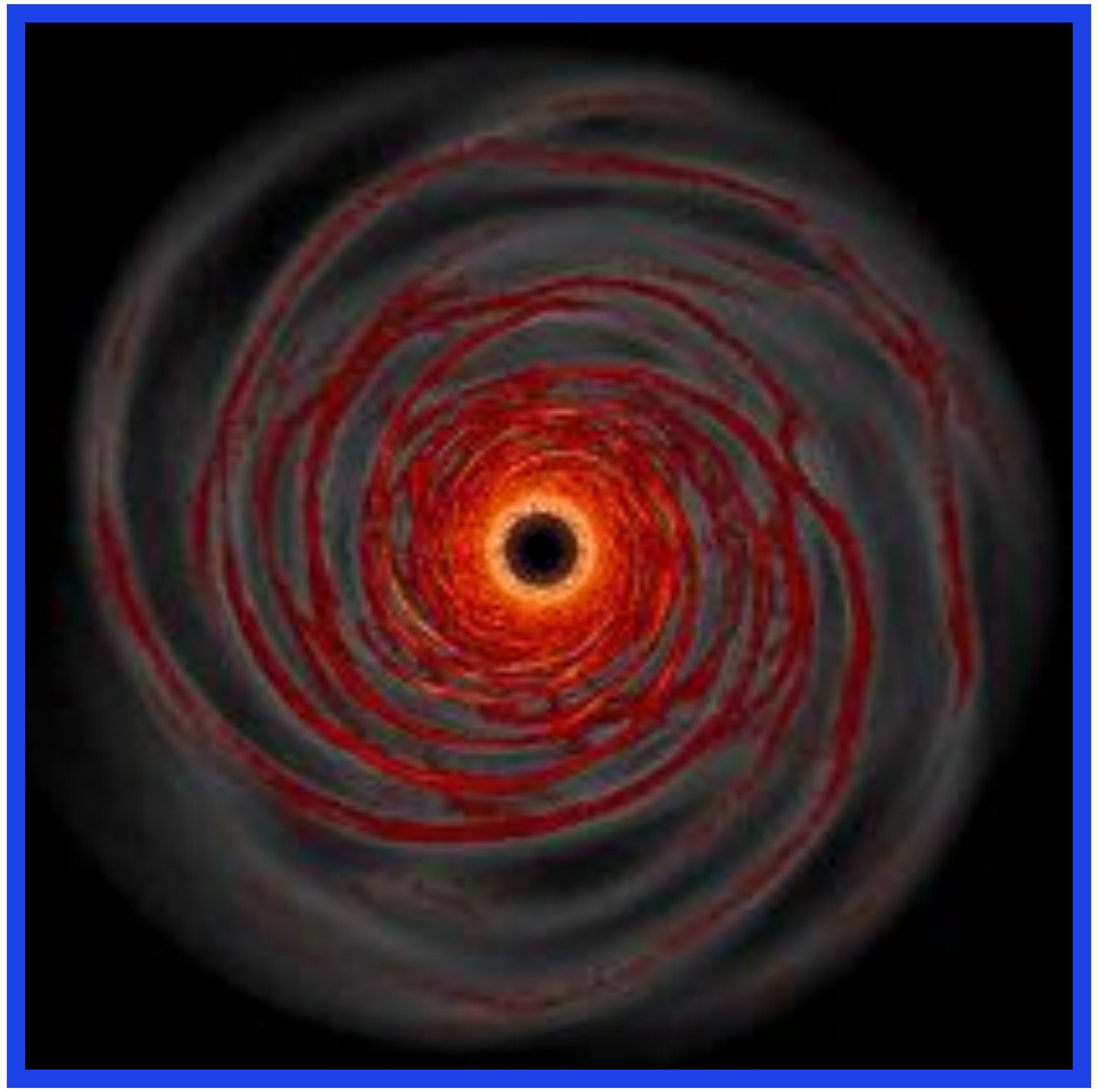




Dust dynamics in GI discs

Complementary to Sahl's one, Thanks for feedbacks and discussion!





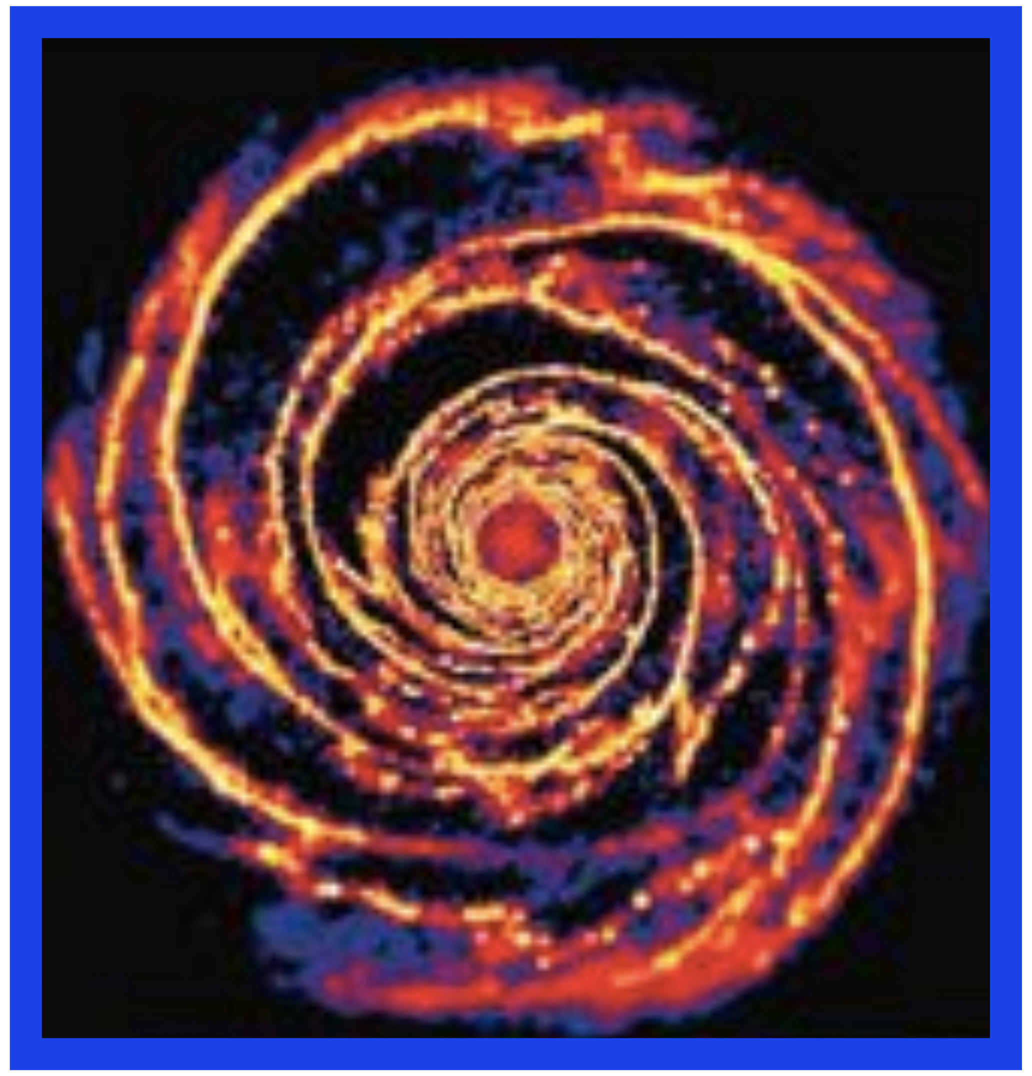
What we know so far

Rice et al. 2004, 2006

First 3D SPH simulations of gas and dust GI discs.

- Efficient dust trapping inside spiral arms
- Dust is so unstable that collapses $\sim 1M_{\oplus}$ planetesimals?

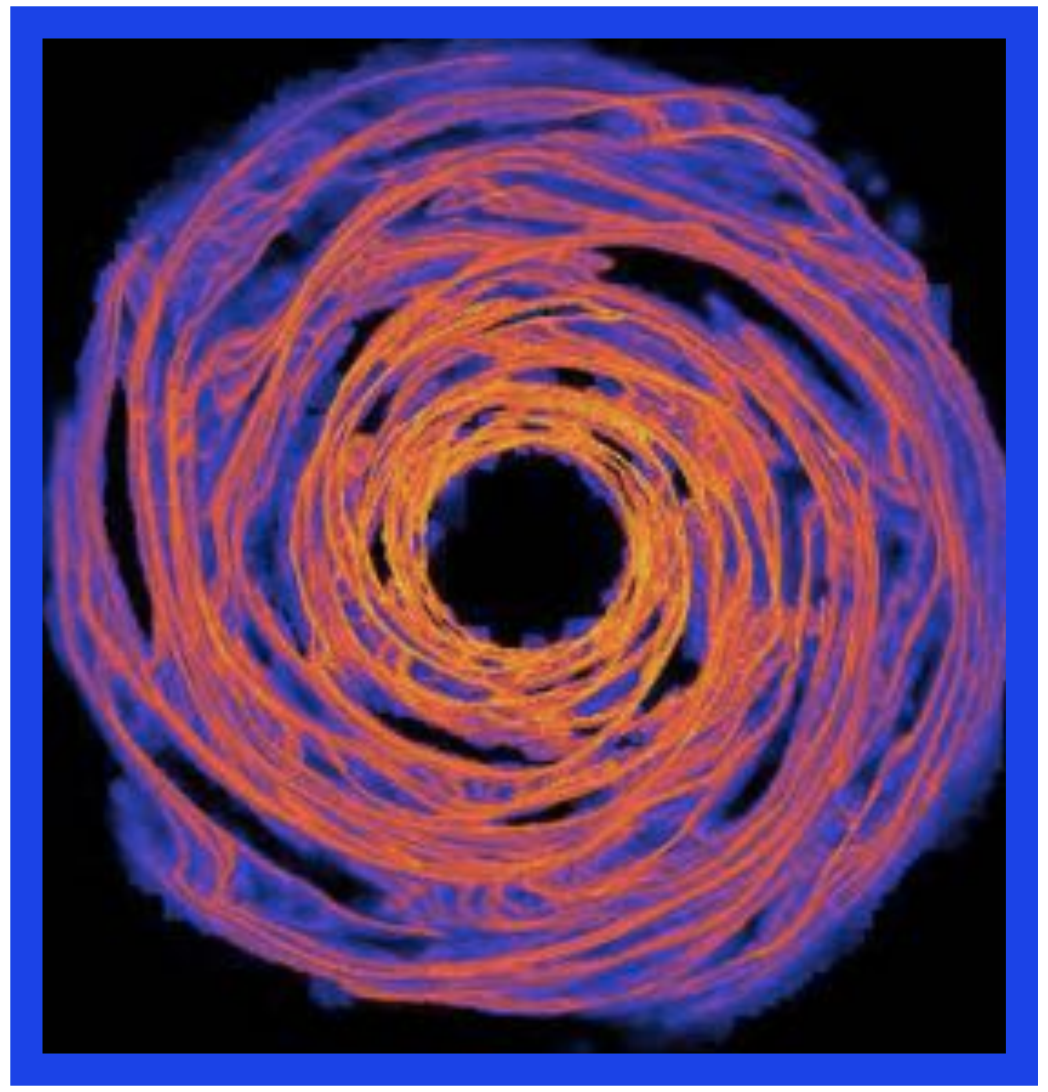
Warning: Low resolution , not able to properly resolve dust clumps



Booth & Clarke 2016

2D SPH simulations of gas and dust GI discs.

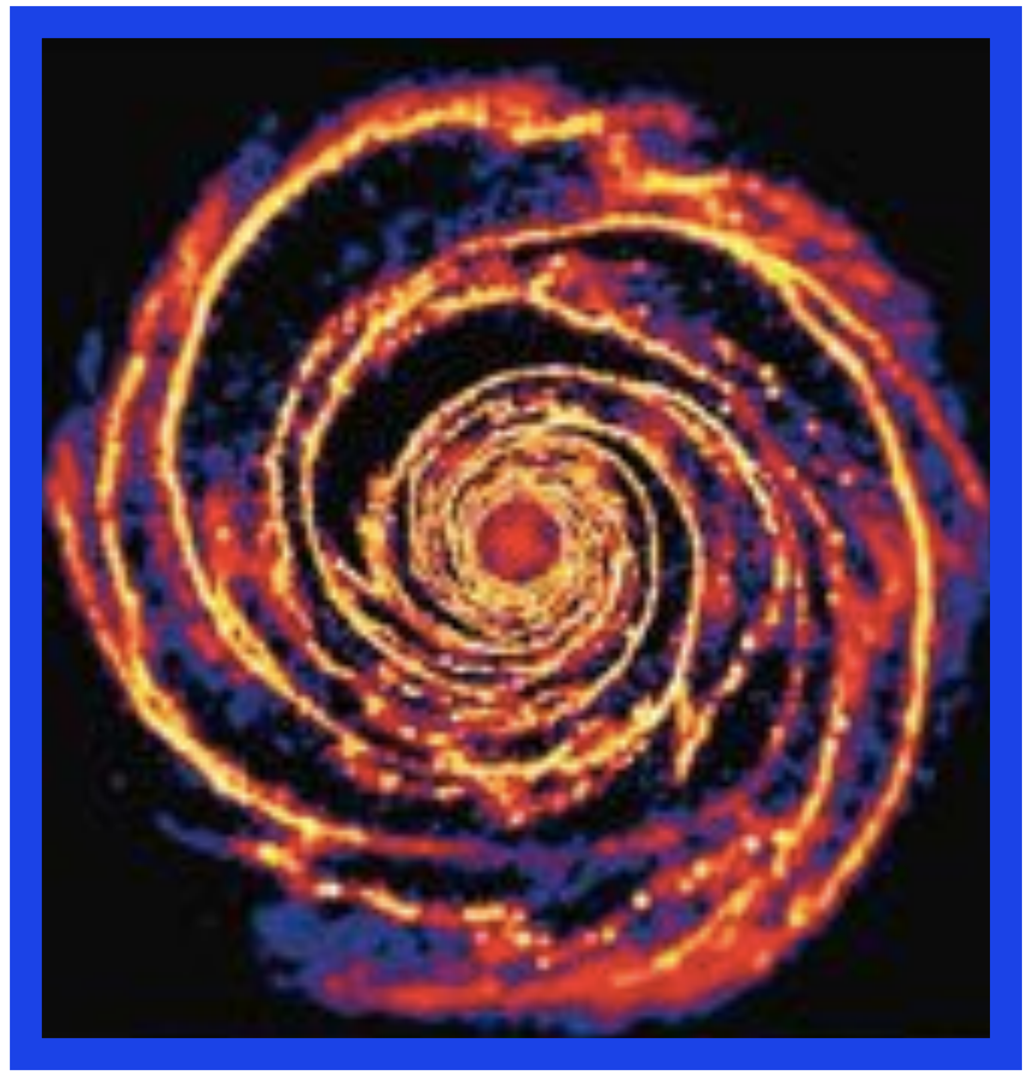
- Important parameter is dust dispersion velocity c_d
- $c_d \propto St^{1/2}\beta^{-1/2}$



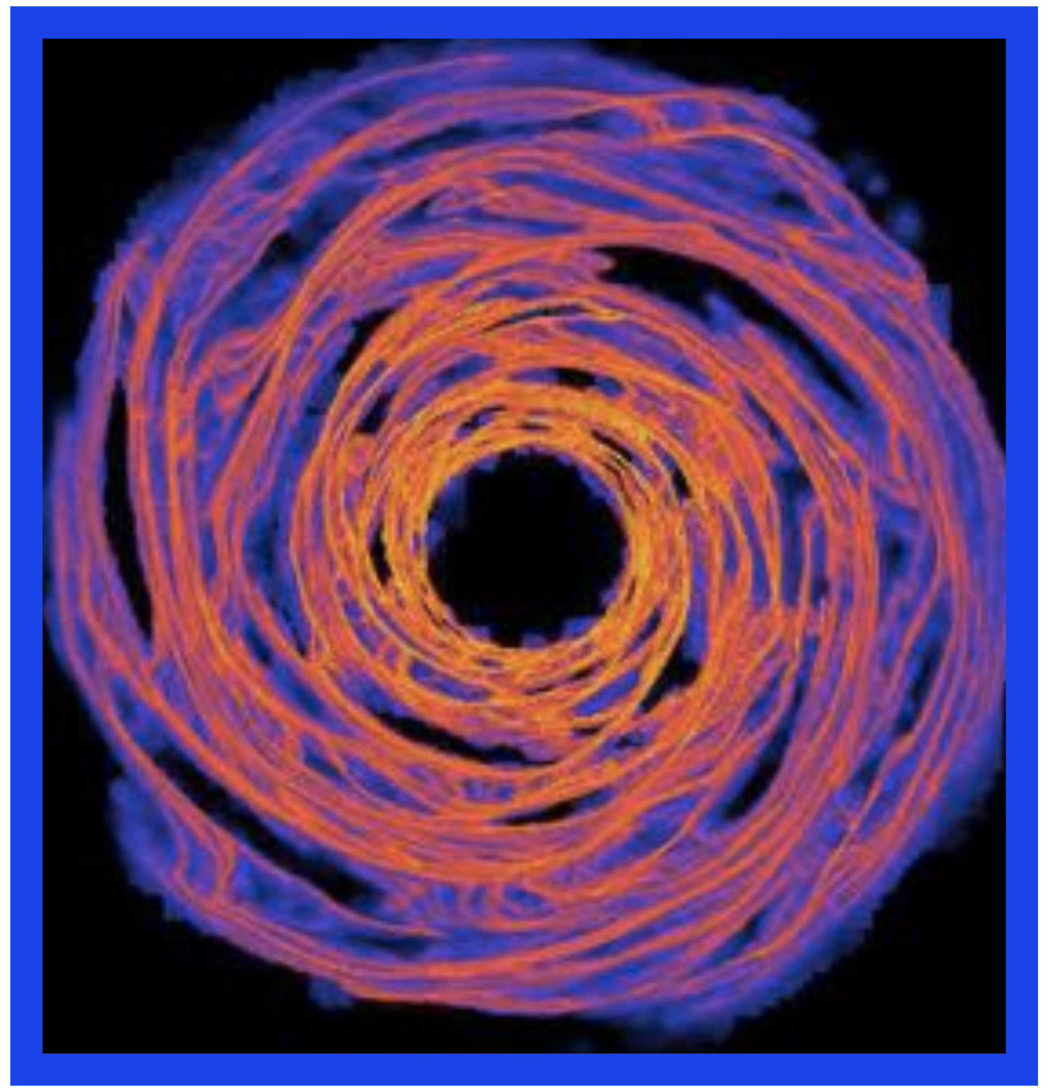
What we know so far

Longarini et al. 2023
Analytical theory: stability of a dusty GI disc

- The presence of dust makes the system more unstable
- If dust is sufficiently cold and abundant, it can drive instability at small wavelengths



What are we doing:
PHANTOM simulations of gas and dust GI discs, 2 fluids implementation, to compare to the linear theory and study parameters space



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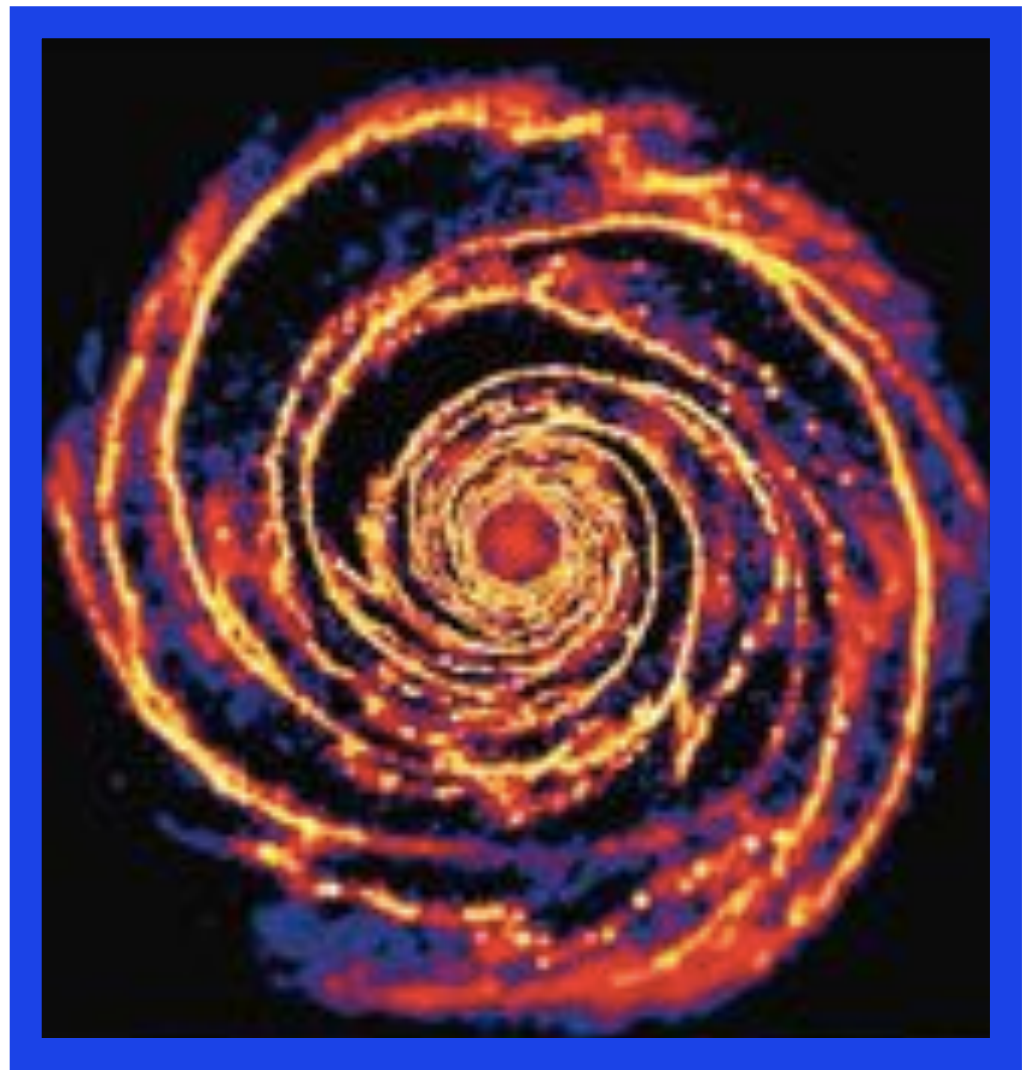

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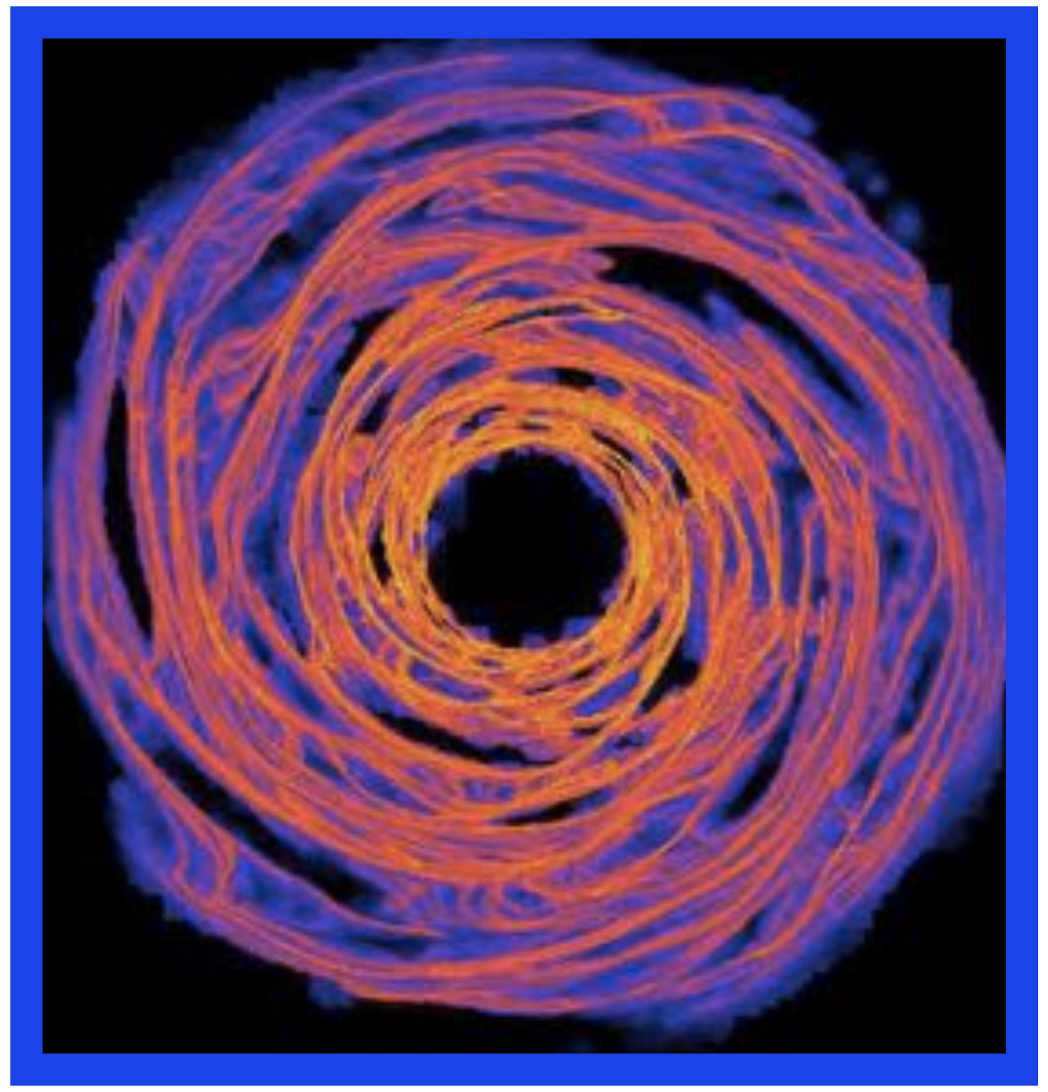
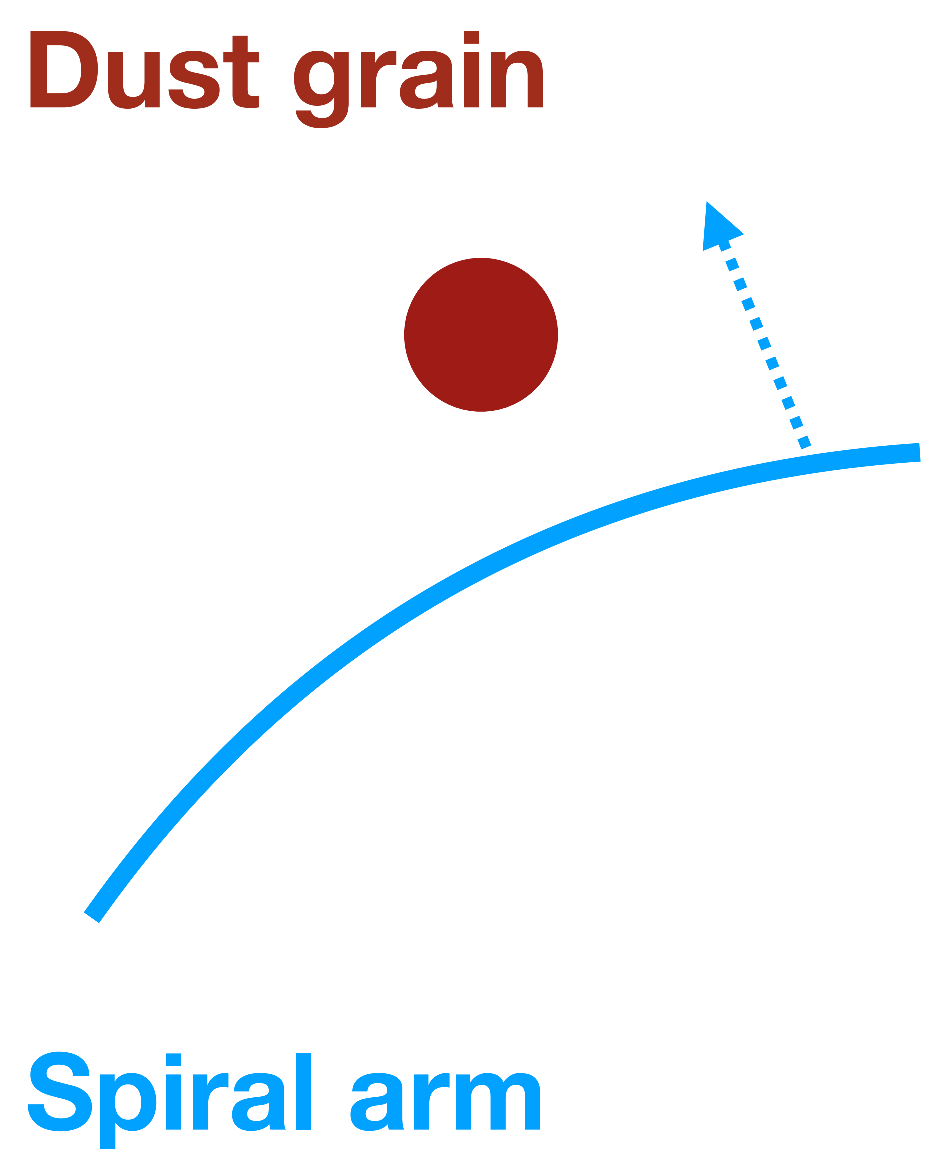


What happens to dust?



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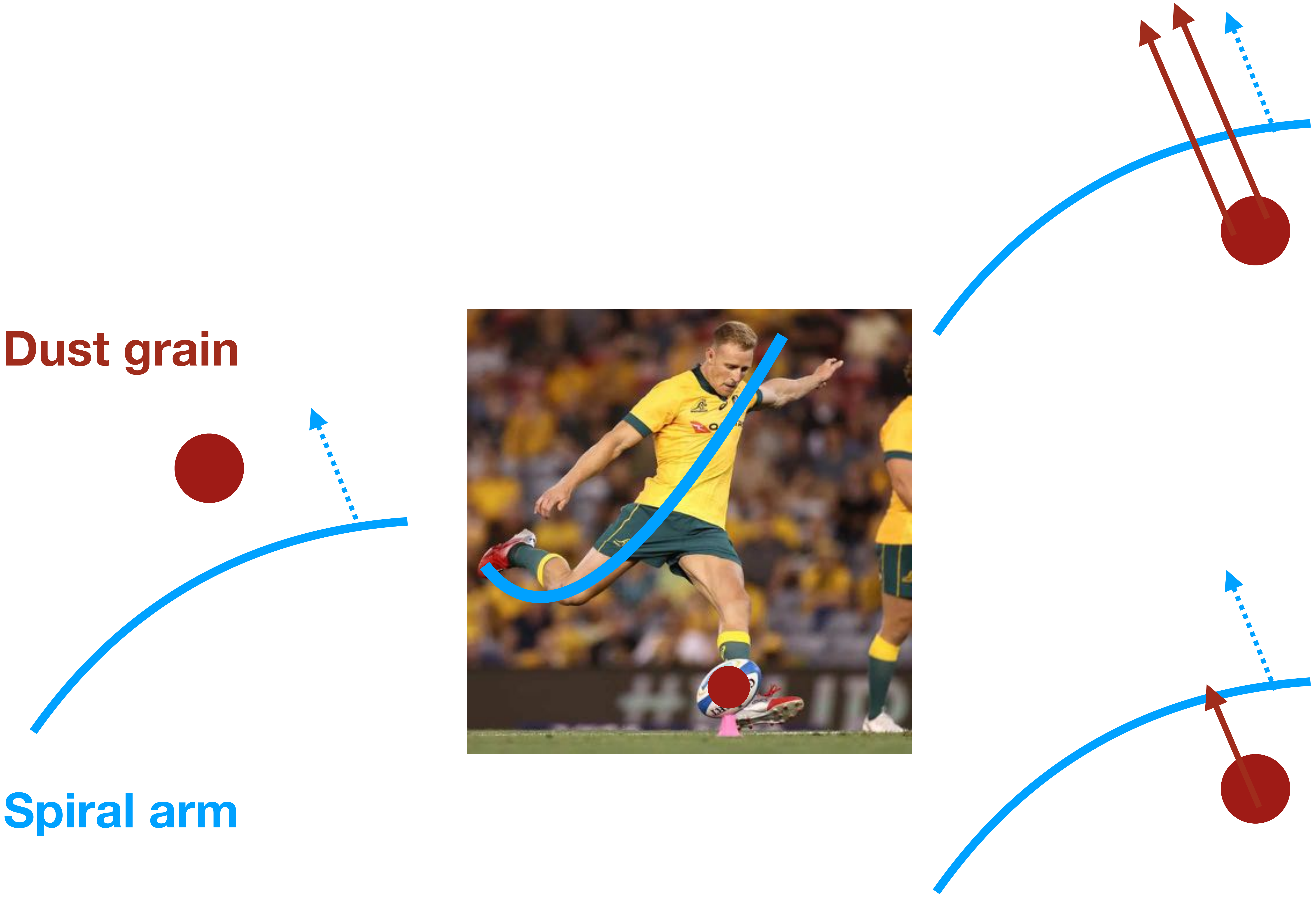
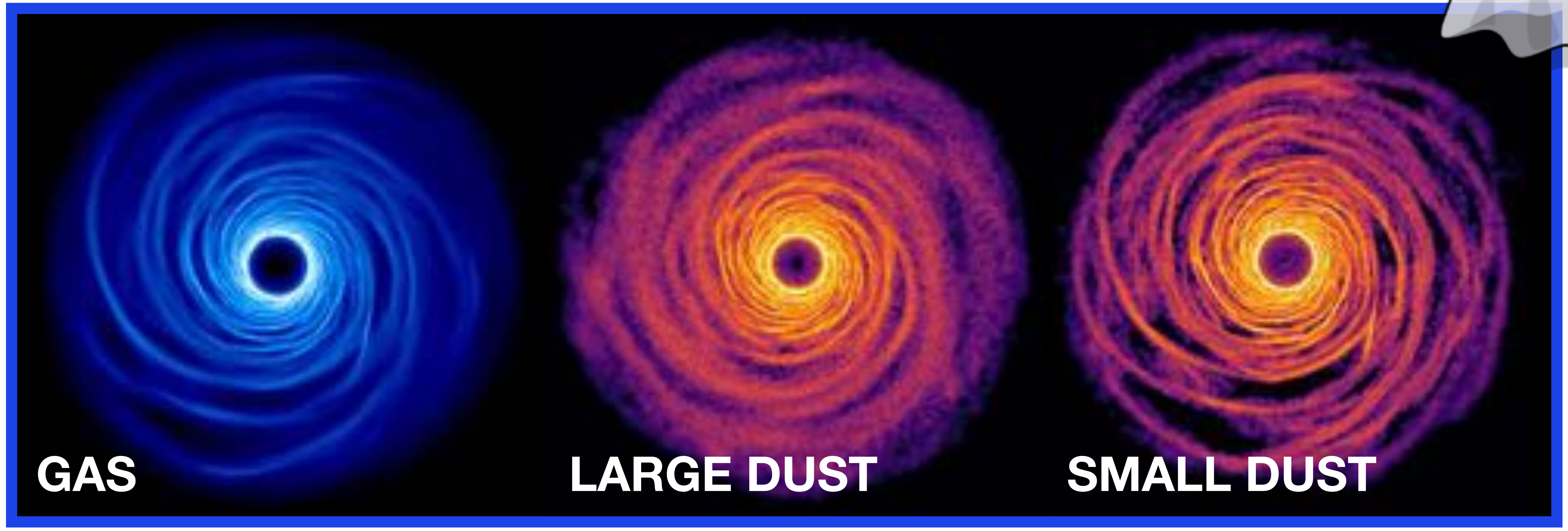


Efficiently excited:
Stronger kick if
- Low β
- Low $M_d/M_\star$
- High St

Not efficiently excited:
Weaker kick if
- High β
- High $M_d/M_\star$
- Low St

Hydro simulations



1M - 2M gas particles
250K - 500K dust particles

Simulation	$M_d/M_\star$	β_{cool}	s [cm]	⟨St⟩	s_{10} [cm]
S1	0.05	8	300	40	3
S2	0.05	10	300	40	3
S3	0.05	15	300	40	3
S4	0.05	8	60	8	0.6
S5	0.05	10	60	8	0.6
S6	0.05	15	60	8	0.6
S7	0.1	8	600	40	6
S8	0.1	10	600	40	6
S9	0.1	15	600	40	6
S10	0.1	8	120	8	1.2
S11	0.1	10	120	8	1.2
S12	0.1	15	120	8	1.2
S13	0.2	8	1500	40	15
S14	0.2	10	1500	40	15
S15	0.2	15	1500	40	15
S16	0.2	8	600	16	6
S17	0.2	10	600	16	6
S18	0.2	15	600	16	6

GI parameters

$\beta, M_d/M_\star$

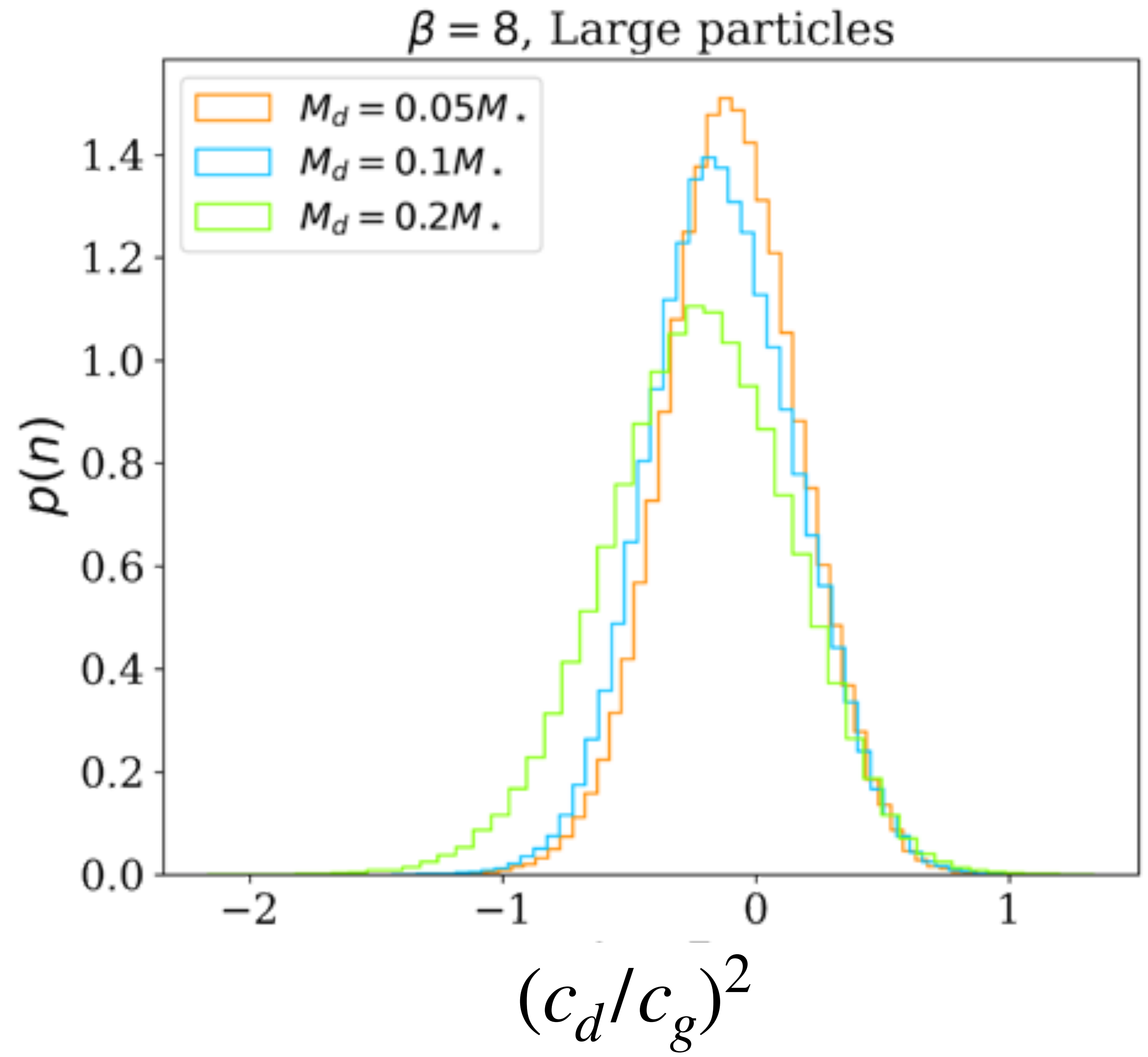
—?→

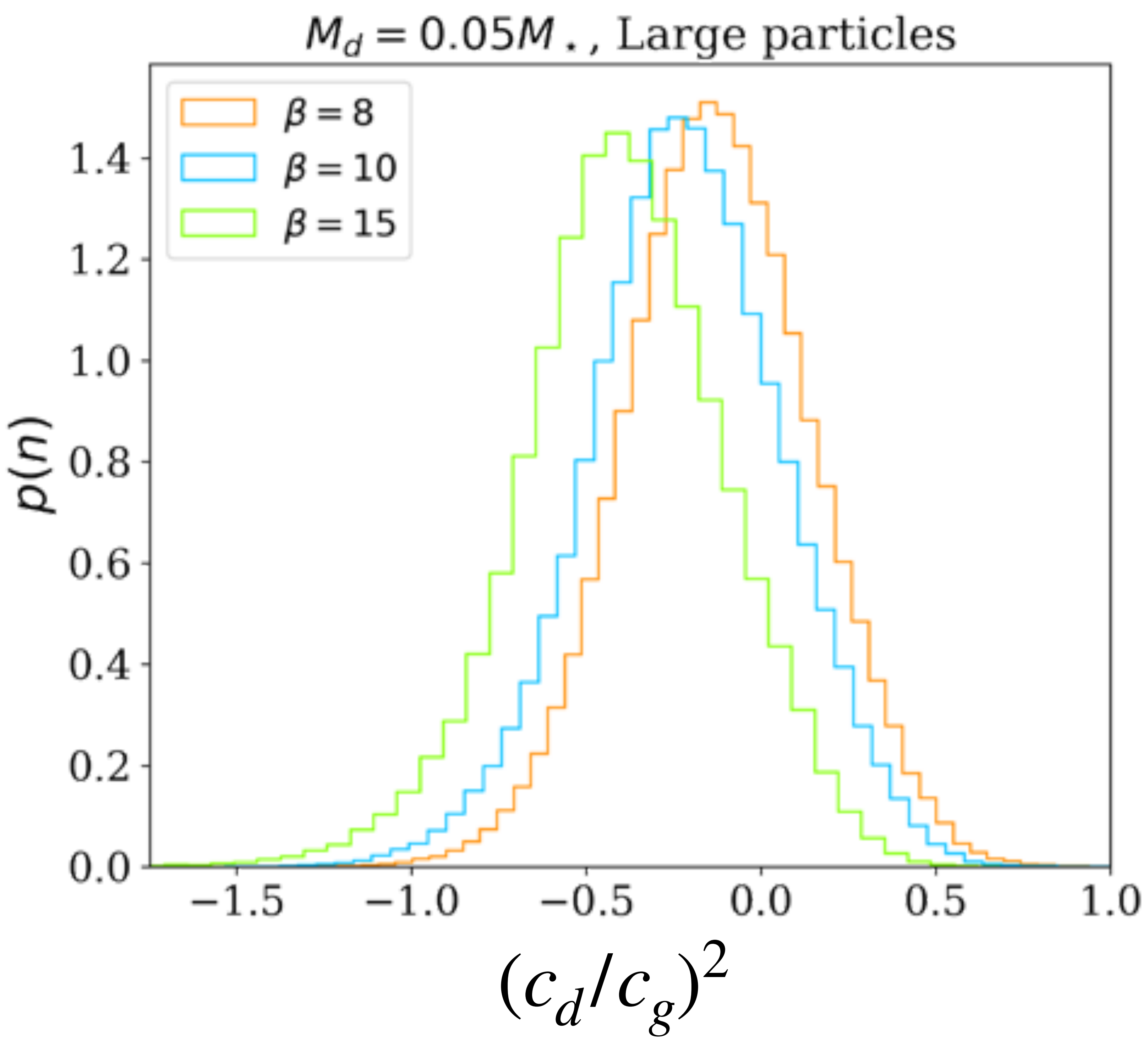
Dust parameters

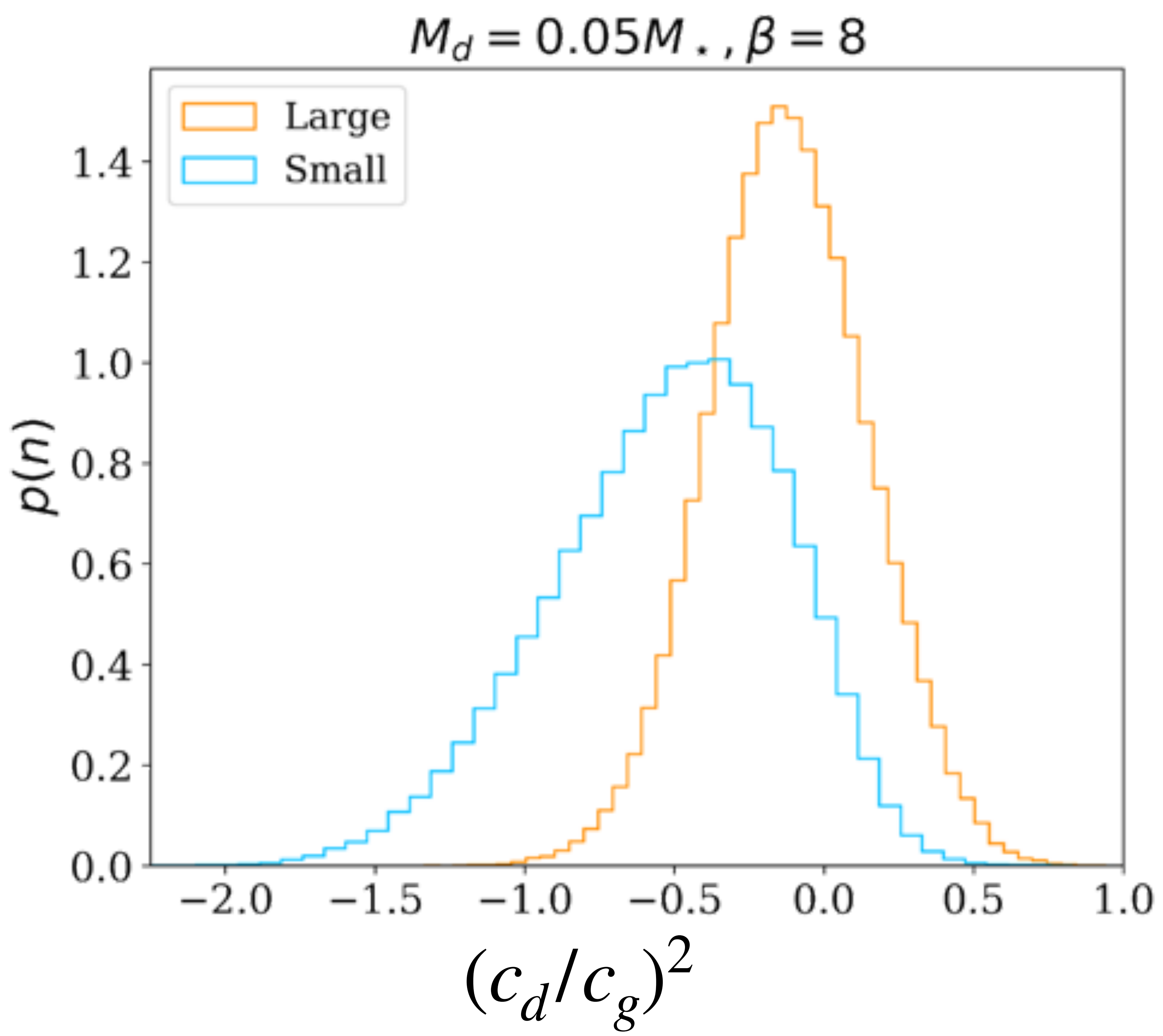
ϵ : dust to gas ratio

c_d : dispersion velocity

Dust dispersion velocity







<u>Higher disc to star mass ratio = less spiral arms</u>

→ Dust receives less kicks from the spiral

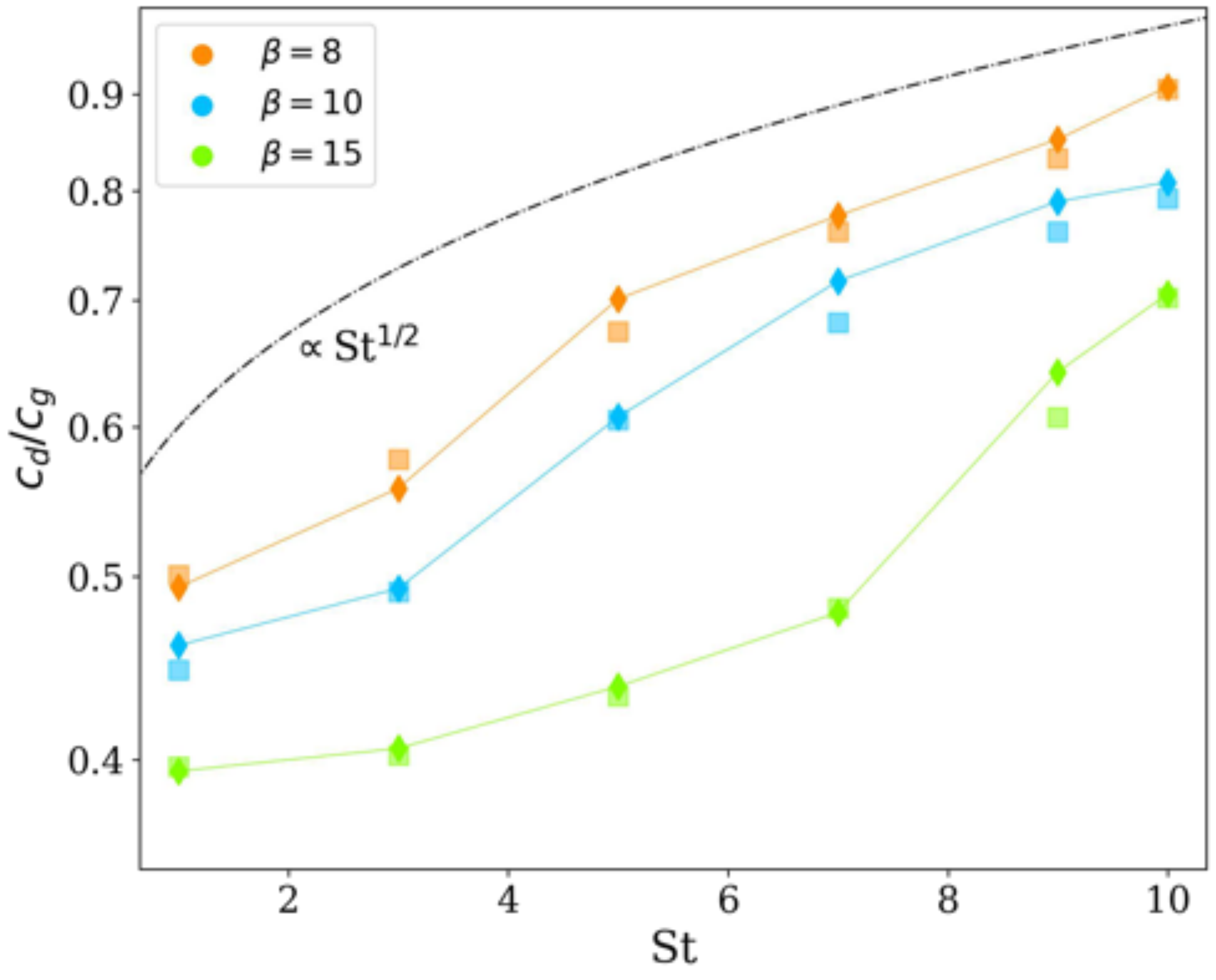
<u>Higher beta-factor = weaker spiral arms</u>

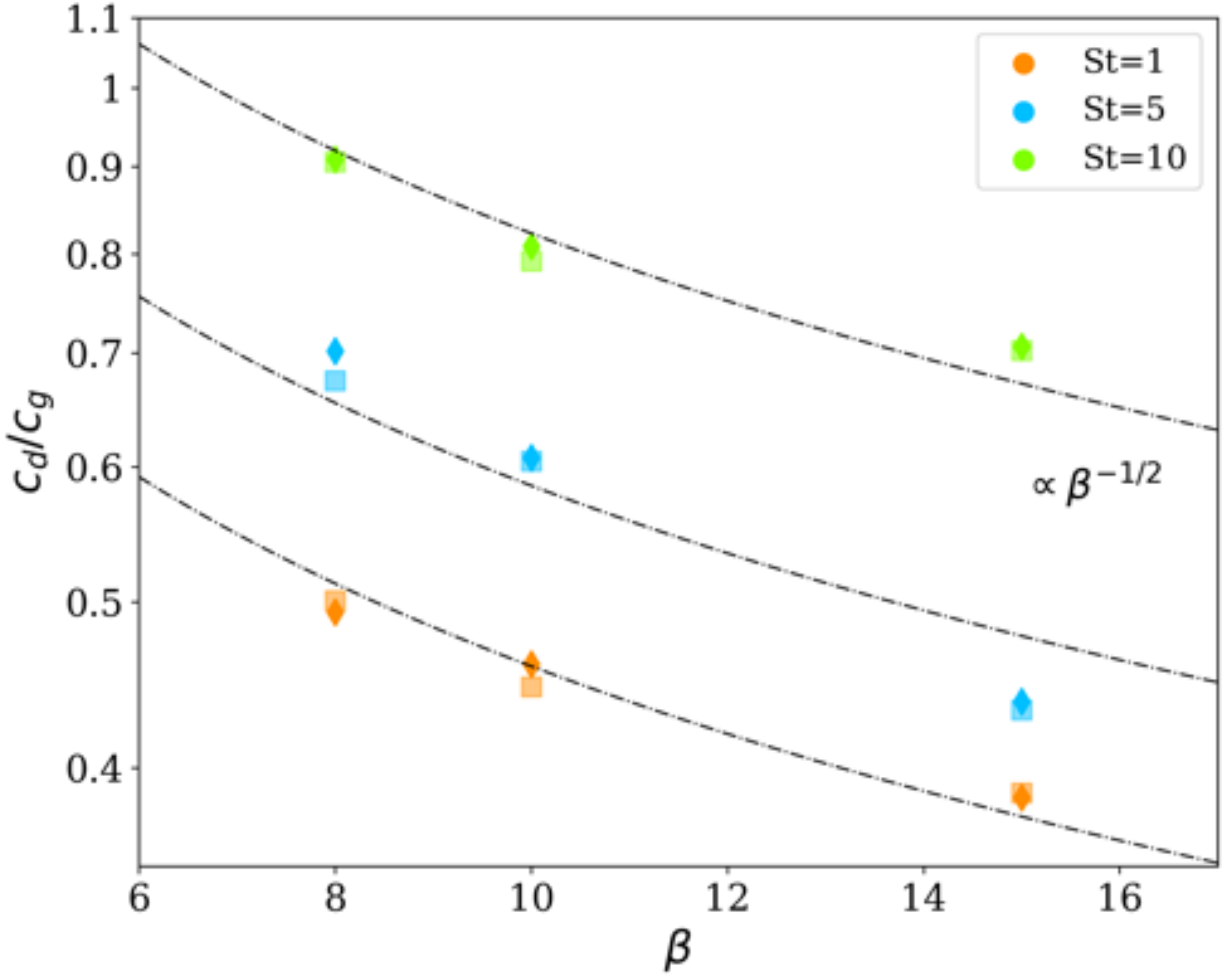
→ Spiral potential well is shallower

<u>Aerodynamical coupling damps the spiral kick</u>

→ Uncoupled particles are excited more

Dust dispersion velocity





$$c_d \propto St^{1/2}\beta^{-1/2}$$

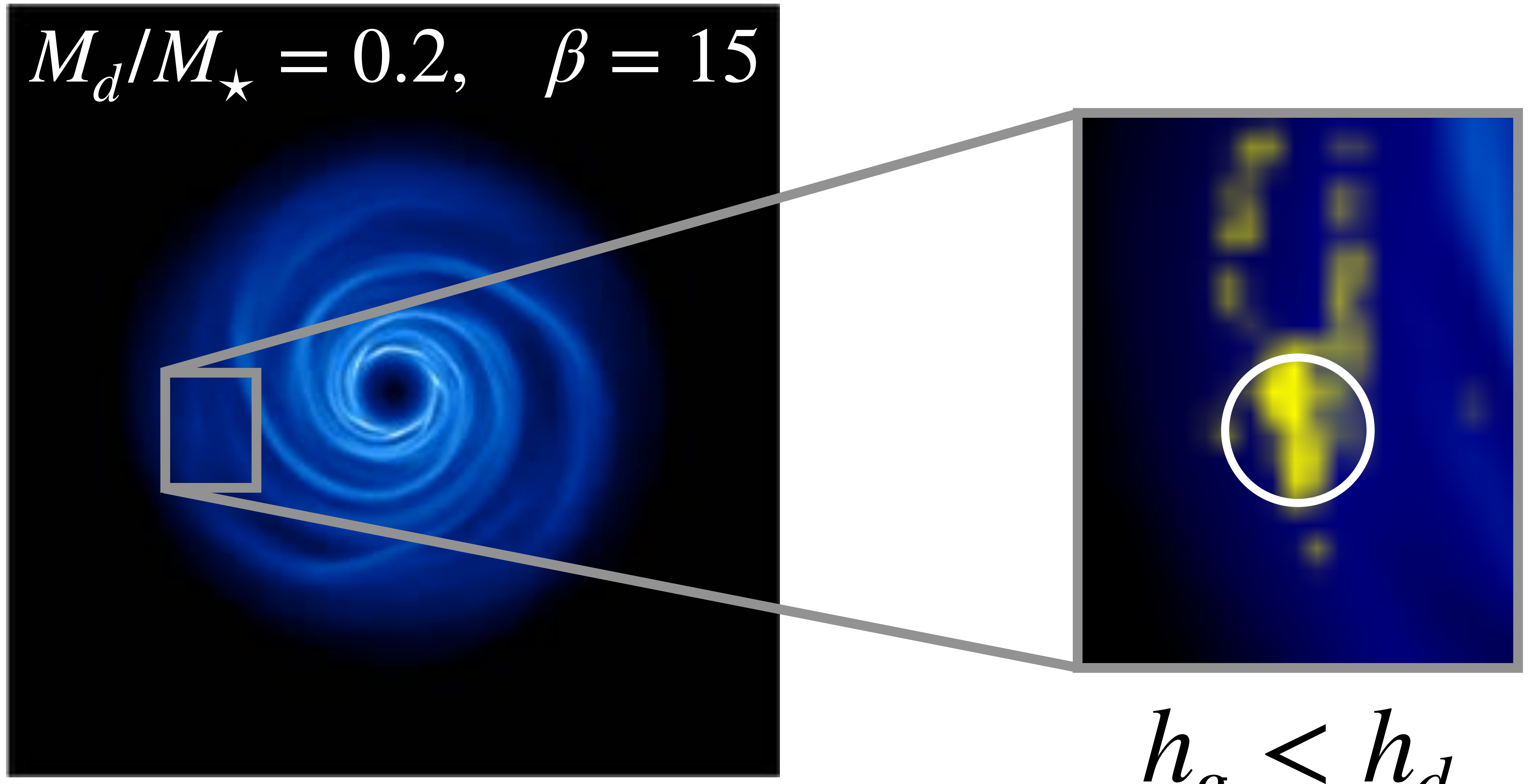
Dust collapse

Sahl's simulations range

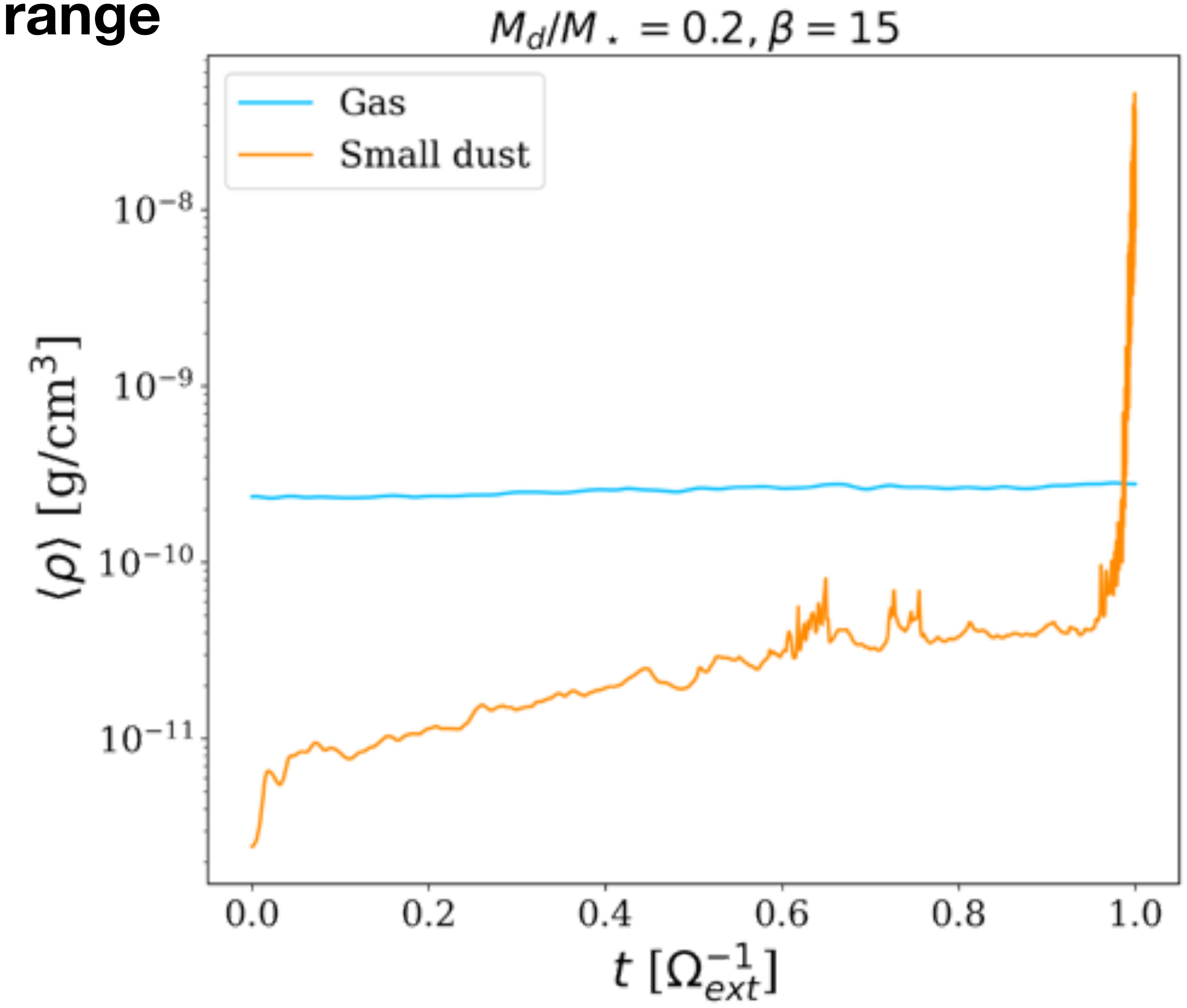
We observe dust collapse only for

- Higher disc to star mass ratio $(M_d/M_\star = 0.2)$
- Long cooling $(\beta = 10 - 15)$
- Small dust particles

Mass of the clump $M_{cl} \simeq 0.8 M_\oplus$



$h_g < h_d$



Only dust is collapsing
Simulation stops (too long computational time…)

Dust collapse

Sahl's simulations range

We observe dust

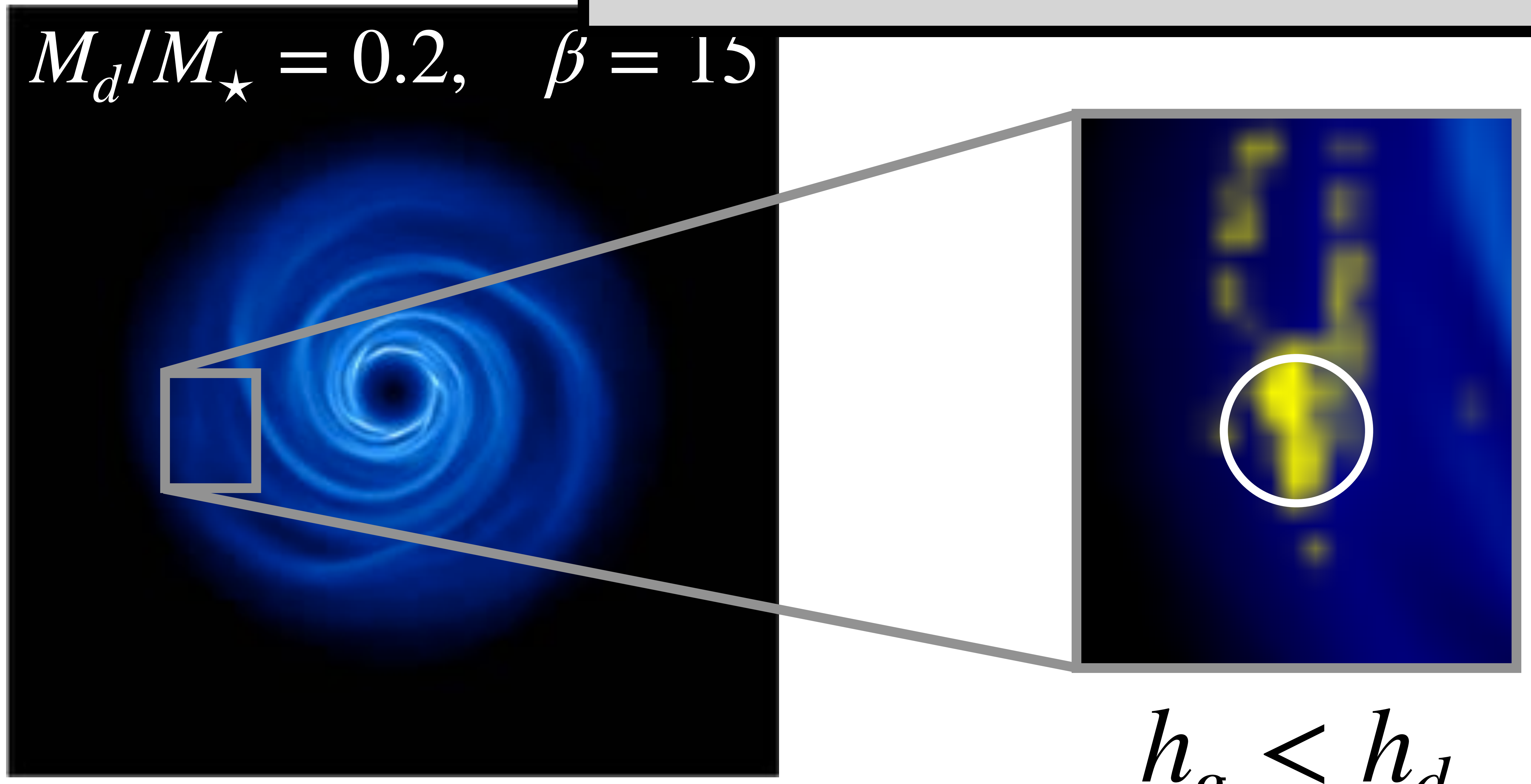
- Higher disc to s
- Long cooling (β
- Small dust part

Mass of the clum

$$t_s \propto (\rho_g + \rho_d)^{-1} \rightarrow 0$$

Stopping time < tilmestep

SINK!



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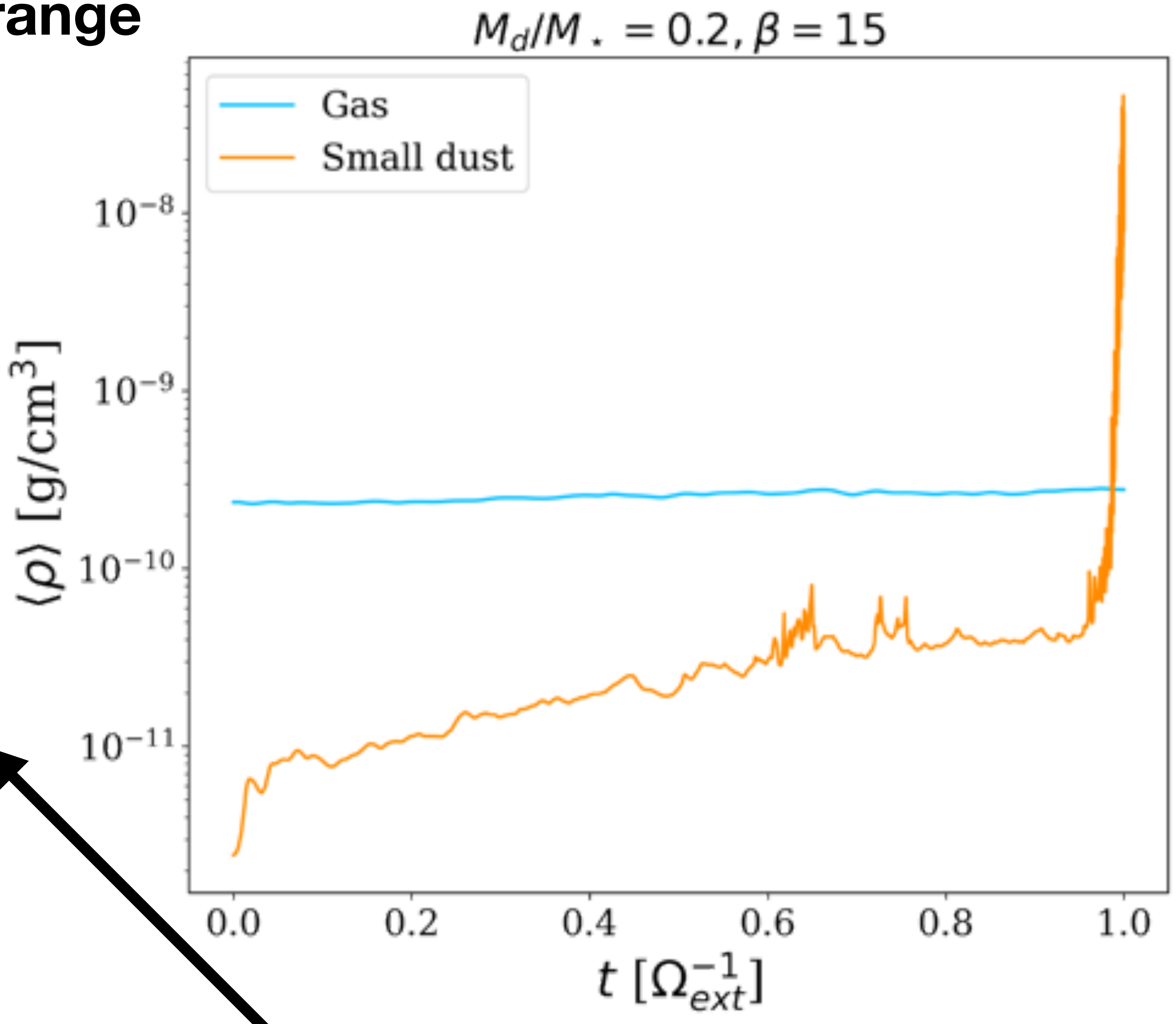

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Summary

- Gravitational instability is significant in Class0-Class1 objects
  → difficult to observe because of cloud contamination

- Kinematic signatures could be a smoking gun for GI
  → connection to angular momentum transport

- Dust in GI scenario could be a way to solve the problem of planetesimals

But, most importantly

Testing theory with numerical simulations is crucial to understand its limits and applicability