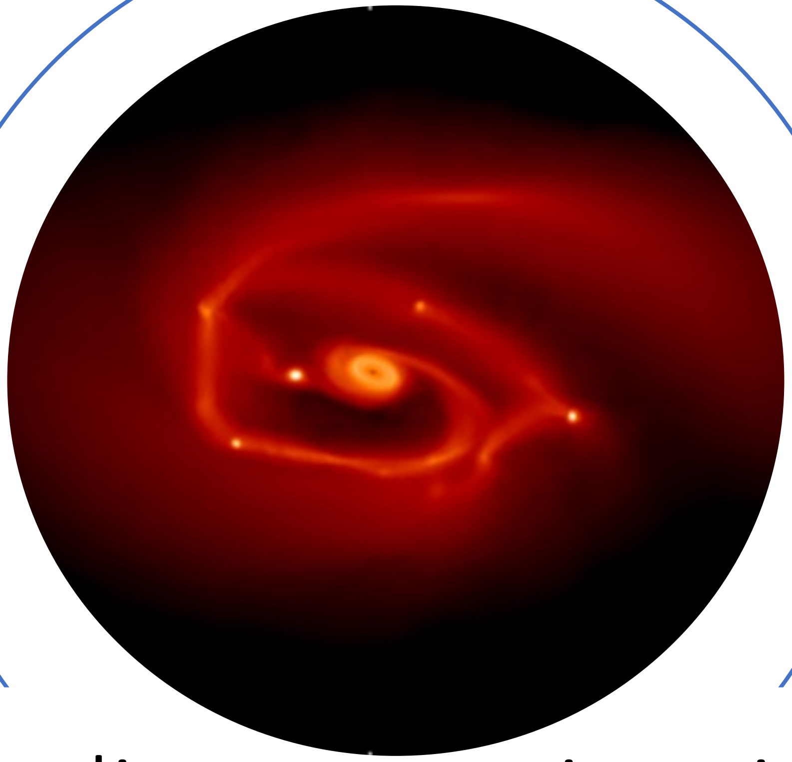


Radiative cooling approximations and the beginning of planet formation

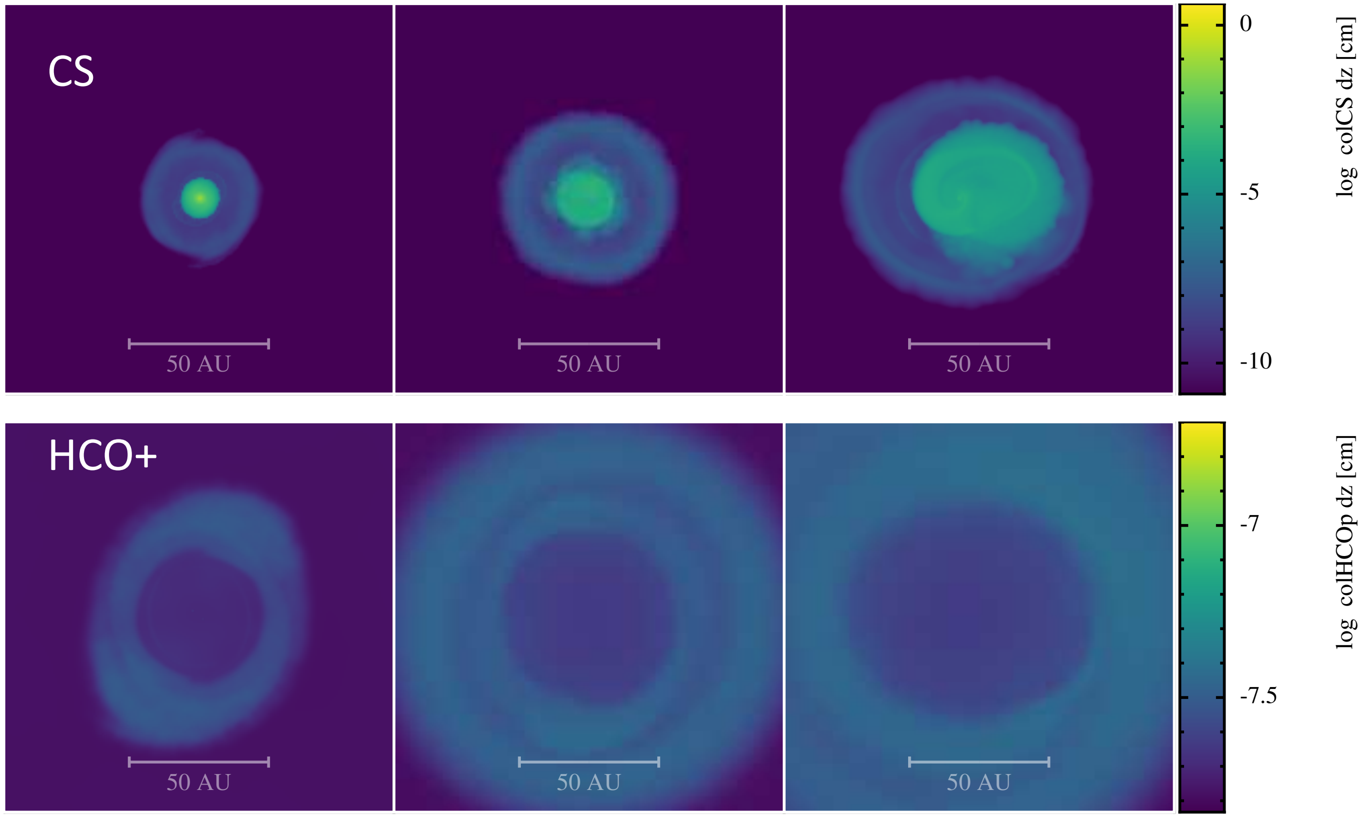
ALISON K. YOUNG, MAGGIE CELESTE, RICHARD BOOTH, KEN RICE, DIMITRIS STAMATELLOS, ETHAN CARTER

But first... who am I?

- Star formation, first hydrostatic cores, chemistry, synthetic observations...

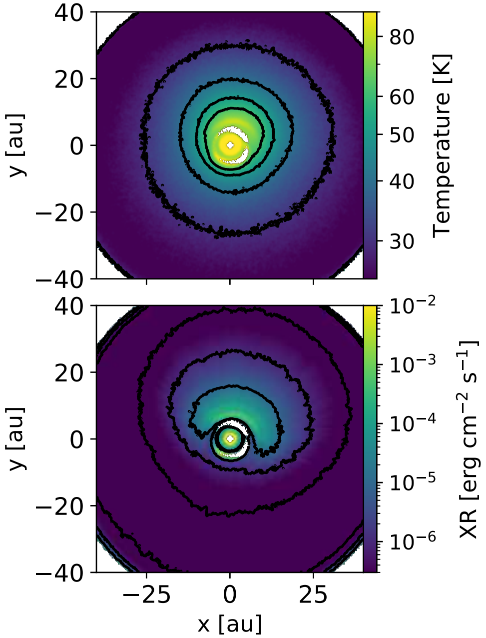
But first… who am I?

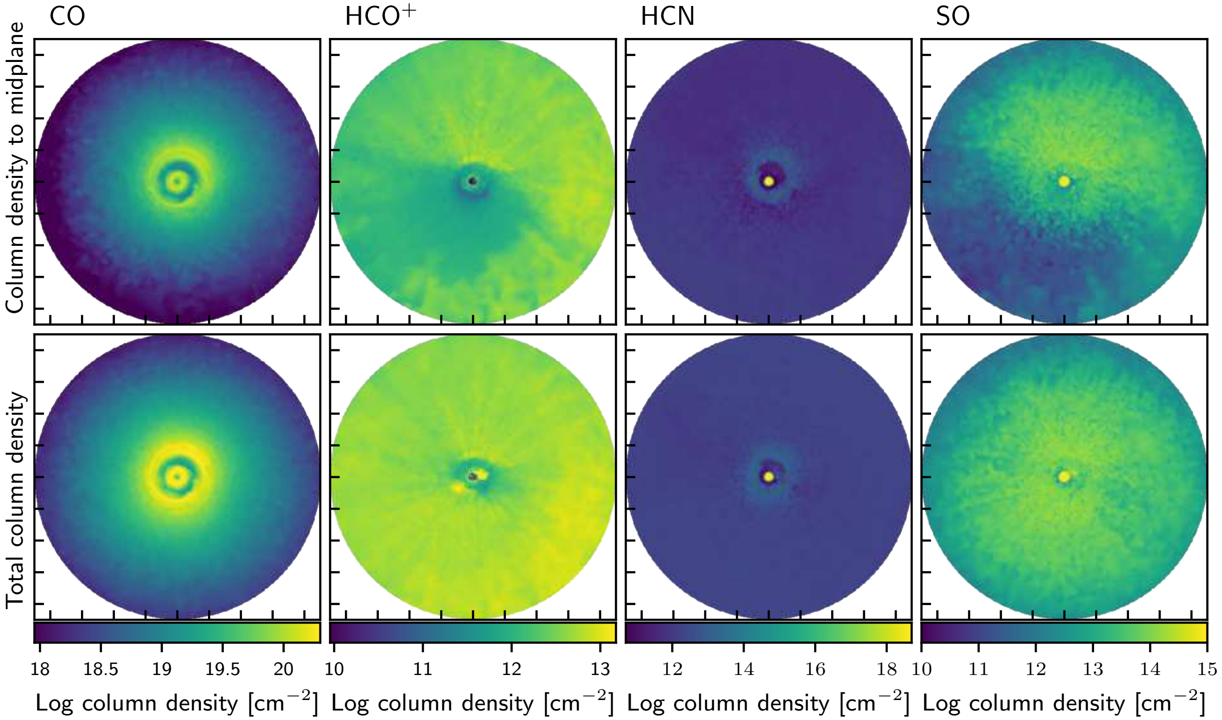
- sink feedback + chemistry



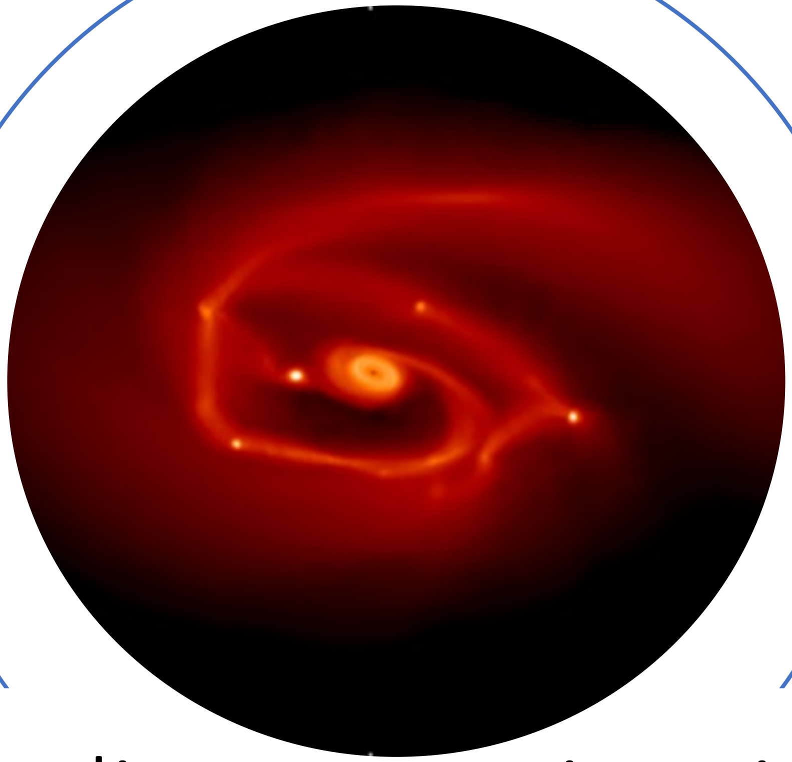
But first… who am I?

- warps & breaks in protoplanetary discs & chemistry too…





Young+ 2021



Radiative cooling approximations and the beginning of planet formation

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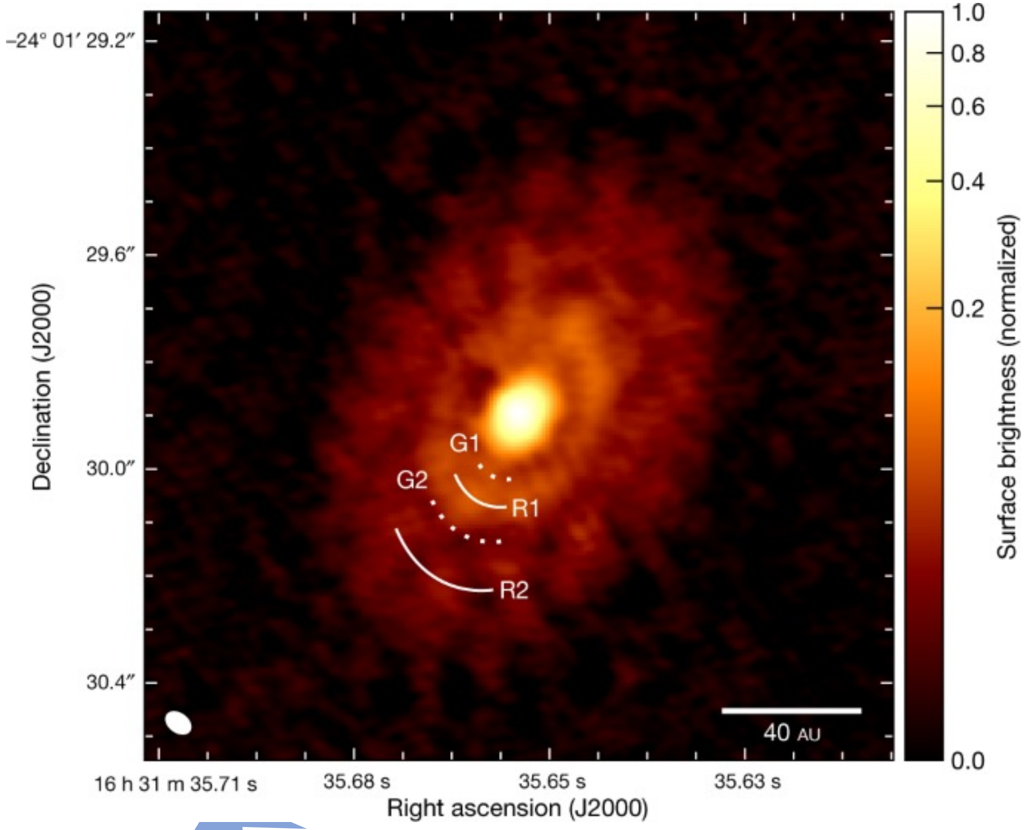
The problem:



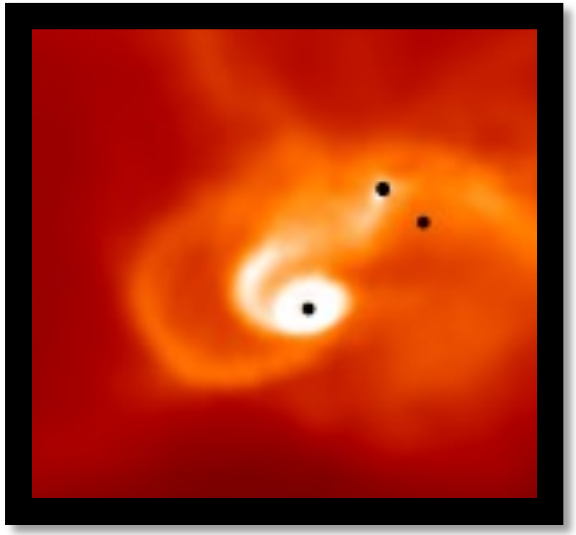
Segura-Cox+ 2020

IRS 63 – no more than 500,000 years old

Planets form early and quickly
What does this mean?

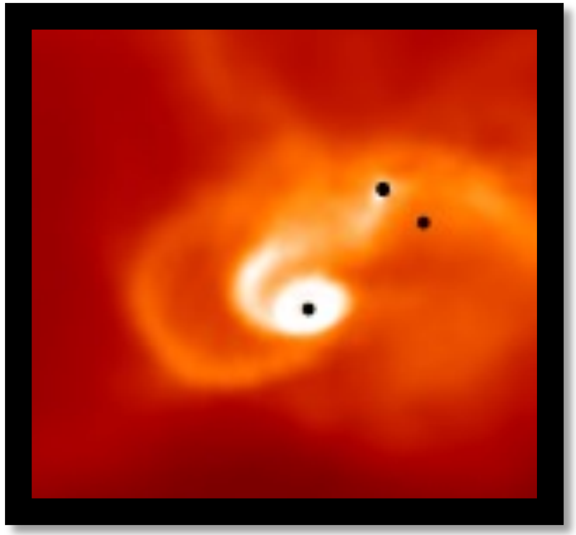
Alison Young

Growing planetesimals quickly



- need to form planetesimals within few 100 kyr
- young discs are more massive and subject to gravitational instability
- how does grain growth occur in turbulent discs?

Modelling young discs - challenges



- more massive and subject to gravitational instability
- internal heating
- stability very sensitive to cooling rate
- accretion streamers
- dynamical interactions with neighbour stars
- What are the “initial conditions” for young discs?

Modelling cooling in massive discs

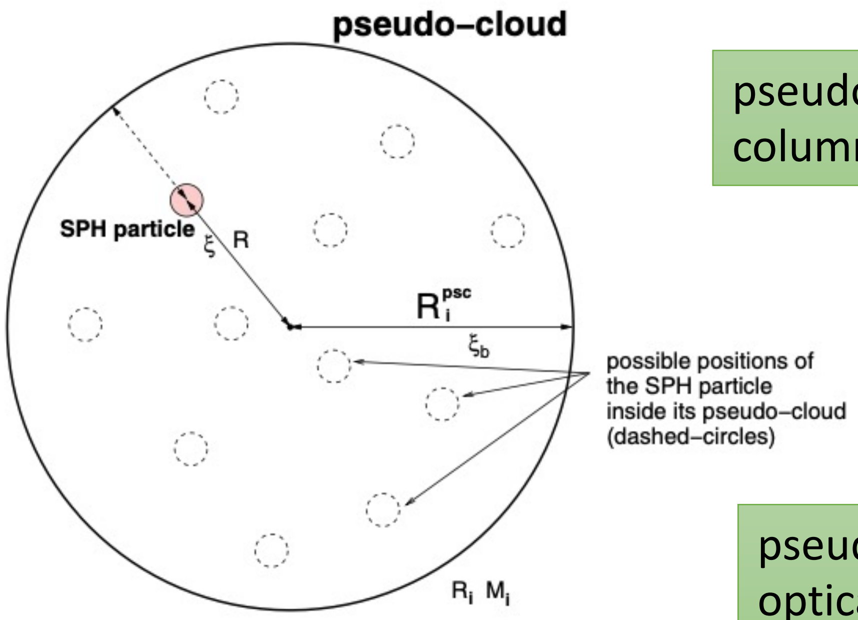
- Gravitational instability is key source of heating and viscosity (turbulence)
- Cooling rate governs whether disc fragments etc.
- Full radiative transfer is prohibitively expensive (except if you're Sahl)
- We need an effective approximate treatment

Current options: beta cooling

- $t_c = {}^{\beta}/_{\Omega}$ Gammie (2001)
- cooling time parameterised as a function of orbital period (radius)
- can't account for disc structures
- doesn't tell us much about the physics

Current options: polytropic cooling v1

Stamatellos+ (2007)



mass- weighted average of Σi(ξ) over all possible dimensionless radii, ξ

pseudo-mean column-density

$$\bar{\Sigma}_i = \zeta_n \left[\frac{-\psi_i \rho_i}{4\pi G} \right]^{1/2},$$

A constant

We already calculate the potential 😀

pseudo-mean optical depth

$$\bar{\tau}_i = \bar{\Sigma}_i \bar{\kappa}_i .$$

Current options: polytropic cooling v1

Stamatellos+ (2007)

$$\left.\frac{du_i}{dt}\right|_{\mathrm{rad}} = \frac{4\sigma\left(T_0^4(\boldsymbol{r}_i) - T_i^4\right)}{\bar{\Sigma}^2\bar{\kappa}_i(\rho_i, T_i) + \kappa_i^{-1}(\rho_i, T_i)},$$

radiative cooling rate

Background temperature

Can set this to ISRF + stellar irradiation

pseudo-mean optical depth

Assume that gas cools/heats towards an equilibrium temperature.

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Current options: polytropic cooling v1

Assume that gas cools/heats towards an equilibrium temperature.

$$\left.\frac{du_i}{dt}\right|_{\rm rad} = \frac{4\sigma\left(T_0^4(\boldsymbol{r}_i) - T_i^4\right)}{\bar{\Sigma}^2\bar{\kappa}_i(\rho_i, T_i) + \kappa_i^{-1}(\rho_i, T_i)},$$

Substitute Teq for Ti
Assume heating/cooling balance to find equilibrium temperature

$$\left.\frac{du_i}{dt}\right|_{\rm hydro} + \left.\frac{du_i}{dt}\right|_{\rm rad} = 0,$$

$$u_{{\rm eq},i} = u(T_{{\rm eq},i}, \rho_i).$$

This gives an equilibrium energy

$$t_{{\rm therm},i} = \left(u_{\rm eq,i} - u_i\right)\left[\left.\frac{du_i}{dt}\right|_{\rm hydro} + \left.\frac{du_i}{dt}\right|_{\rm rad}\right]^{-1}.$$

with thermal timescale

Stamatellos+ (2007)

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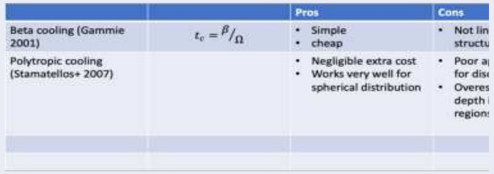
Current options: polytropic cooling v1

$$t_{\mathrm{therm},i} = \left(u_{\mathrm{eq,i}} - u_i\right)\left[\left.\frac{du_i}{dt}\right|_{\mathrm{hydro}} + \left.\frac{du_i}{dt}\right|_{\mathrm{rad}}\right]^{-1}.$$

Cooling (heating) rate is then:

$$\left.\frac{du_i}{dt}\right|_{\mathrm{cool}} = \frac{1}{\delta t}\left[u_i \exp\left(\frac{-\delta t}{t_{\mathrm{therm,i}}}\right) + u_{\mathrm{eq,i}} \exp\left(\frac{-\delta t}{t_{\mathrm{therm,i}}}\right)\right] + \left.\frac{du_i}{dt}\right|_{\mathrm{hydro}}. \quad (12)$$

Current options: polytropic cooling v1

		Pros	Cons
Beta cooling (Gammie 2001)	$t_c = {}^{\beta}/_{\Omega}$	• Simple • cheap	• Not linked to structure or physics
Polytropic cooling (Stamatellos+ 2007)		• Negligible extra cost • Works very well for spherical distribution	• Poor approximation for discs • Overestimates optical depth in low density regions

Current options: polytropic cooling v2

Lombardi+ (2015)

Pressure scaleheight → $H_{p,i} = \frac{P_i}{|\nabla P_i|}$

We already know this – grad P is calculated for dv/dt 😀

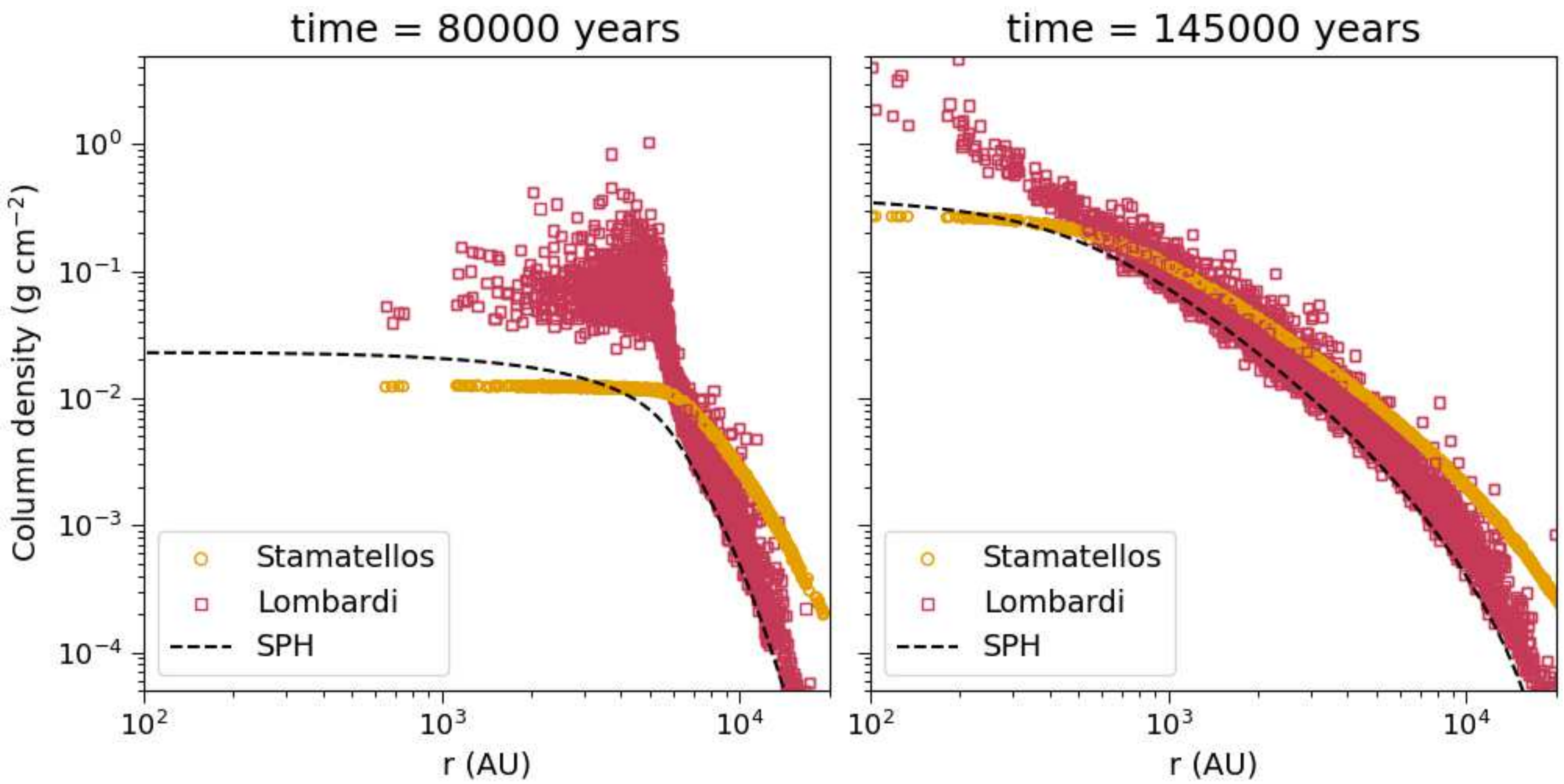
pseudo-mean column-density → $\bar{\Sigma}_i = \zeta' \rho_i H_{p,i}$

A constant → ζ'

pseudo-mean optical depth → $H_{p,i} = \frac{P_i}{|\nabla P_i|}$

Current options: polytropic cooling v2

Collapsing sphere test



Stamatellos method overestimates column at lower density
Lombardi method overestimates in centre of cloud.

Current options: polytropic cooling v2

		Pros	Cons
Beta cooling (Gammie 2001)	$t_c = {}^{\beta}/_{\Omega}$	• Simple • cheap	• Not linked to structure or physics
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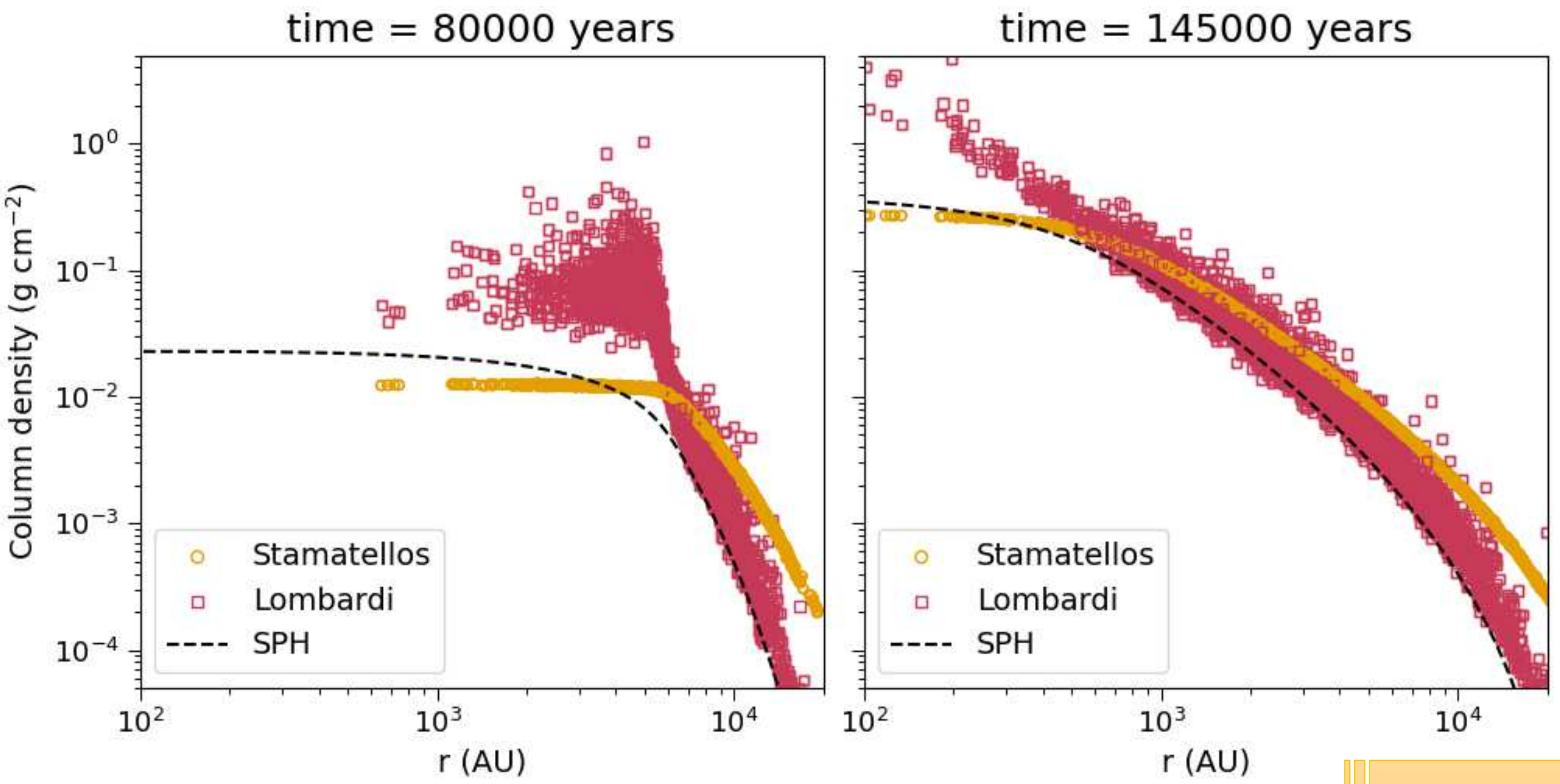
Polytropic cooling (Lombardi+ 2015)	$H_{p,i} = \frac{P_i}{\lvert\nabla P_i\rvert}$	• Negligible extra cost • Works ok for spherical & discs	• Poor when ∇P is small

New: ”Combined Method”

Collapsing sphere test

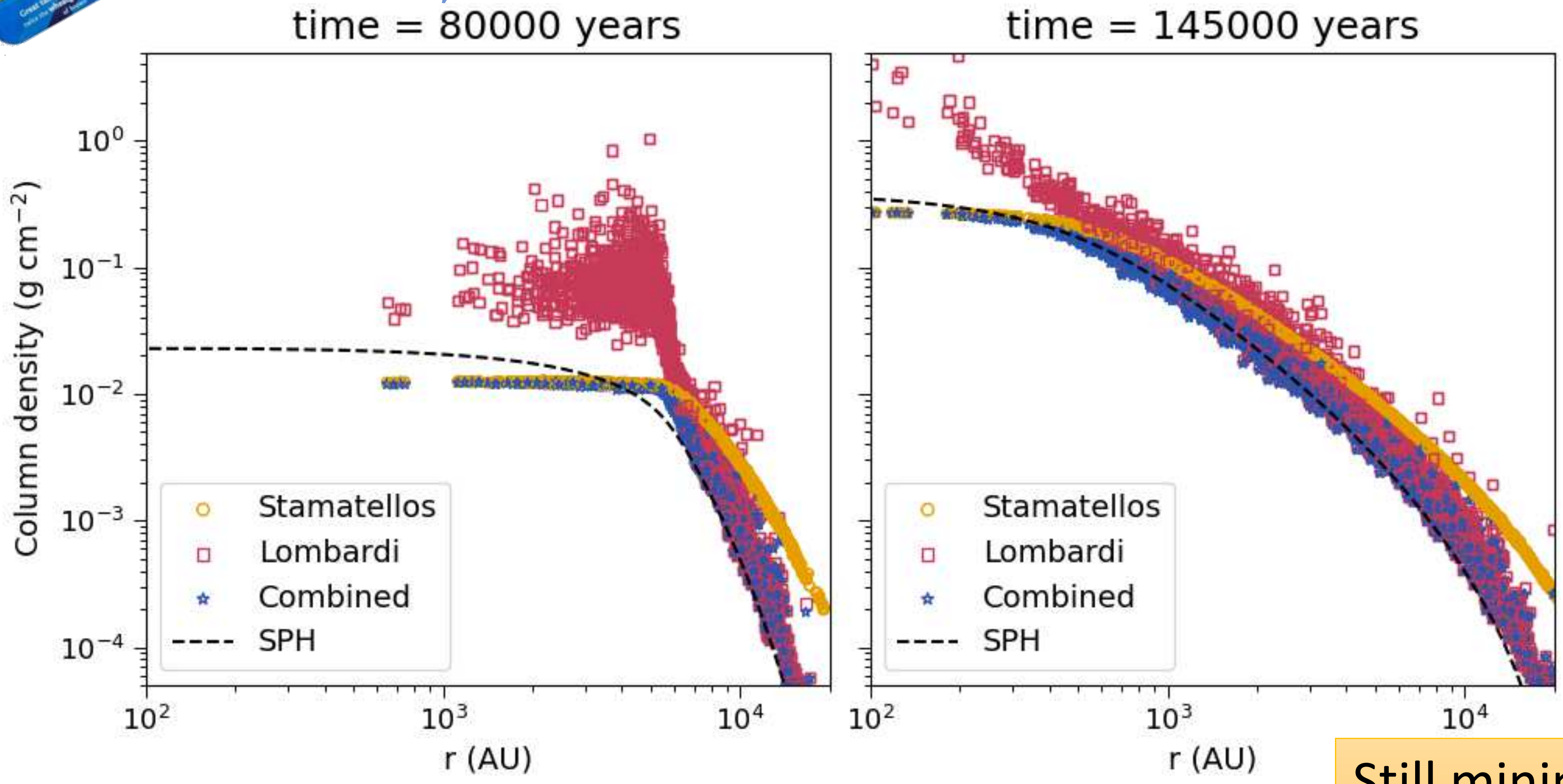


Stamatellos method overestimates column at lower density
Lombardi method overestimates in centre of cloud.

Try combining them

New: "Combined Method"

Collapsing sphere test



$$H_S = \frac{\zeta_n}{\zeta'}\left[\frac{-\psi}{4\pi G\rho}\right]^{1/2},$$

$$H_L = H_{p,i} = \frac{P_i}{|\nabla P_i|}$$

$$H_c = \sqrt{\frac{1}{H_S^{-2} + H_L^{-2}}}$$

Still minimal extra cost 😀

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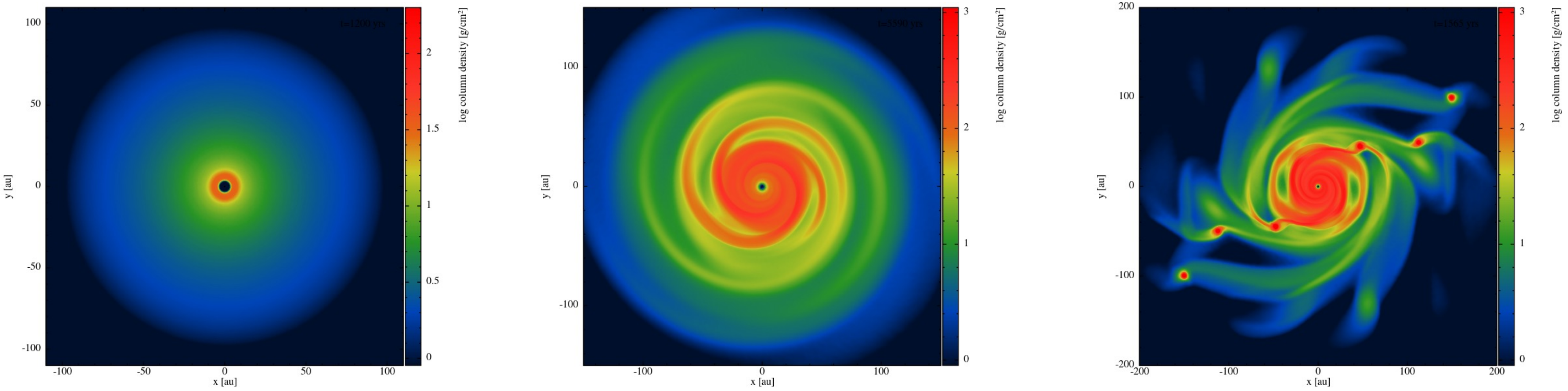
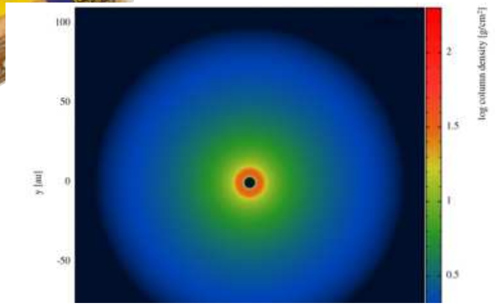
New: ”Combined Method”

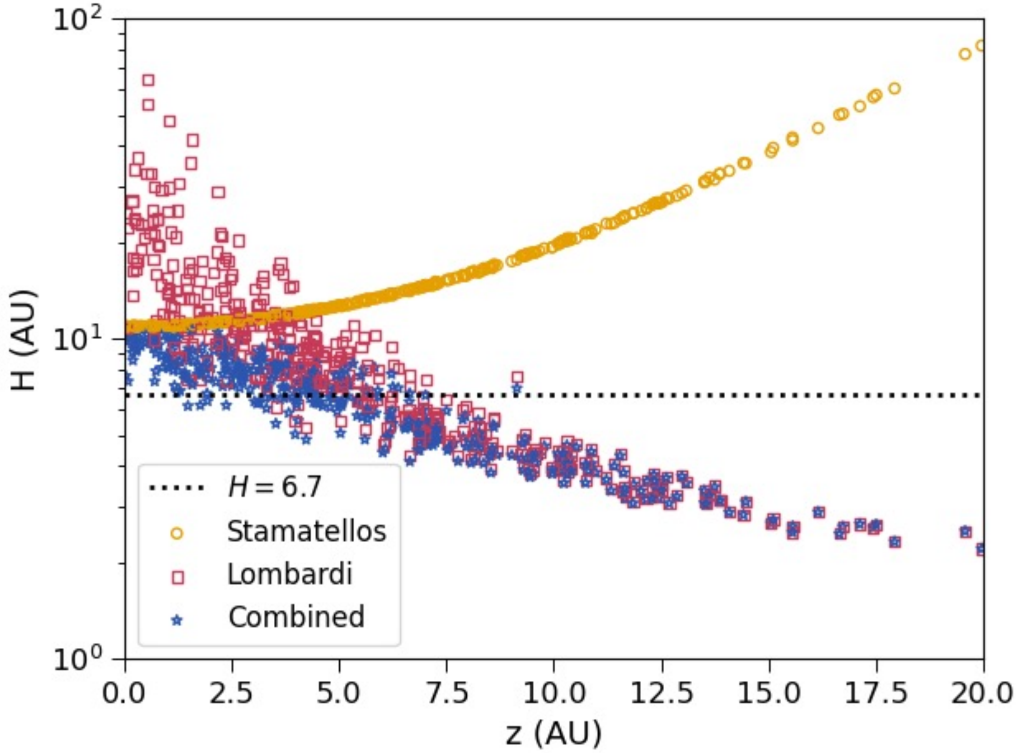
low and high mass discs tests

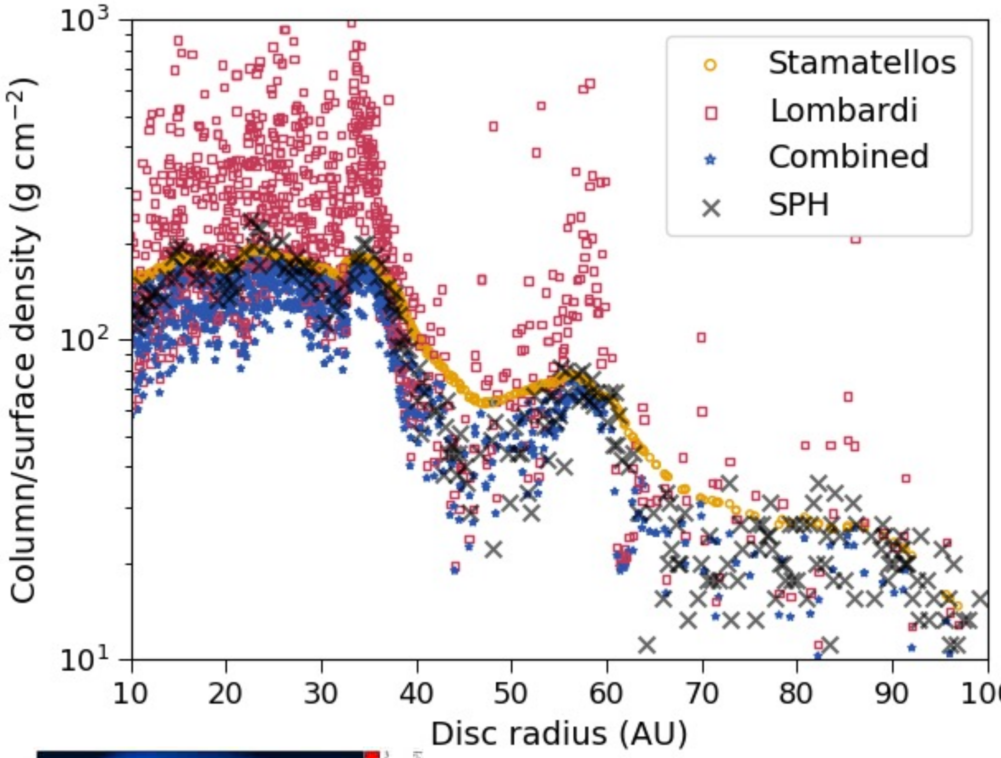


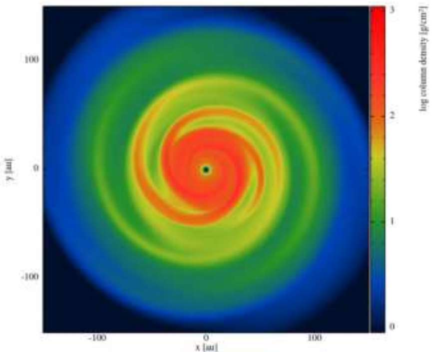
New: "Combined Method"

low and high mass discs tests









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New Combined Method	$H_c = \sqrt{\frac{1}{{H_S}^{-2} + {H_L}^{-2}}}$	• Negligible extra cost • Better than predecessors	• Still poor at centre • Still not great for discs

New: "Modified Lombardi" Method

Can we do even better?

Yes!

In a disc geometry, the mid-plane density, scale-height and column density are related by $\Sigma = \sqrt{\frac{\pi}{2}} H_*$ where $H_* = {}^{c_s}/_{\Omega}$

But if the disc's self-gravity compresses the gas further:

$$H_0 = \frac{H_* t}{\sqrt{1 + 1/(tQ_{3D})}}$$

$$t = \sqrt{\pi/2}$$

From vertical hydrostatic equilibrium and substituting

$$Q_{3D} = {}^{\Omega^2}/_{4\pi G\rho(0)}$$

$$H_{ML} = \sqrt{\frac{1}{H_L^{-2} + H_0^{-2}}}$$

Still minimal extra cost 😀

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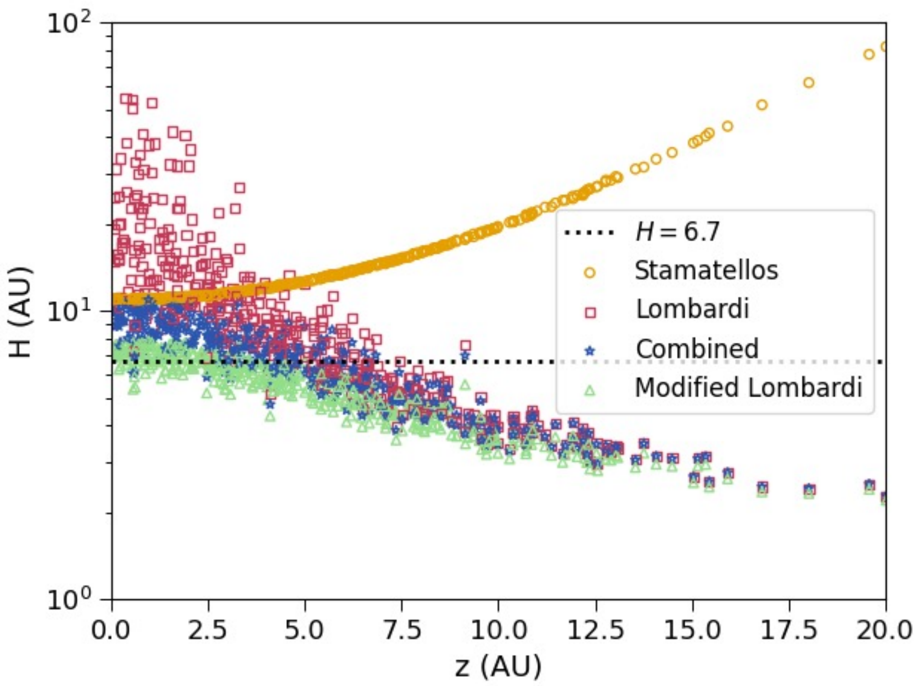
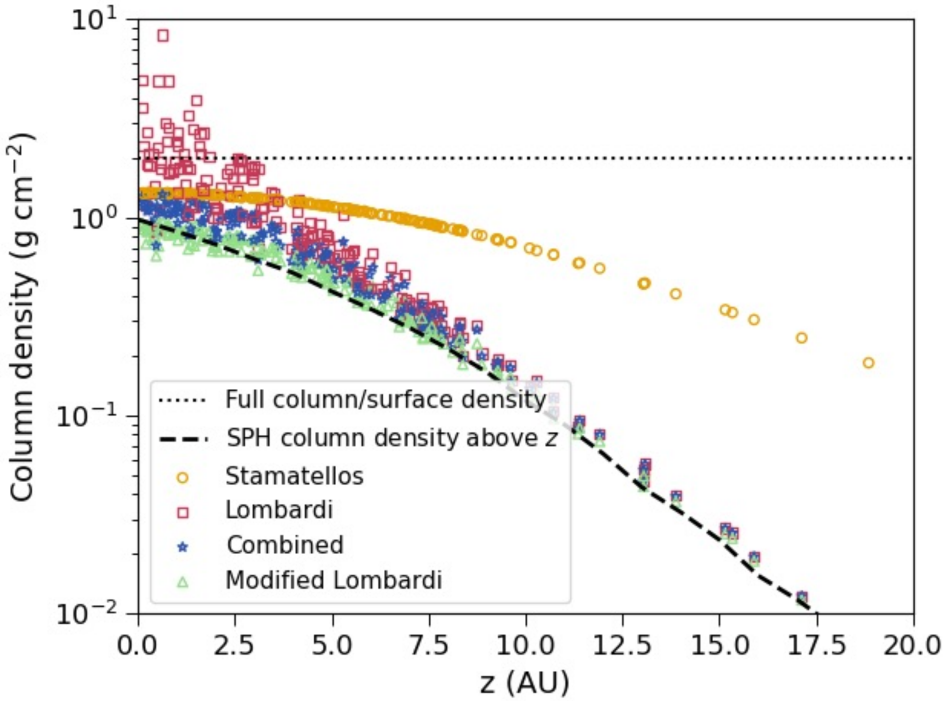
New: "Modified Lombardi" Method





$$H_0 = \frac{H_* t}{\sqrt{1 + 1/(tQ_{3D})}}$$

$$t = \sqrt{\pi/2}$$

$$H_{ML} = \sqrt{\frac{1}{{H_L}^{-2} + {H_0}^{-2}}}$$

Still minimal extra cost 😀

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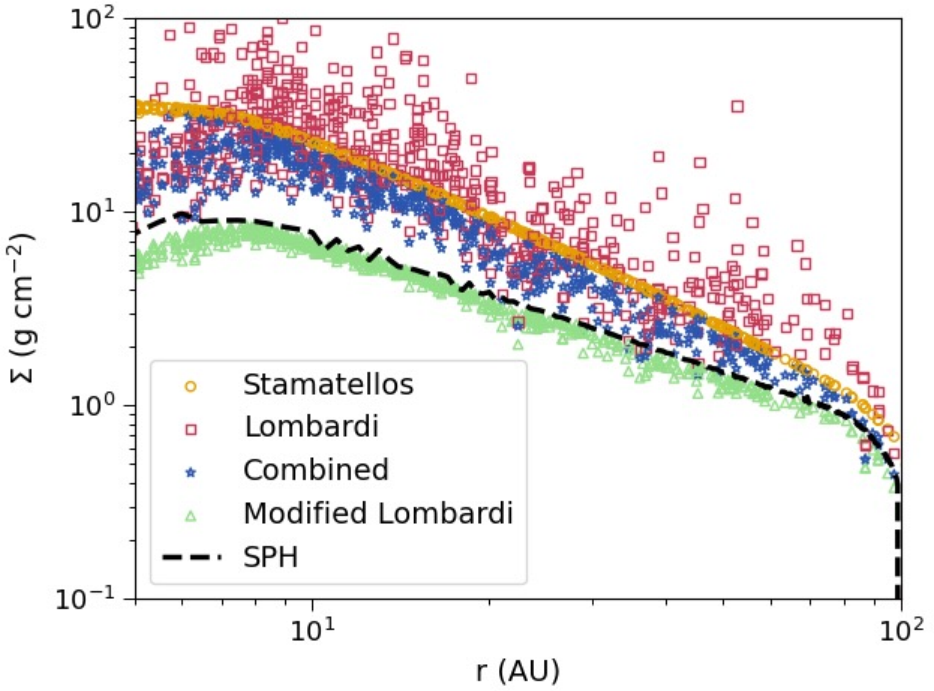
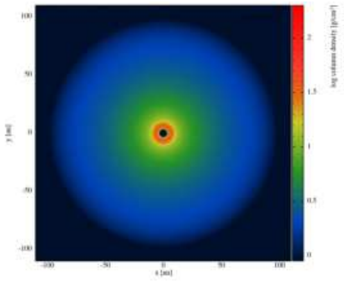
New Modified Lombardi	$H_0 = \frac{H_* t}{\sqrt{1 + 1/(tQ_{3D})}}$	• Negligible extra cost • Better than predecessors	• Still can't do e.g. shadows

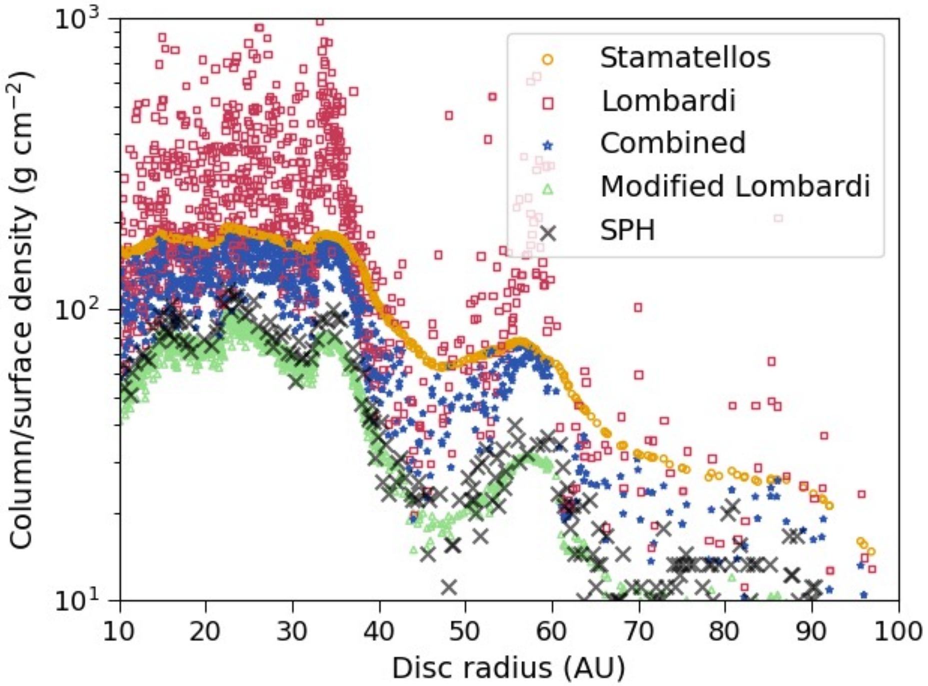
Does it work?

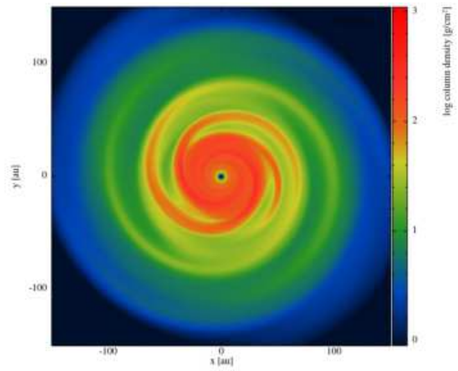
Low mass disc





High mass disc 1





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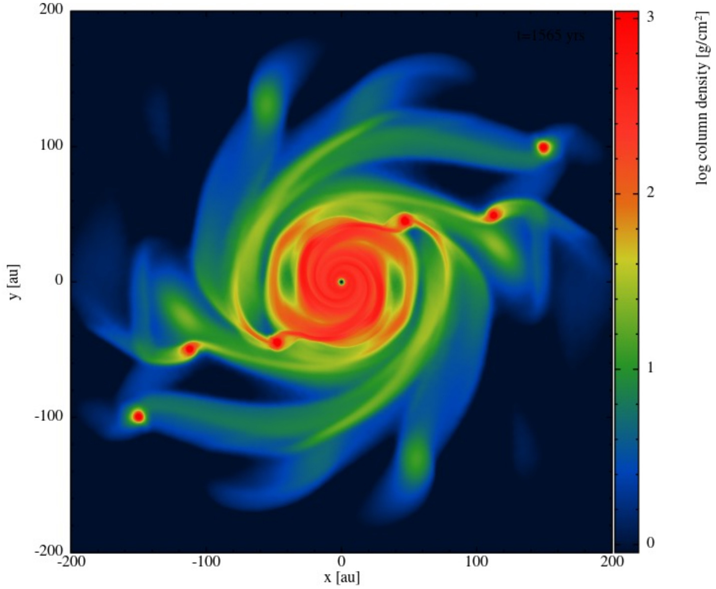
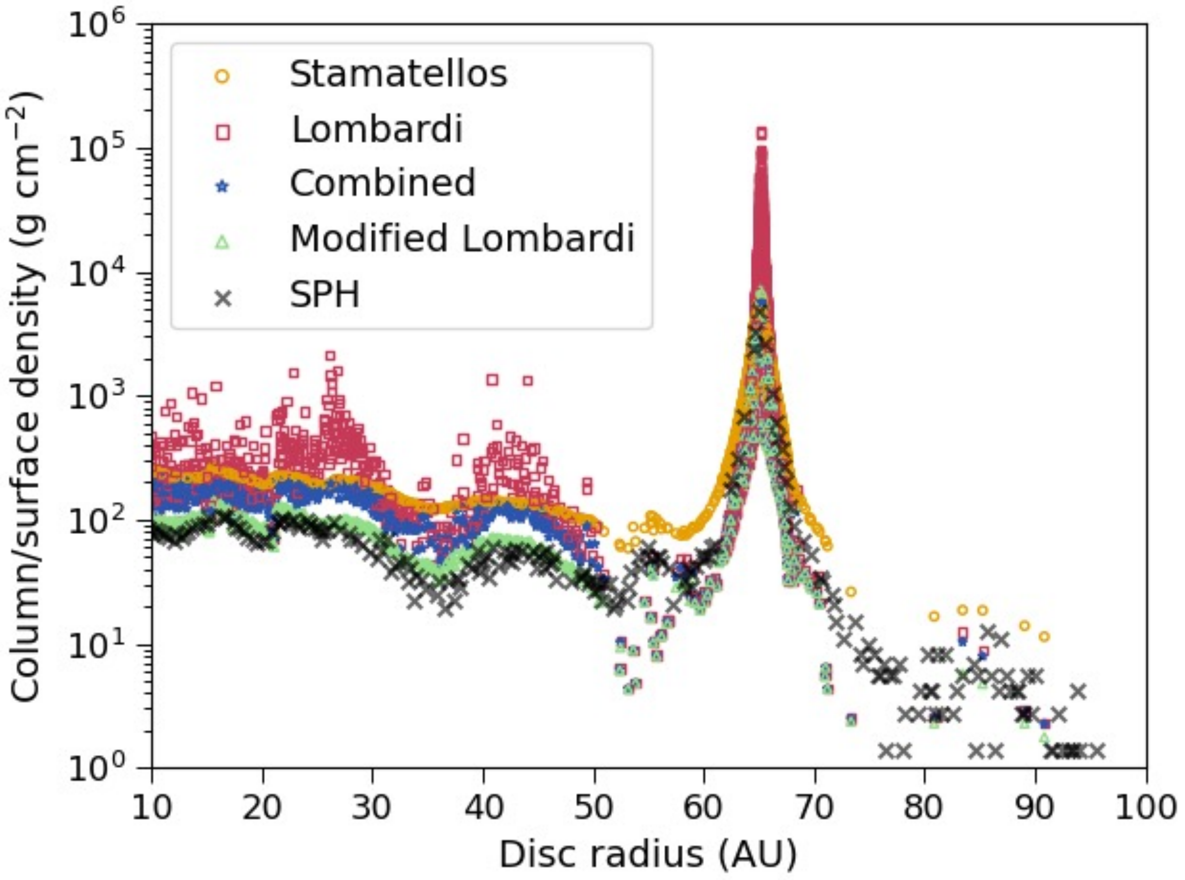
Does it work?

High mass disc 2

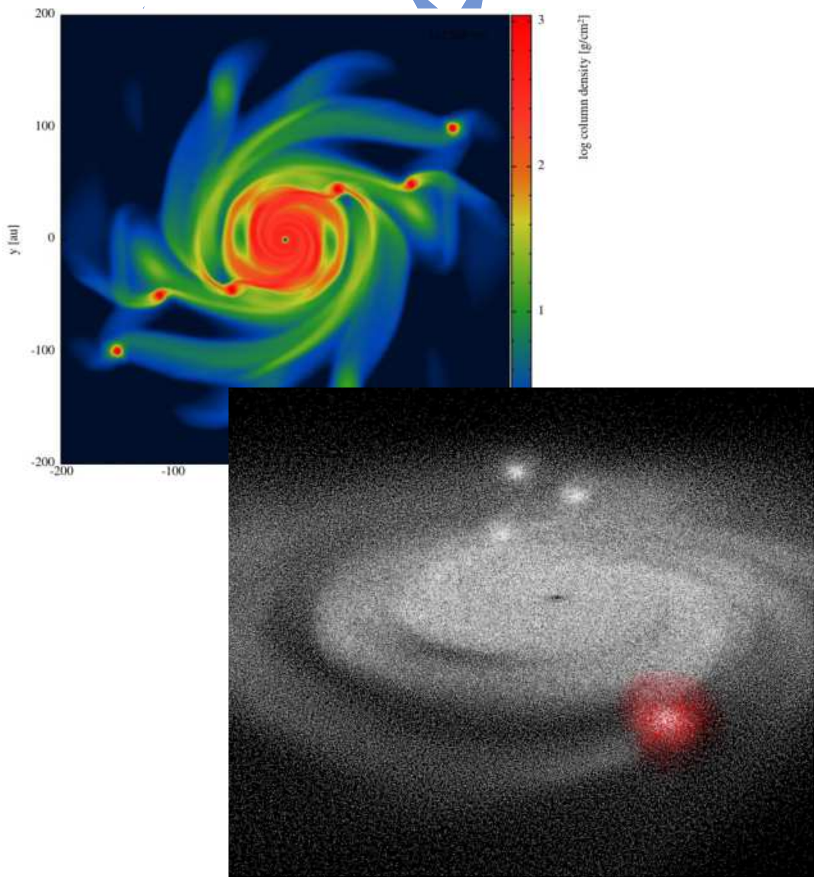


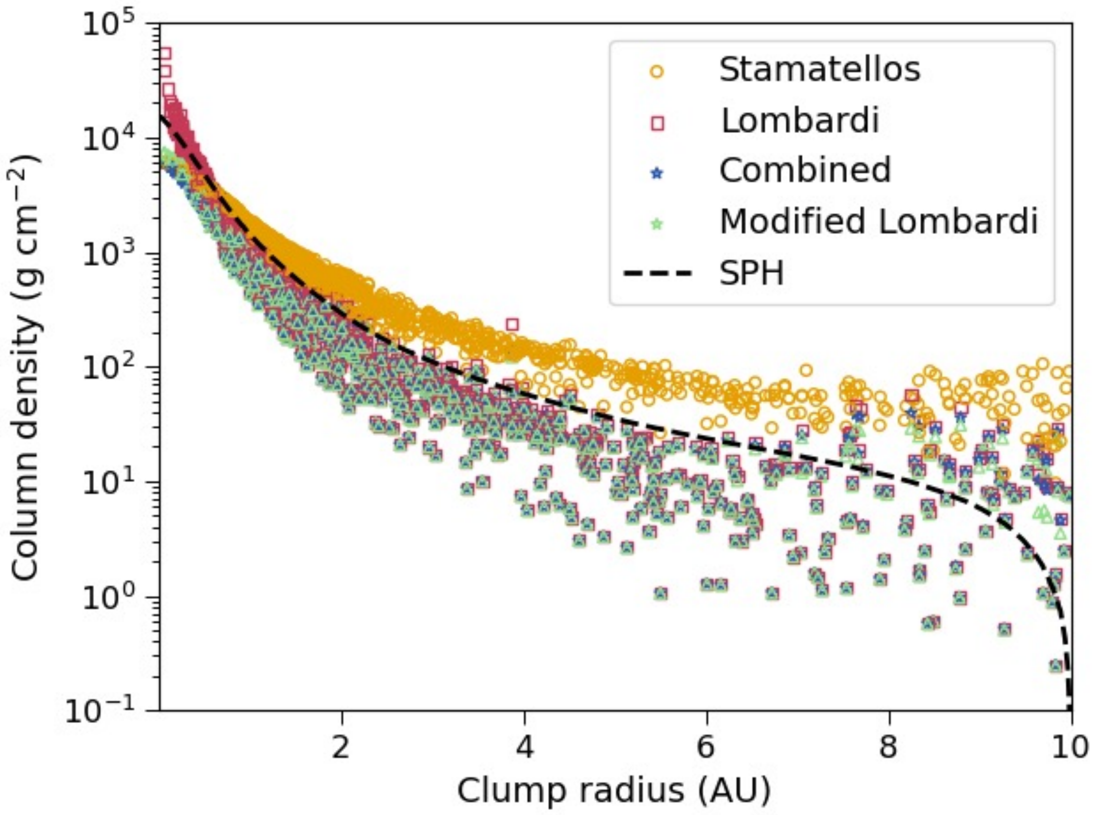


New methods demonstrate far better estimates of column density and scale height.

Does it work?

Higher mass disc 2

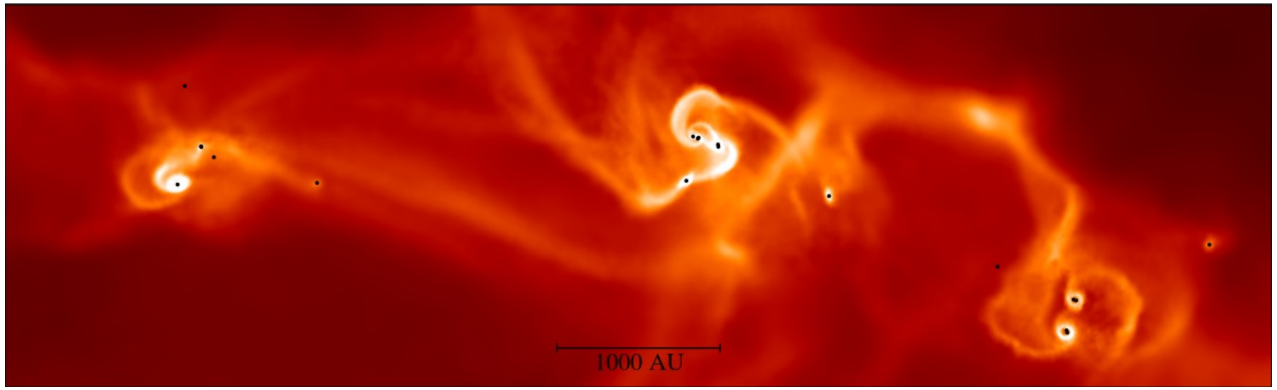




New methods demonstrate far better estimates of column density and scale height.

What next?

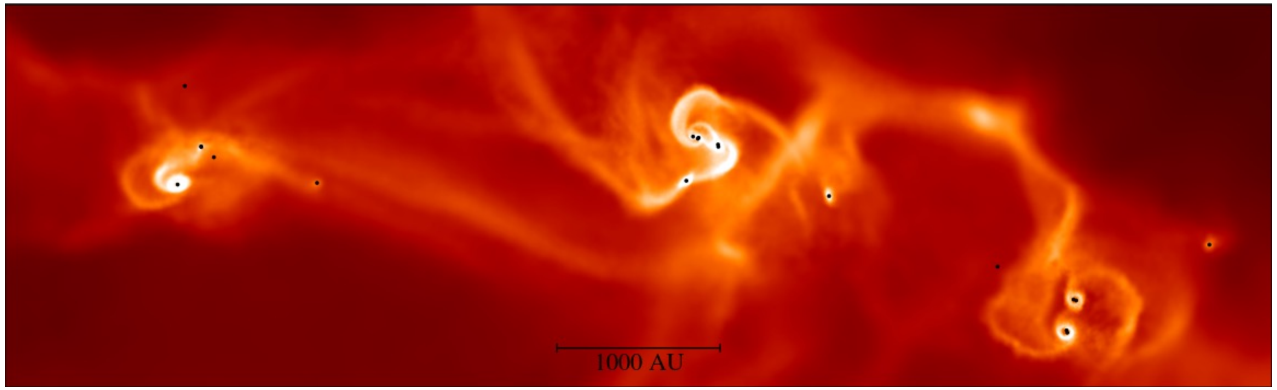
- submit paper!
- fragmentation in young discs



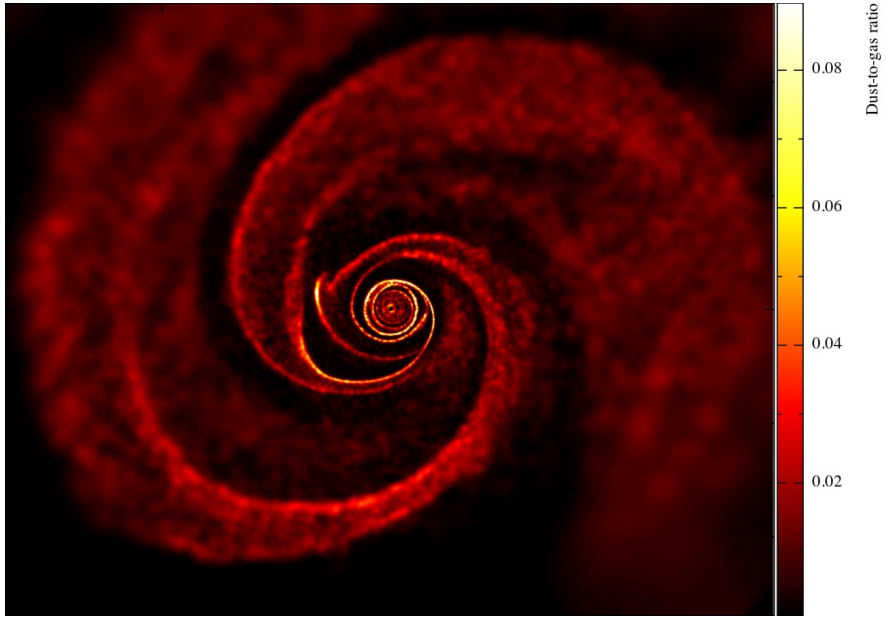
- synthetic observations of very young discs

+ Tom Bending (Exeter)

What next?



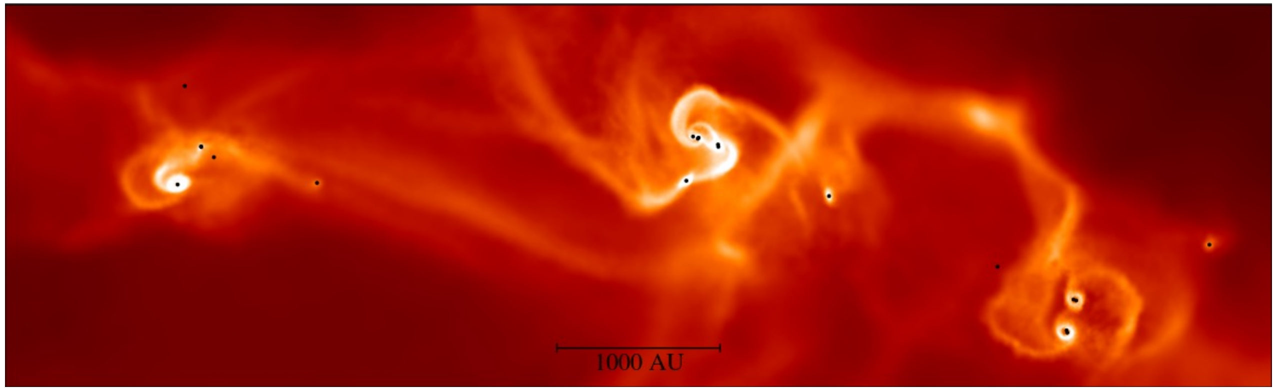
- Dust transport in 'realistic' young discs
- Merge with main repo …?!



Projected dust-to-gas mass ratio in a quasi-stable disc extracted from the Bate (2012) simulation. 50 cm dust grains were injected into the gaseous disc with an initial dust fraction of 0.01 and allowed to evolve taking drag and self-gravity into account.

Alison Young

Summary



- When cooling is important – use these new methods ;)
- radiative cooling need not be expensive
- Can couple with flux-limited diffusion for radiative transfer approximation
- Can prescribe a spatially varying background temperature for e.g. stellar heating.

Thanks to Adam Koval for making many figures!