Surfing GI waves with PHANTOM

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2nd European PHANTOM code family users workshop

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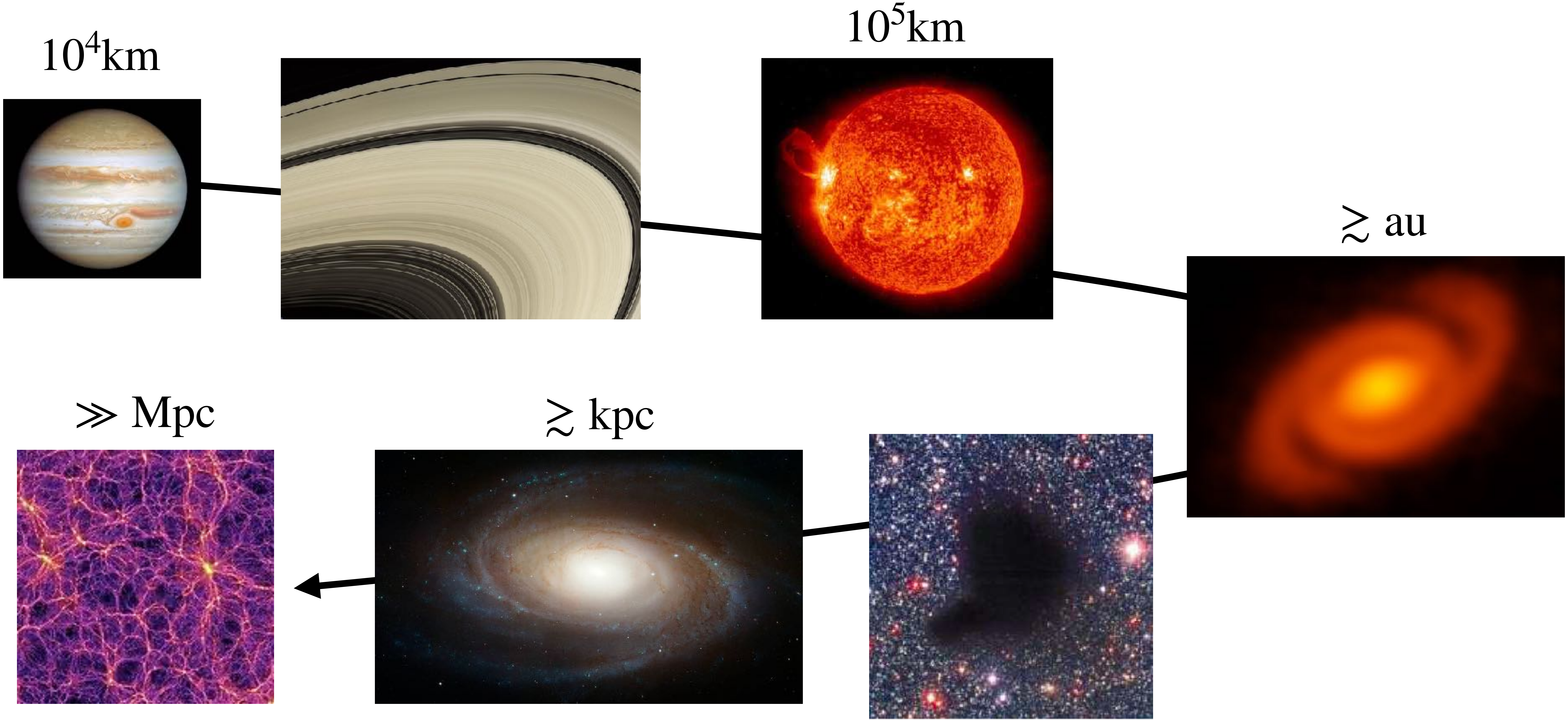
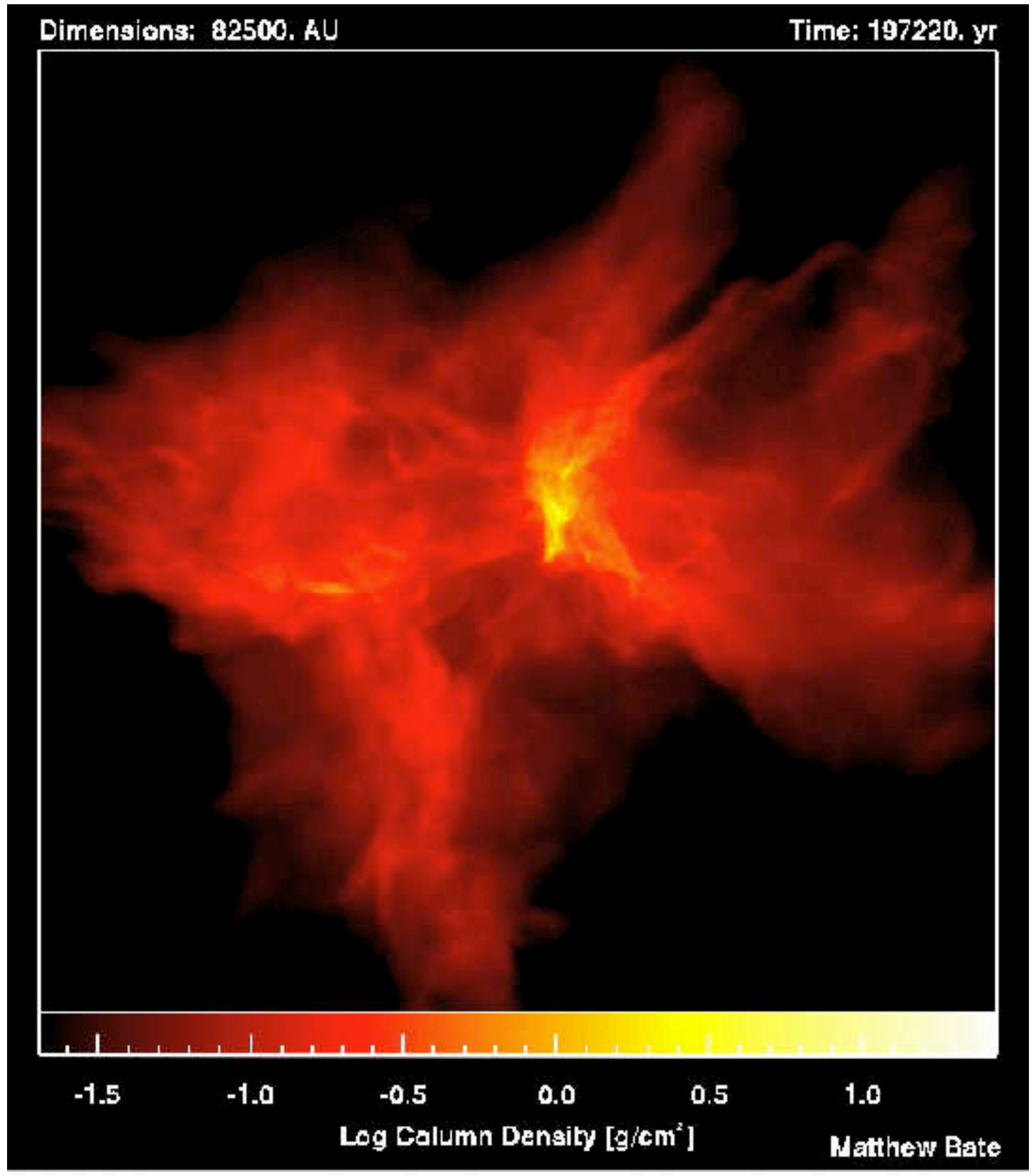
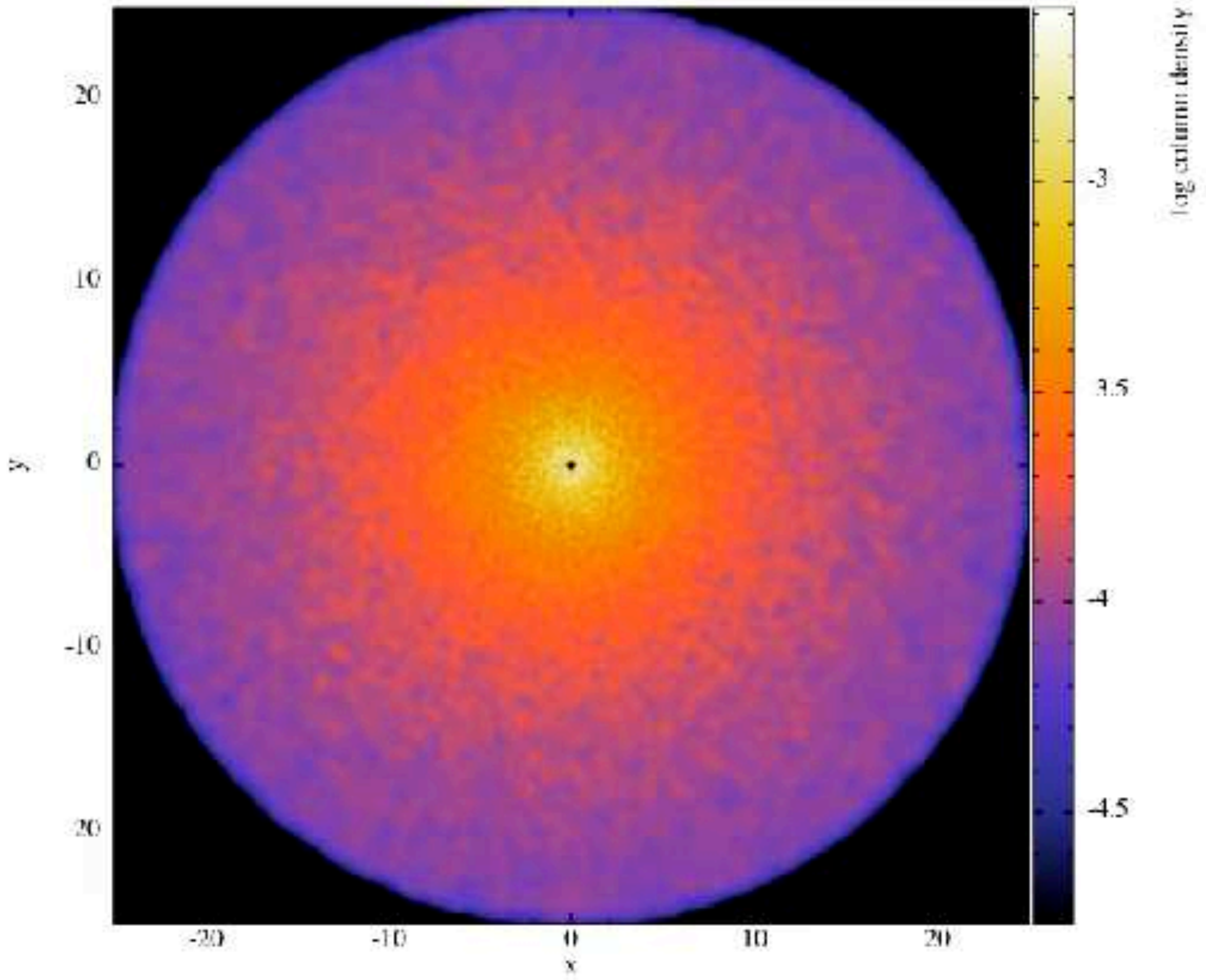
P. Armitage, R. Booth, S. Ceppi, C. Leedham

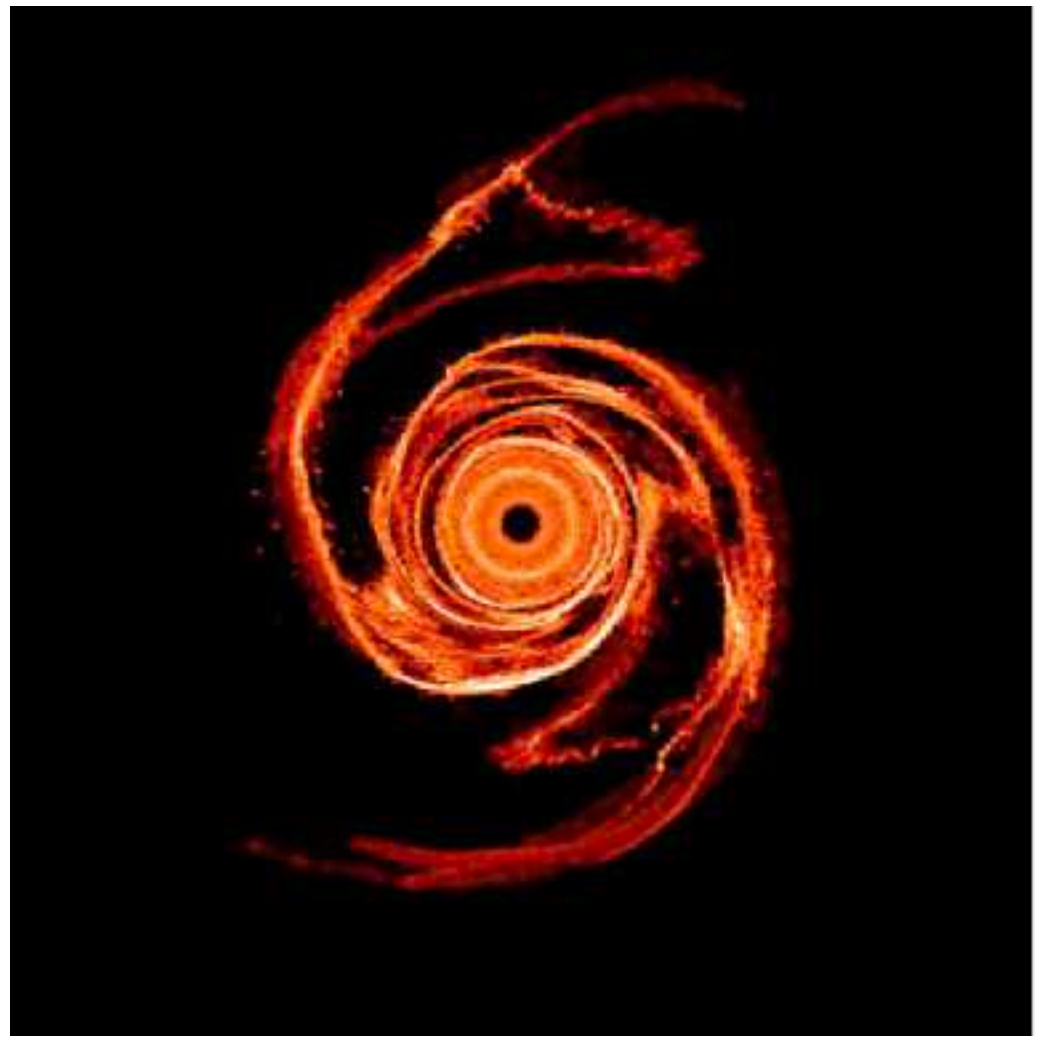
Self-gravity at different scales

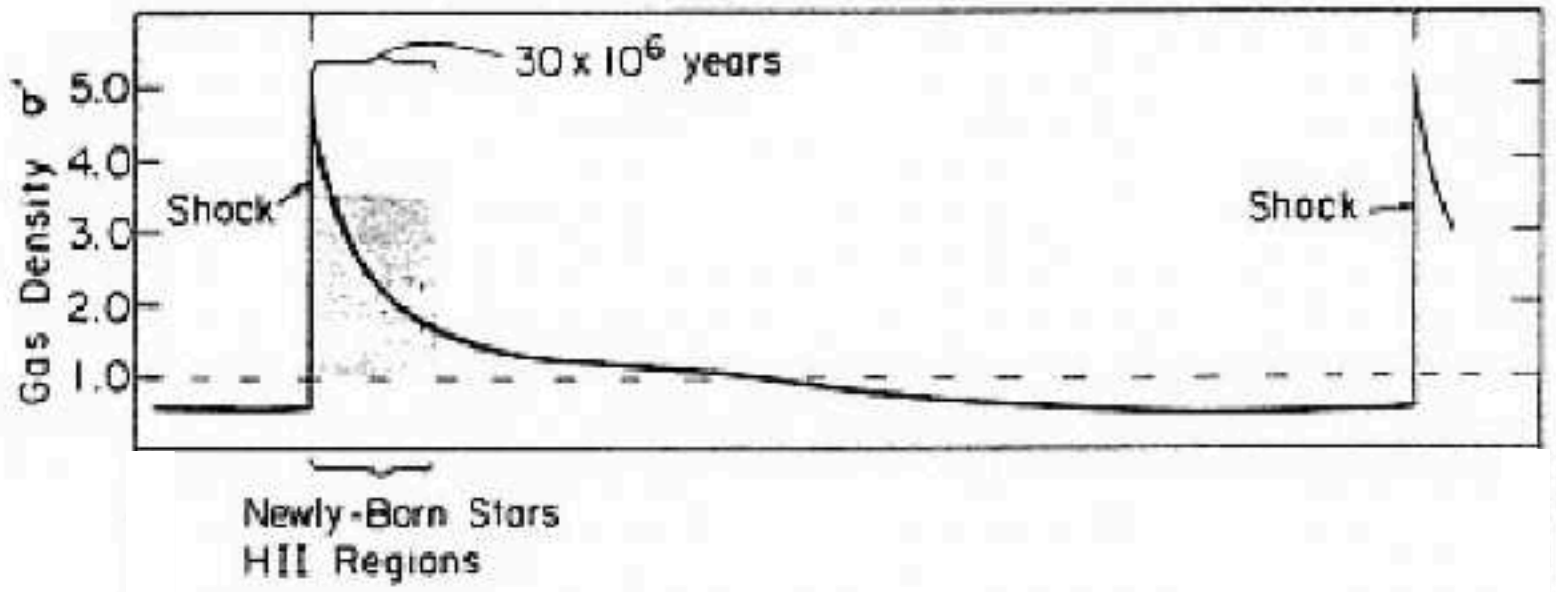


Gravitational instability









All you need to know about GI (in discs!)

Dispersion relation

$$m^2(\Omega_p - \Omega)^2 = c_s^2 k^2 - 2\pi G \Sigma |k| + \kappa^2$$

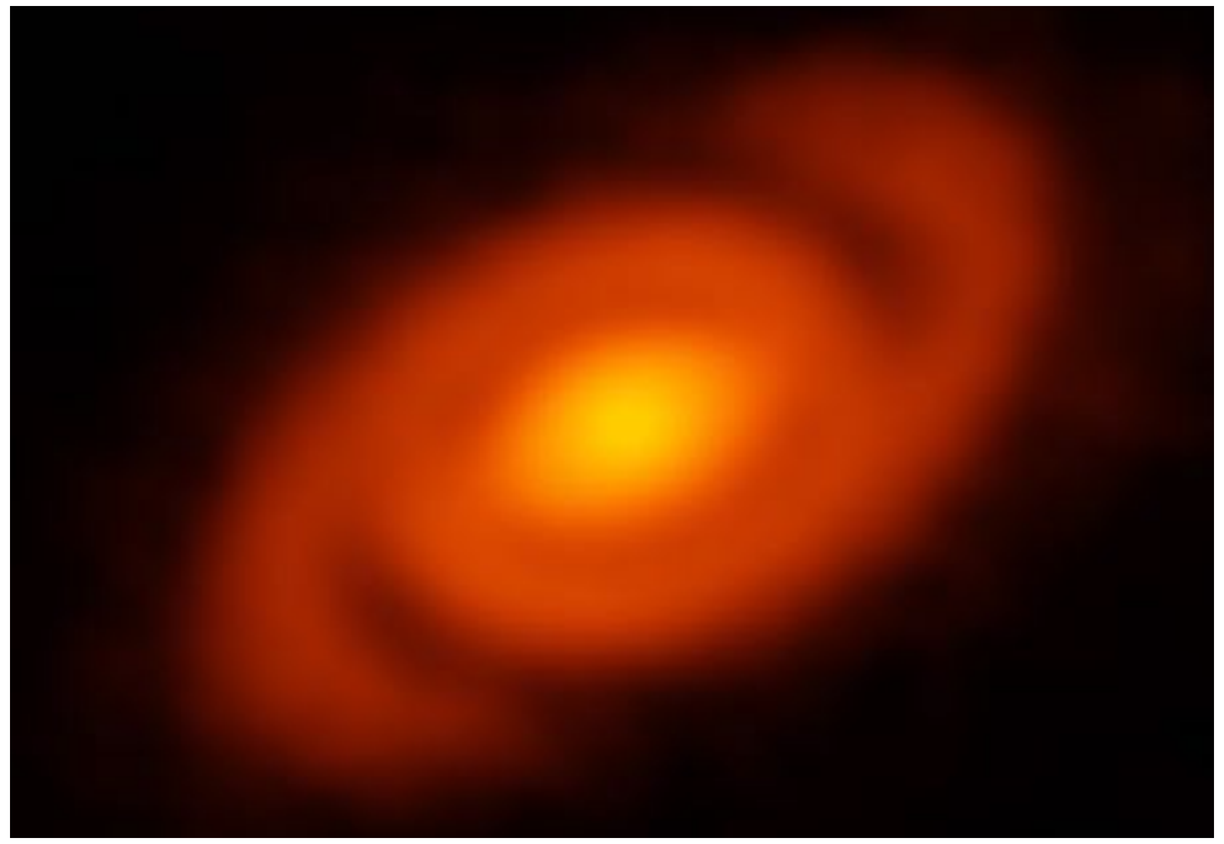
Perturbation frequency

Sound speed
stabilising

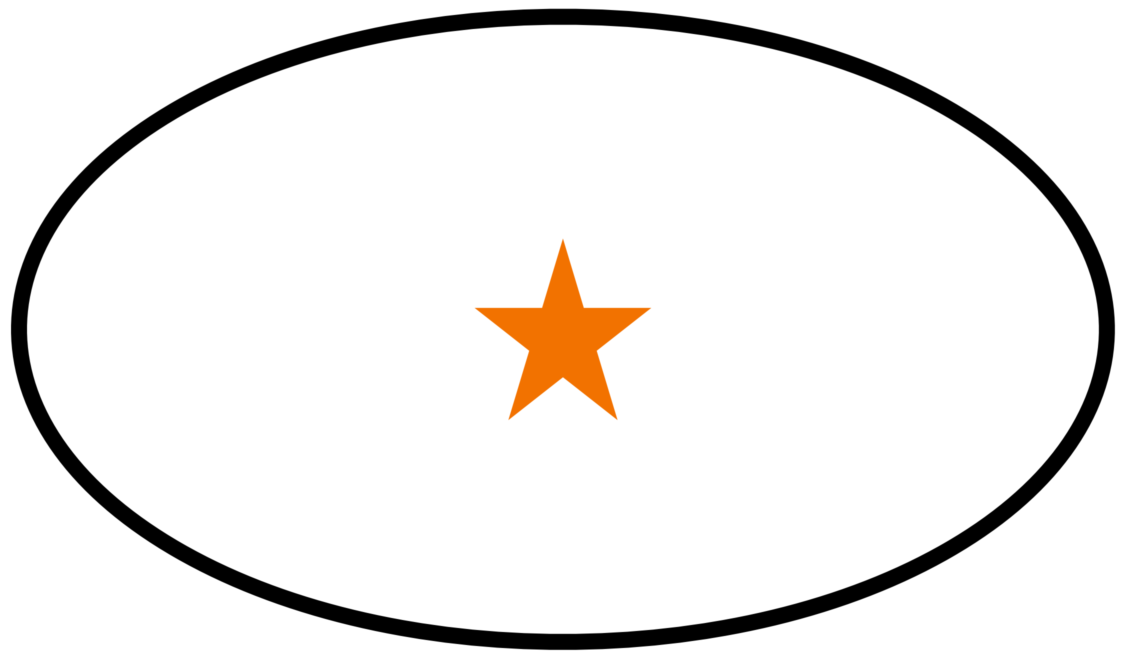
Surface density
destabilising

Epicyclic frequency
stabilising

$$Q = \frac{c_s \kappa}{\pi G \Sigma}$$

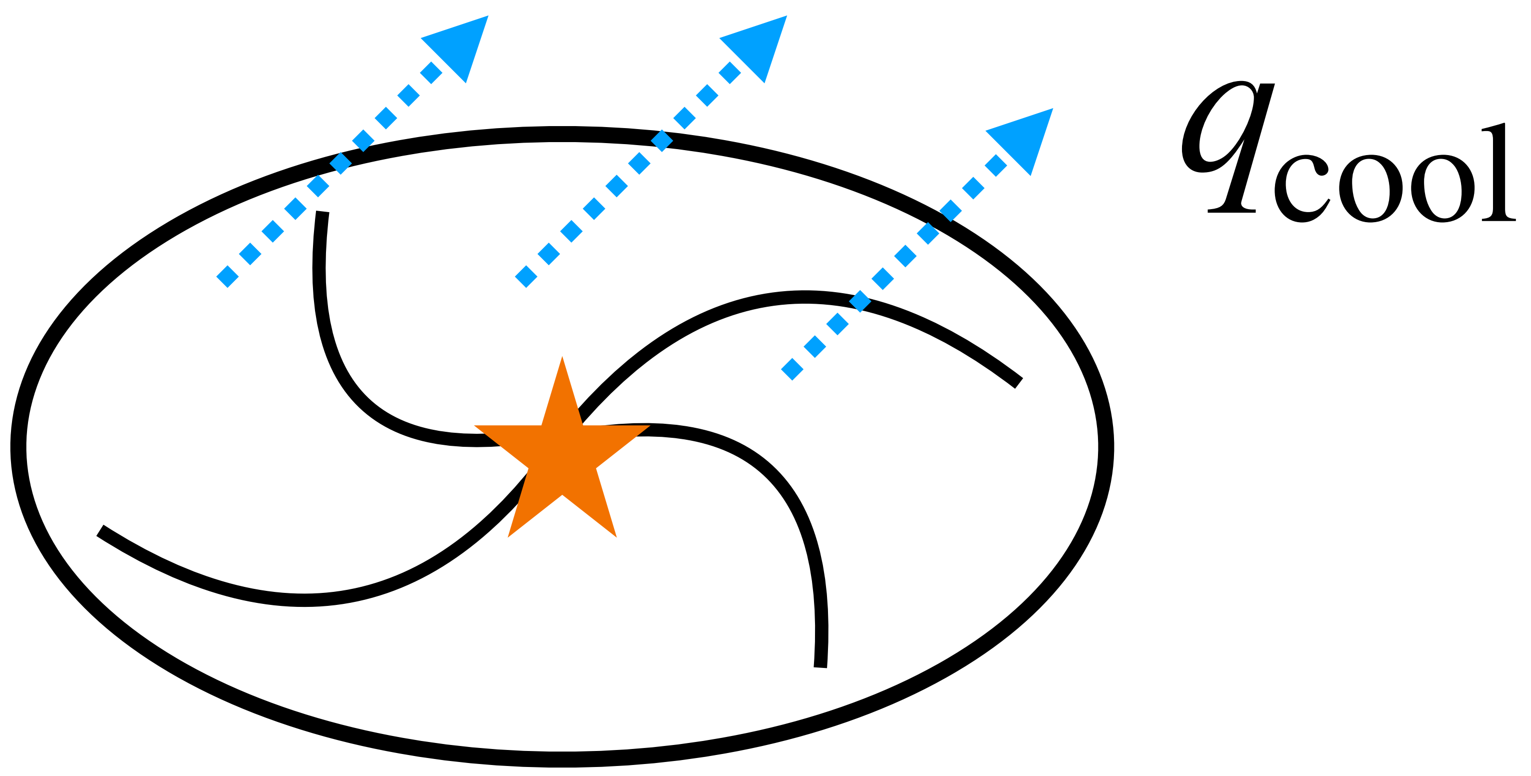


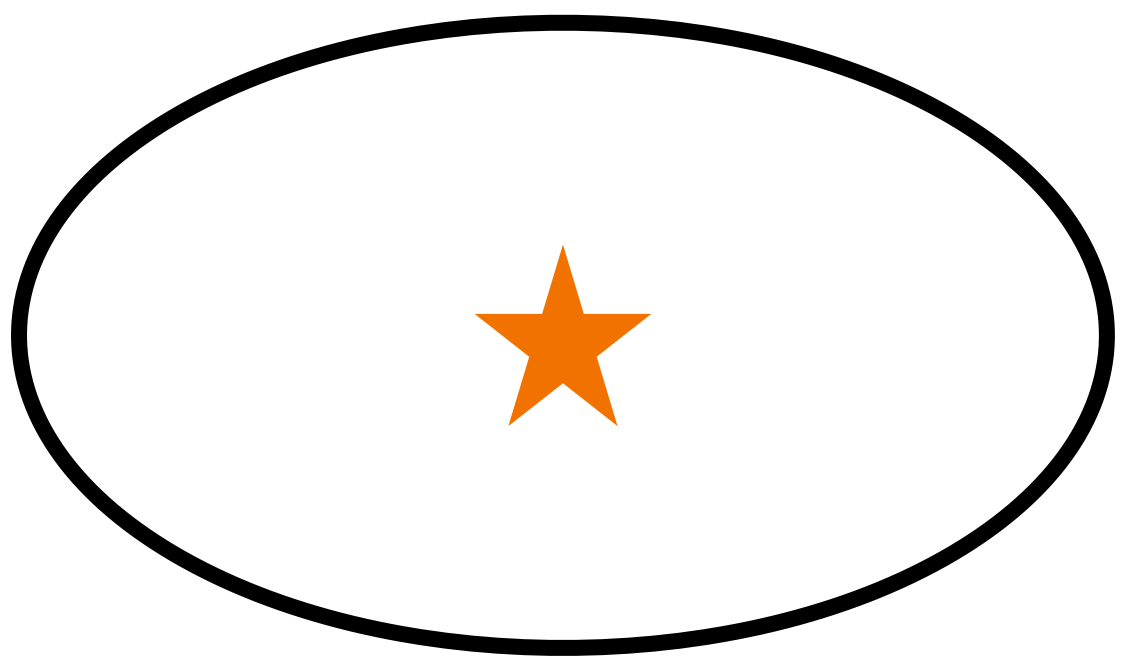
Path to GI



$$Q >> 1$$

Path to GI



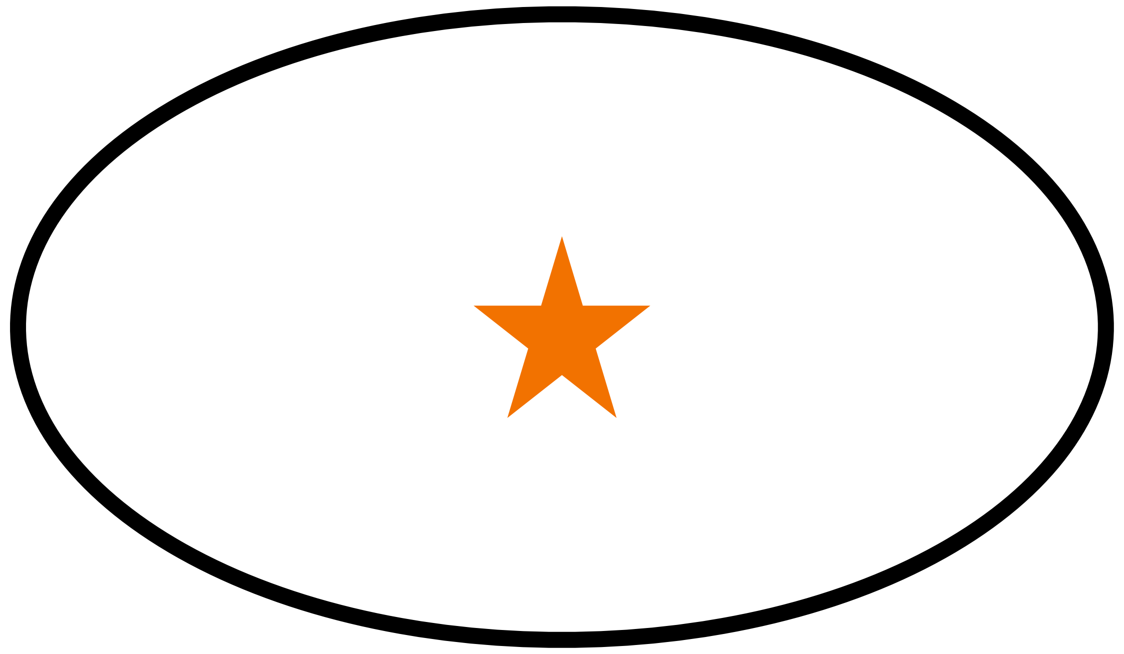


$Q >> 1$

Temperature regulated

Radiative cooling : Q decreases

Path to GI



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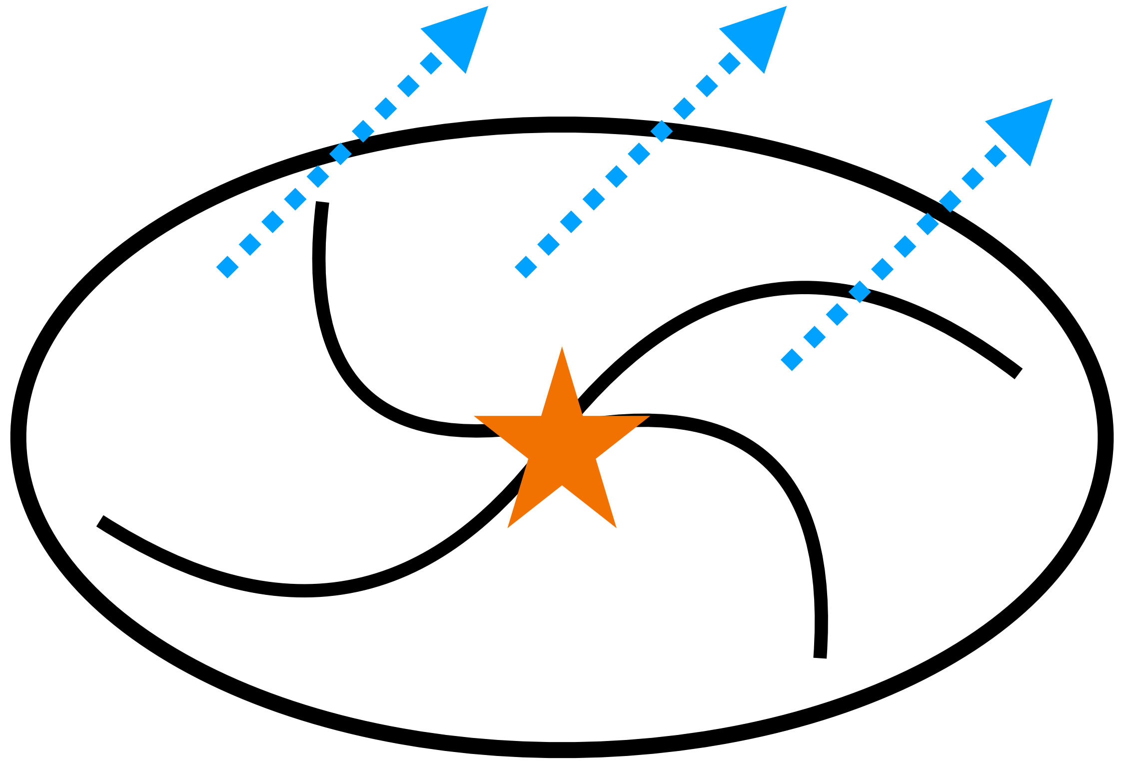
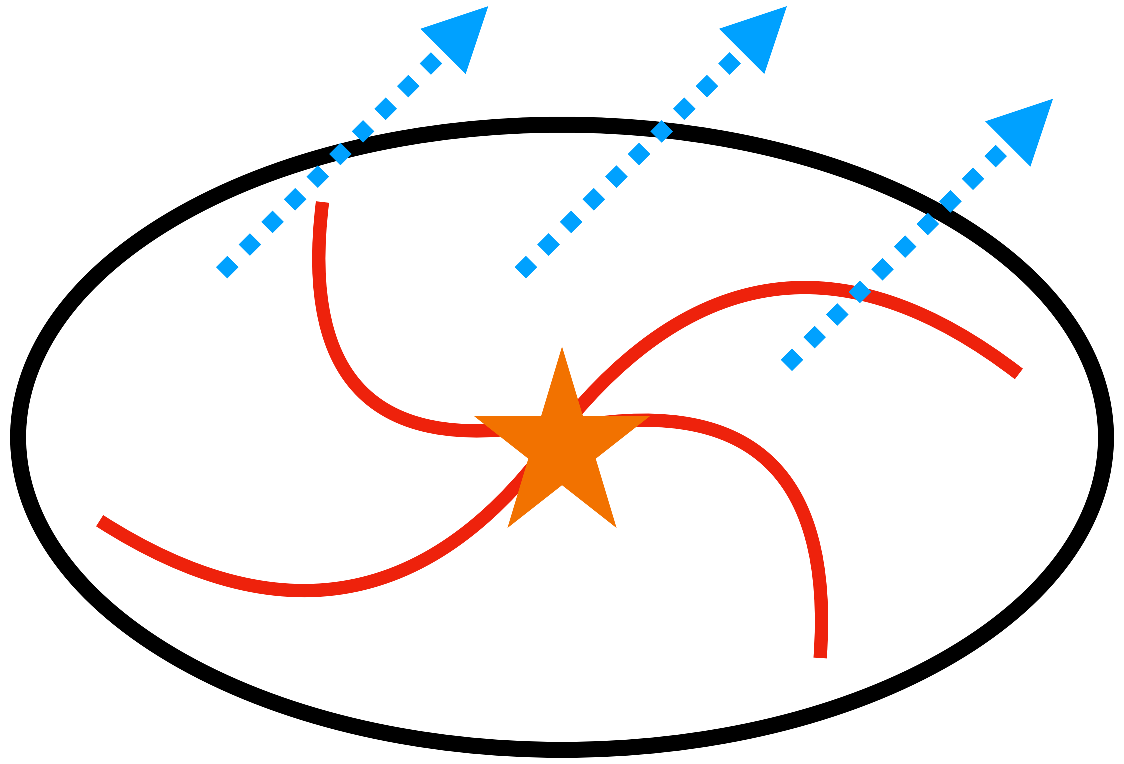


$q_{\rm cool}$



$q_{\rm heat}$

$Q = 1$

Temperature regulated

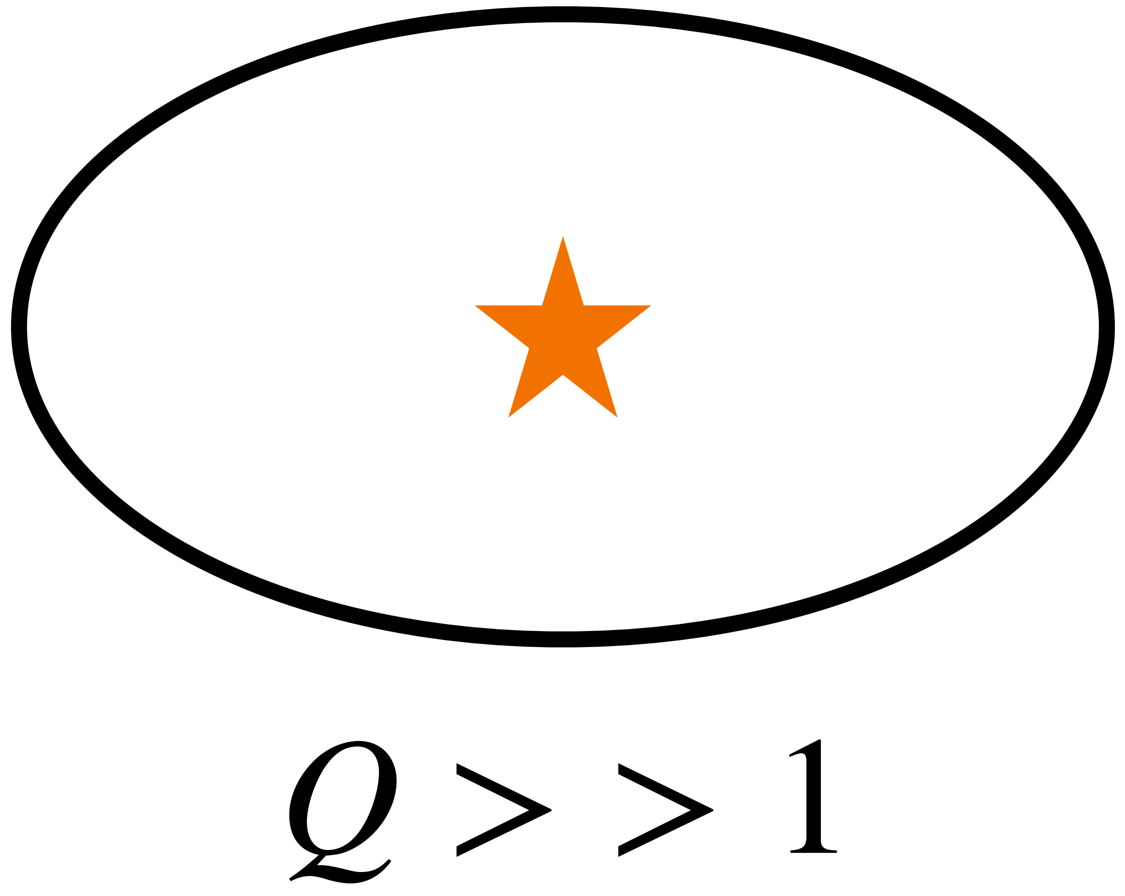
Radiative cooling : Q decreases

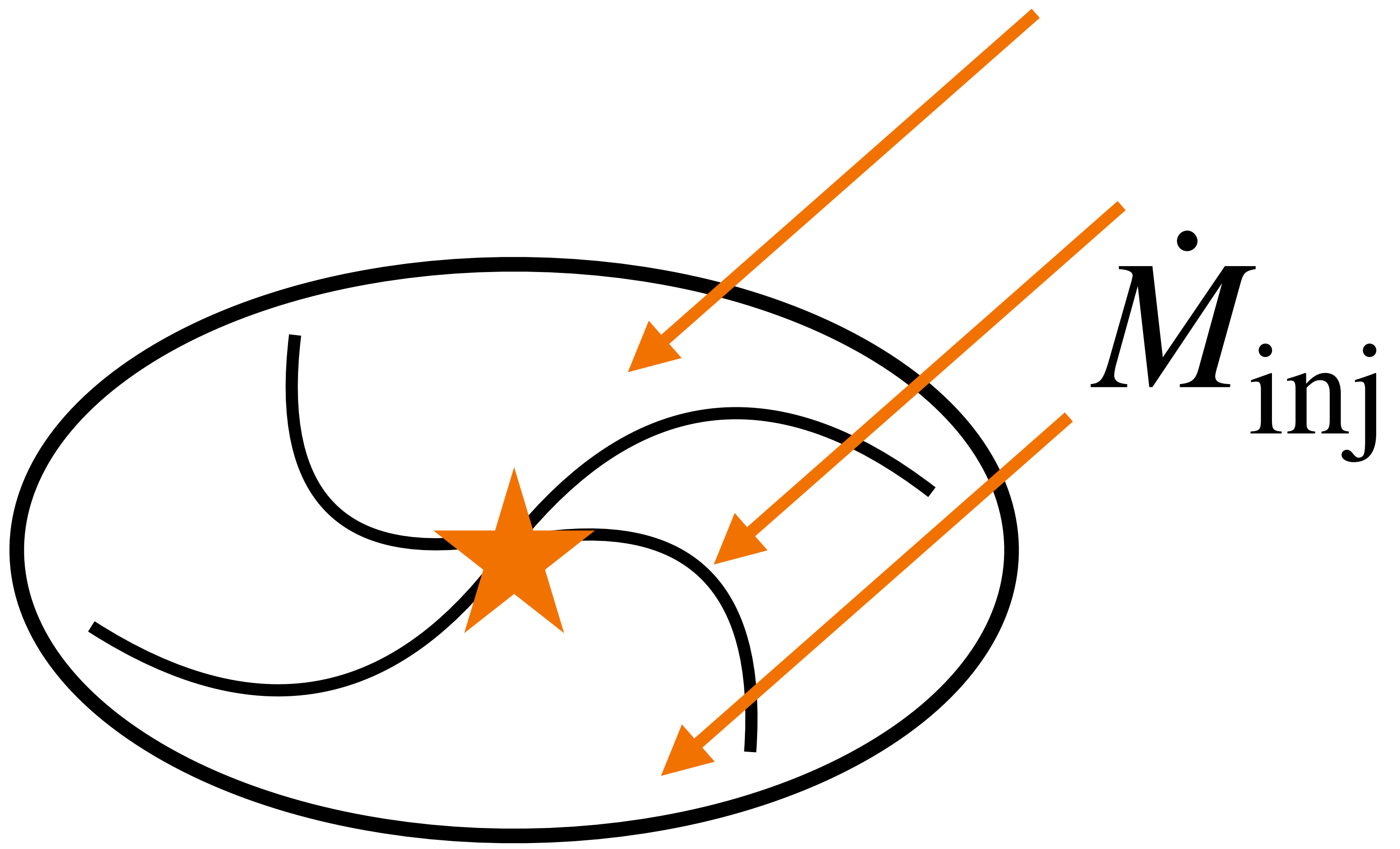
Shock heating from spiral arms : Q = 1

Path to GI

Mass regulated

Mass injection : Σ increases and Q decreases





Path to GI

Longarini et al. subm.

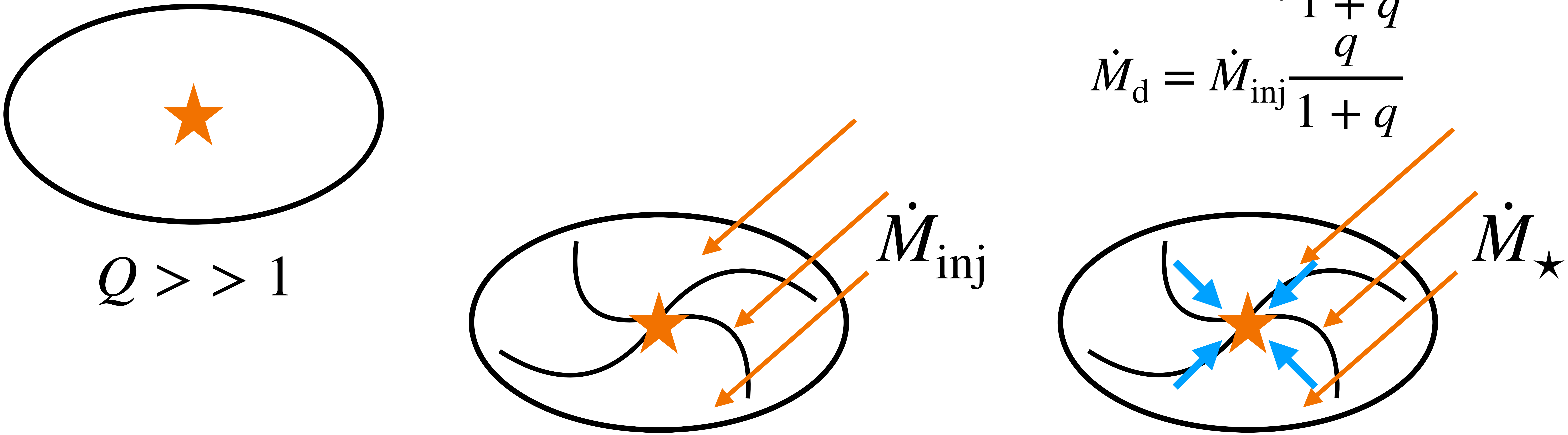
Mass regulated

Mass injection : Σ increases and Q decreases

Mass accreted : Q = 1

$$\dot{M}_\star = \dot{M}_{\rm inj}\frac{1}{1+q}$$

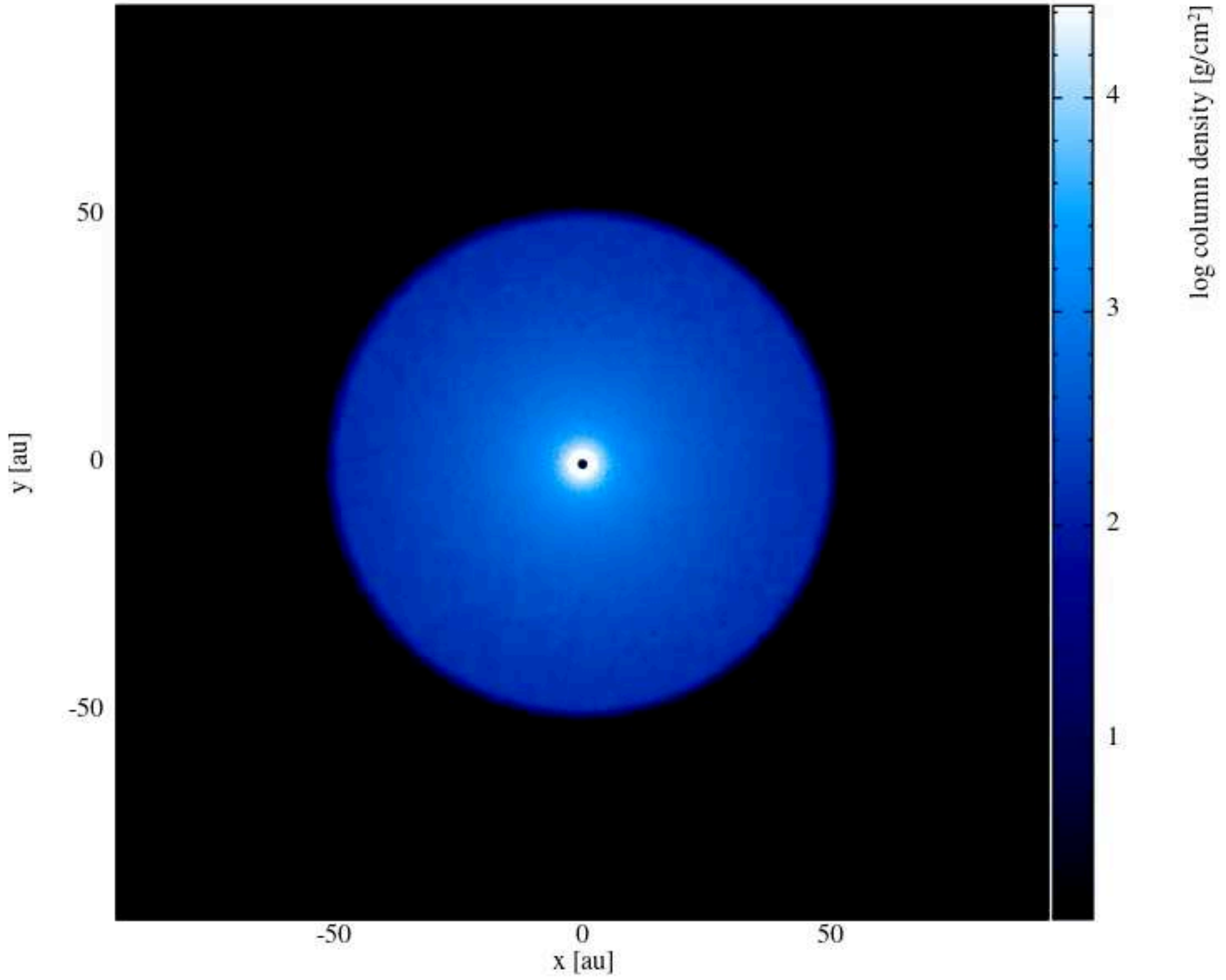
$$\dot{M}_{\rm d} = \dot{M}_{\rm inj}\frac{q}{1+q}$$



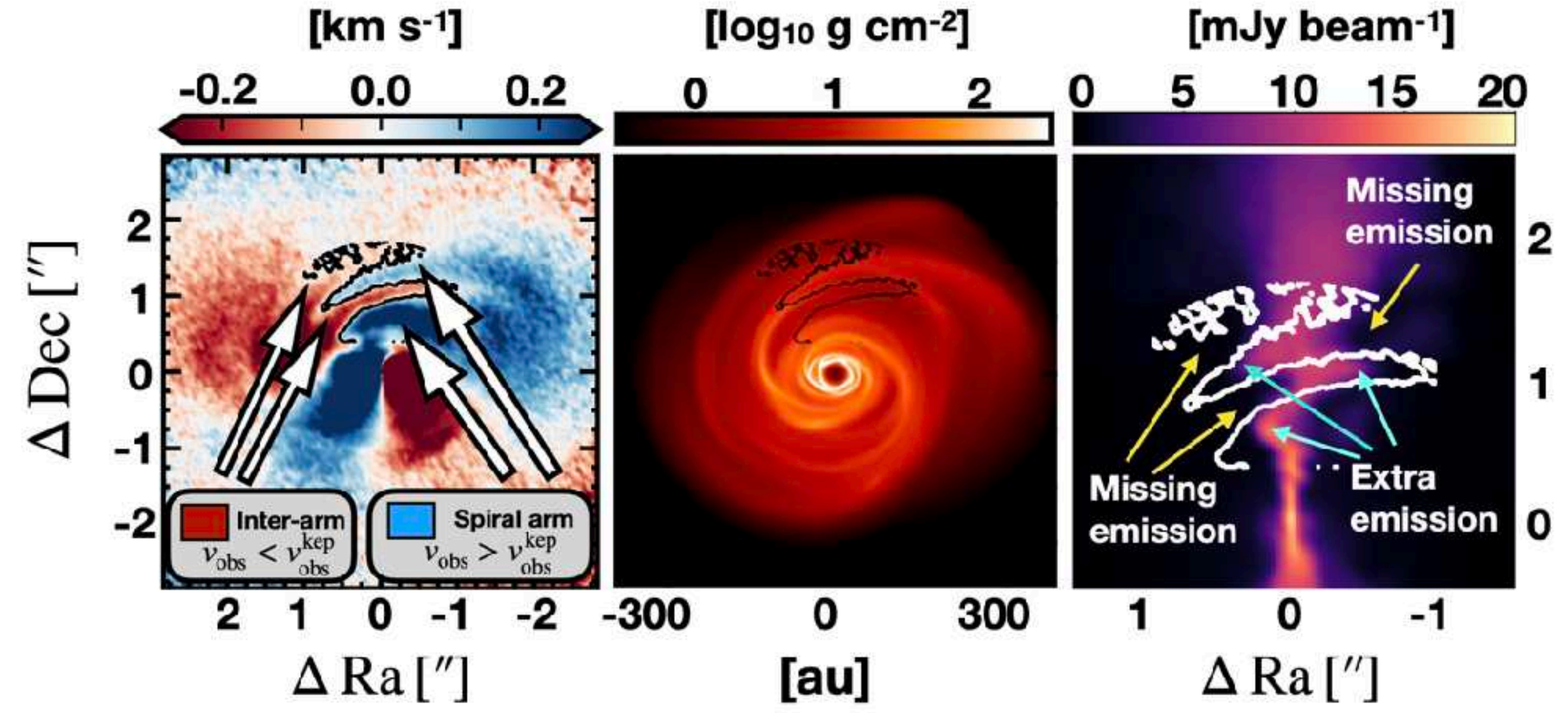
$Q = 1$

GI and cooling

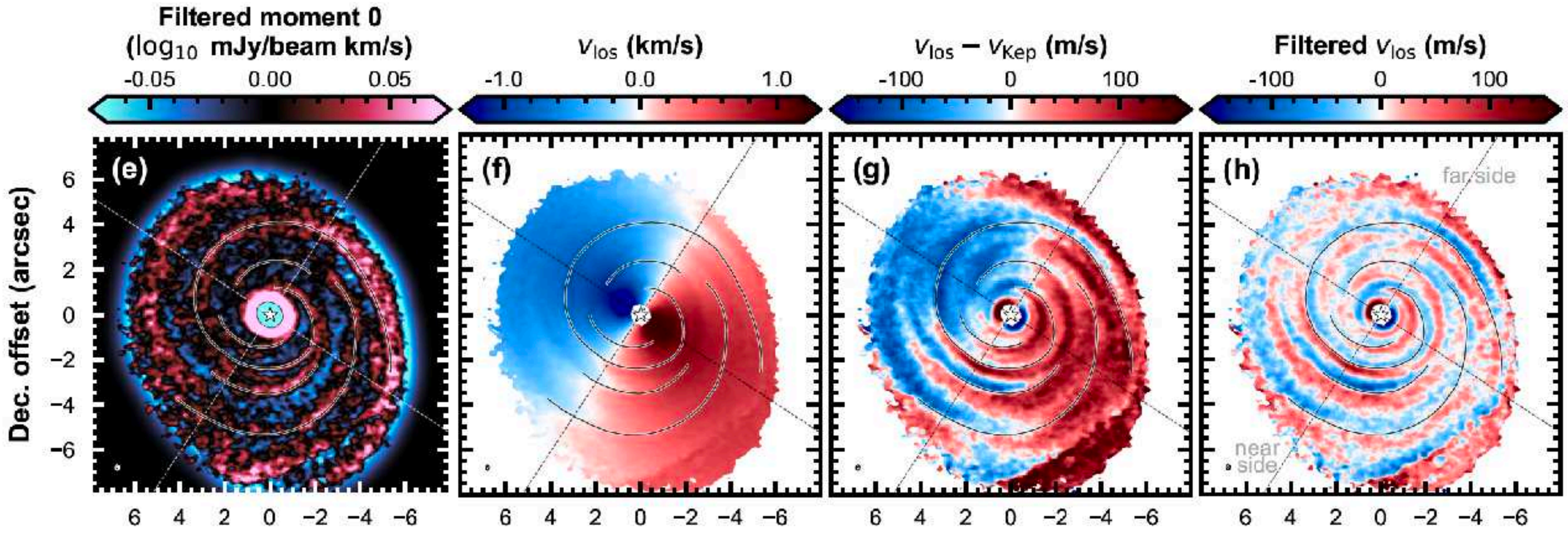
- Cooling factor determines angular momentum transport
  $\alpha_{\mathrm{GI}} \propto (\delta\Sigma/\Sigma)^2 \propto \beta^{-1}$
- Disc to star mass ratio determines the number of spiral arms
- Instantaneous pattern speed is $\propto \Omega$, with waves quickly dissipating after creation because of shocks



GI and cooling: kinematics

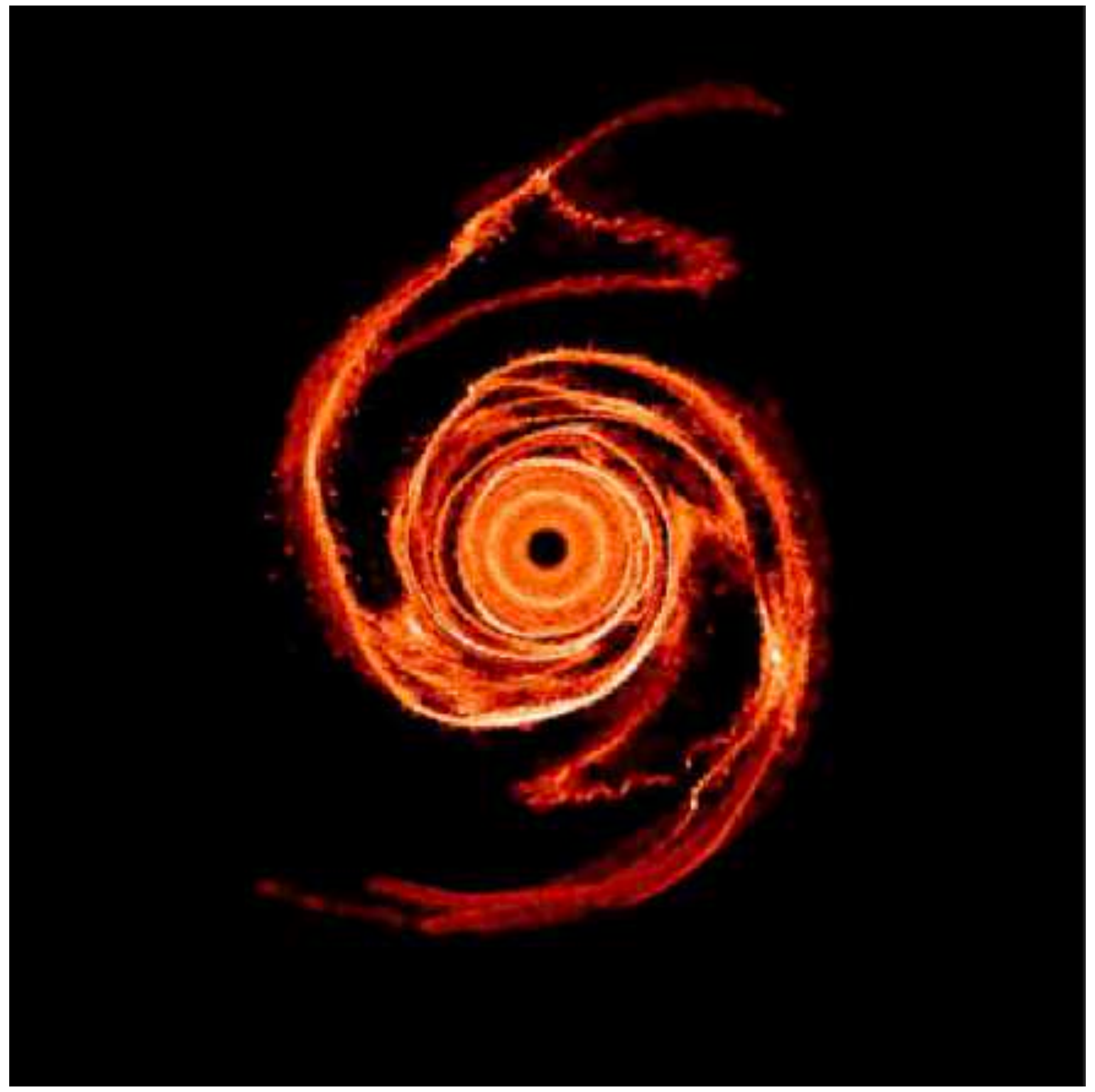


Hall et al. 2020



Speedie et al. 2024

GI and cooling: planet formation



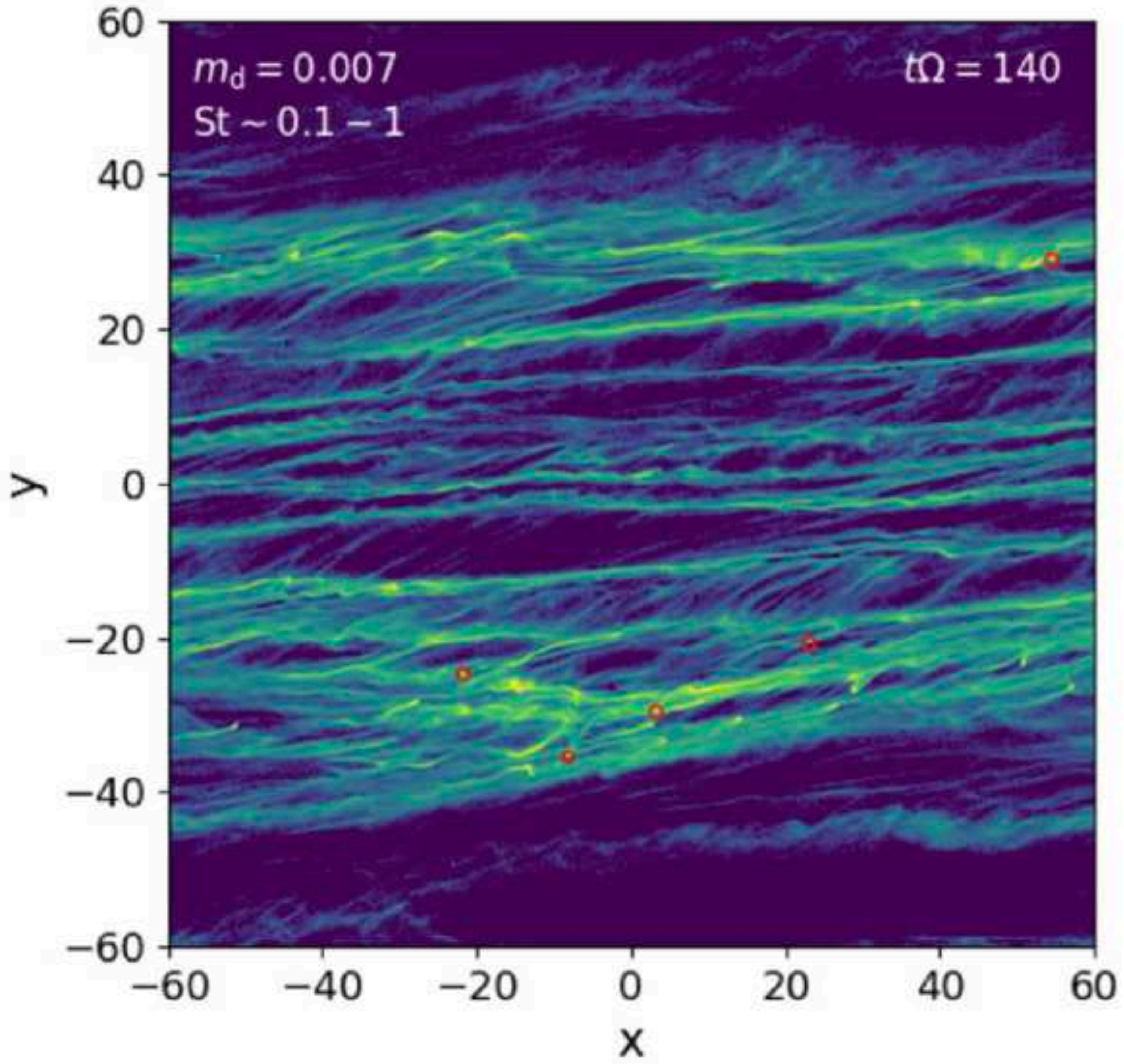
Rowther et al. 2024a

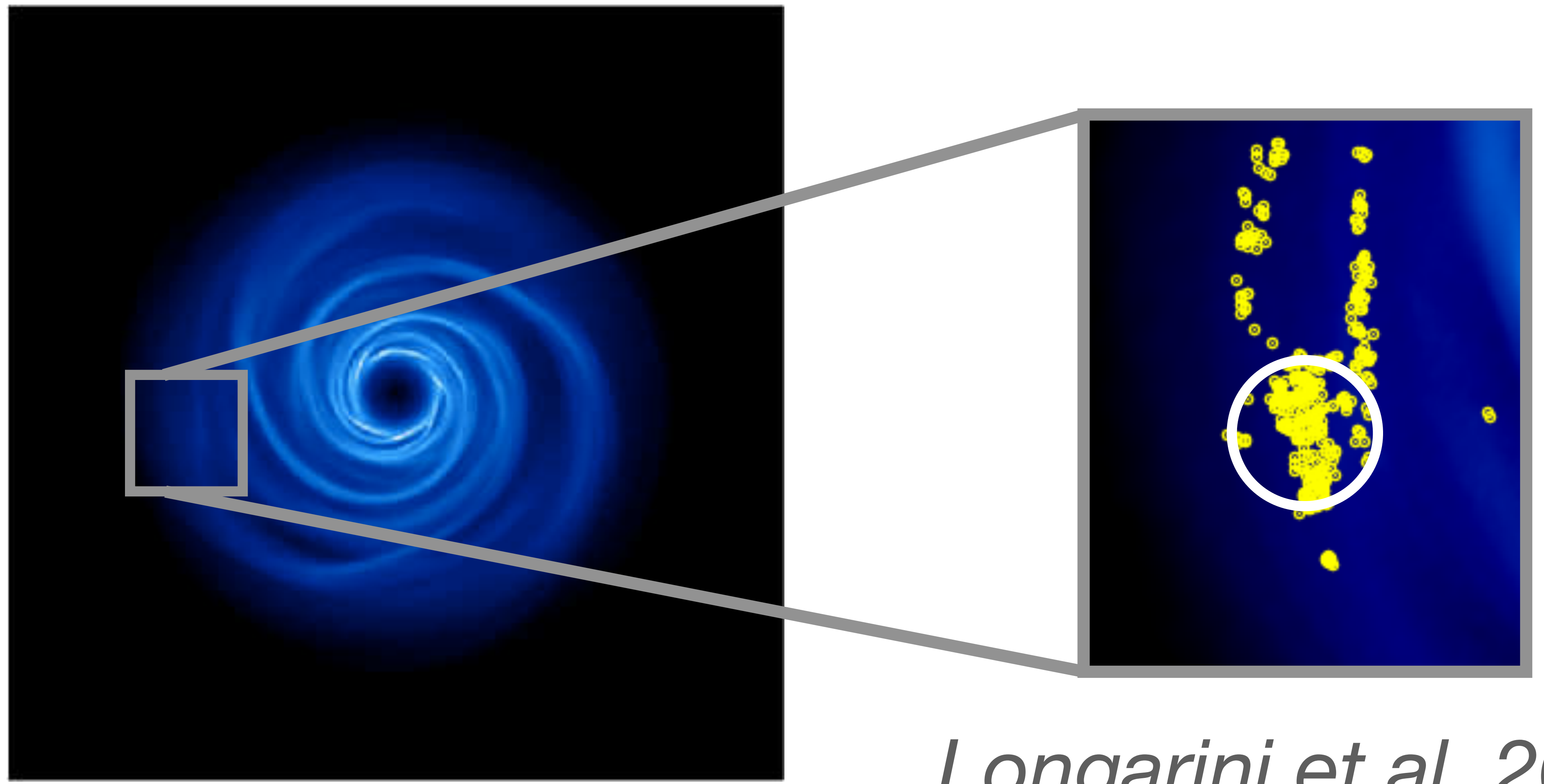
$St \in [0.1, 1]$

Dust collapse is possible

Can dust grow up to those sizes fast enough?

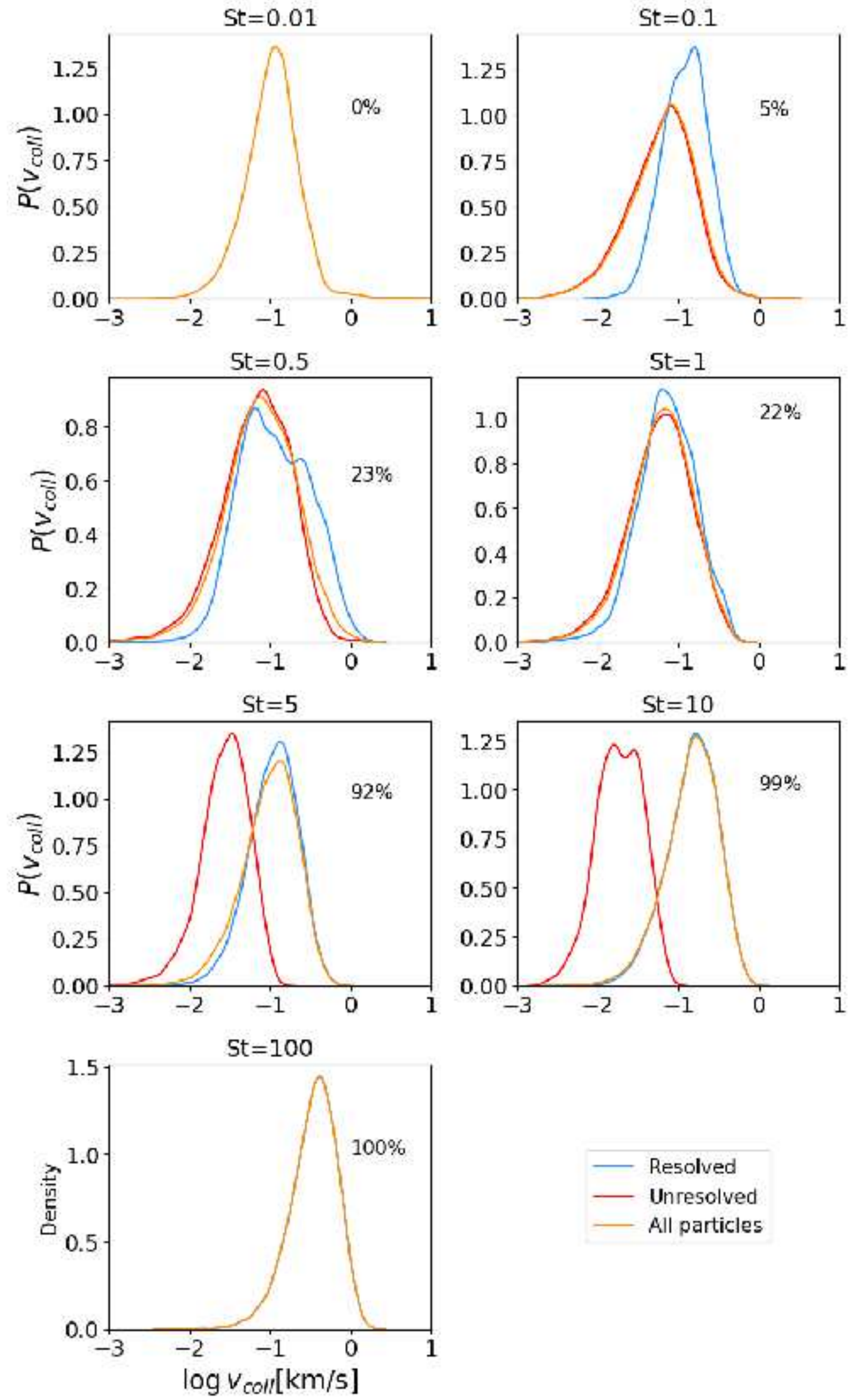
Rice et al. 2025
After 2004!!





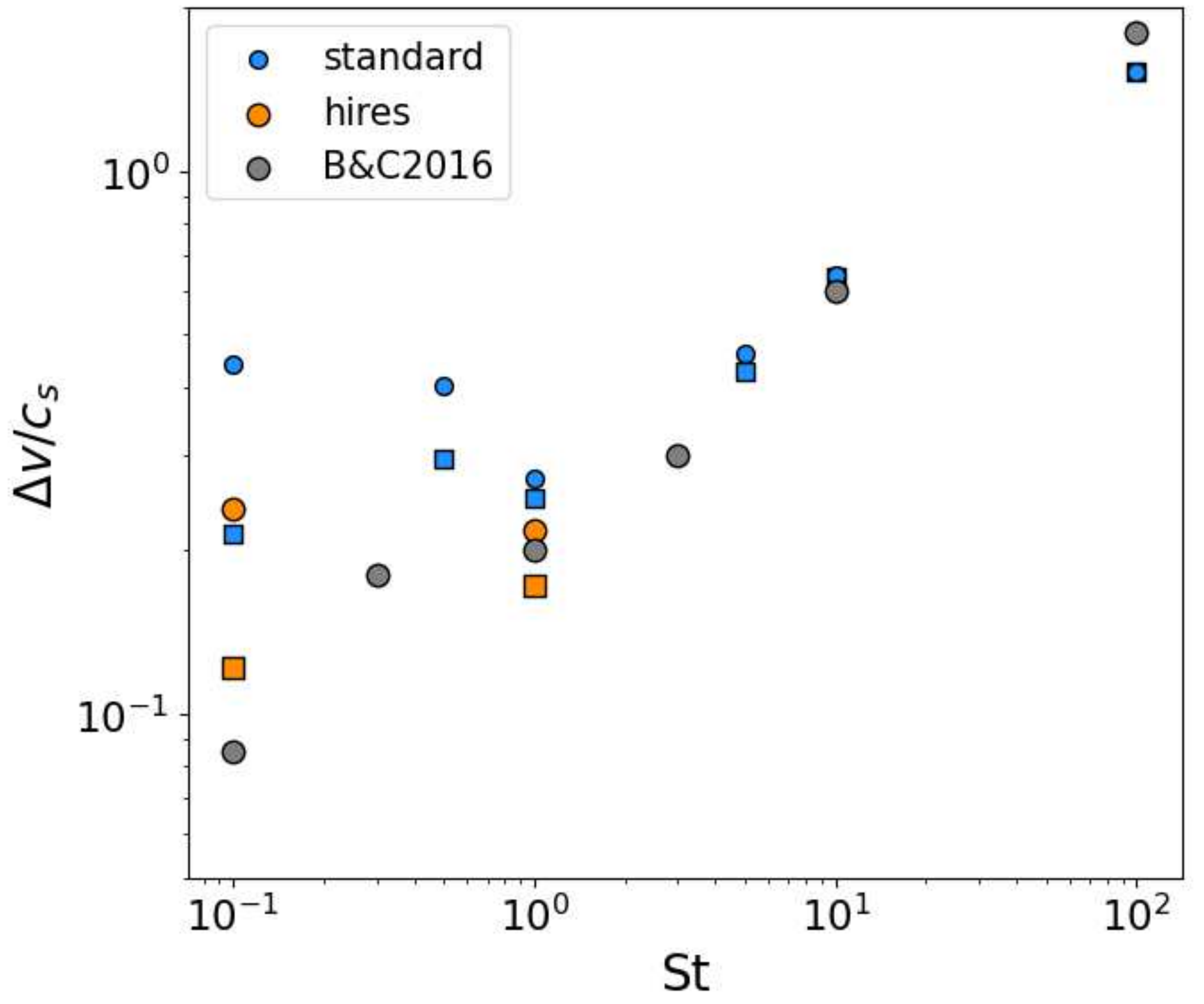
Longarini et al. 2023b

GI and cooling: planet formation



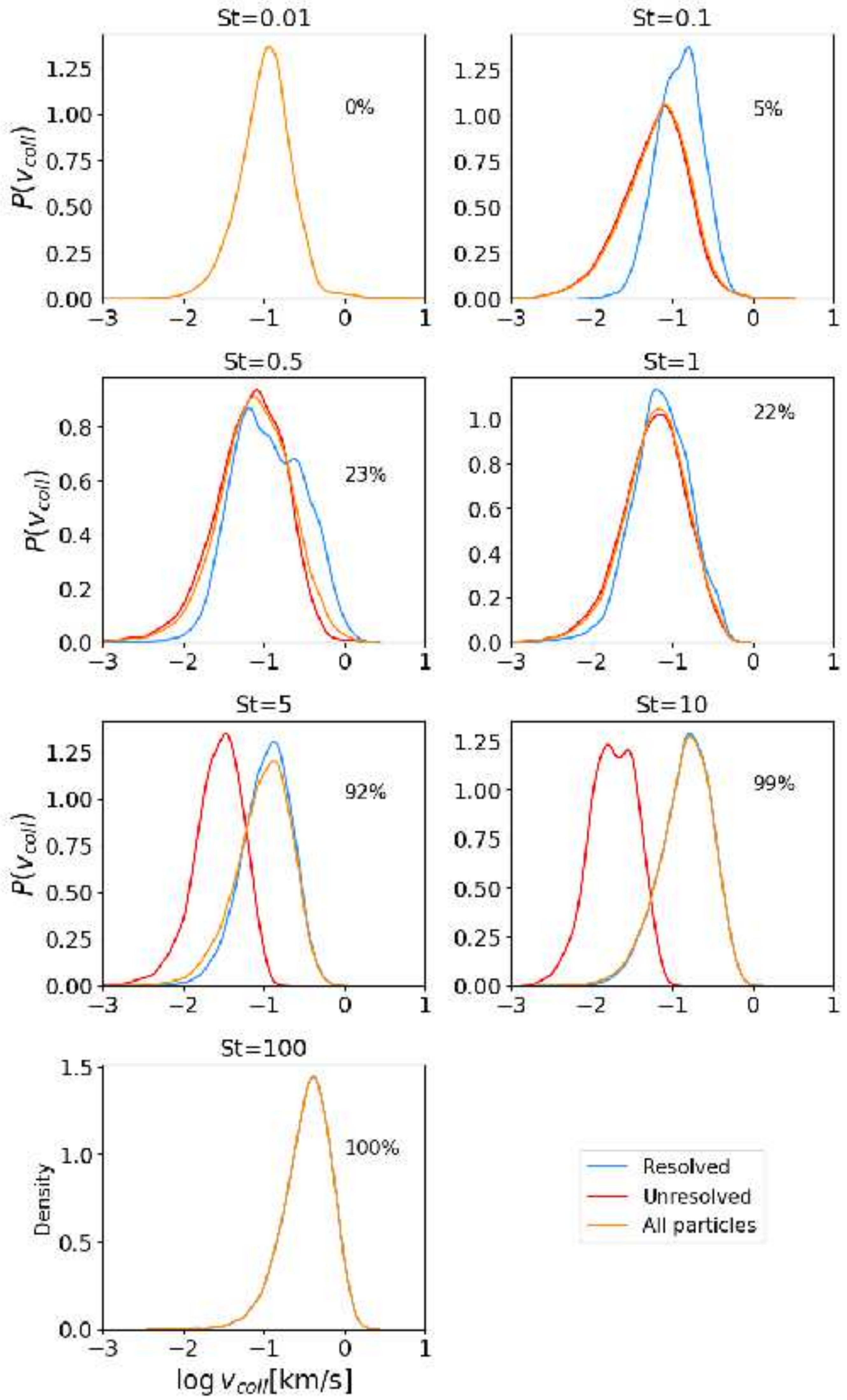
$$\Delta R_{1,2} < t_s \Delta v_{1,2}$$

Very high resolution
is required to resolve
collisions



Longarini et al. in prep

GI and cooling: planet formation



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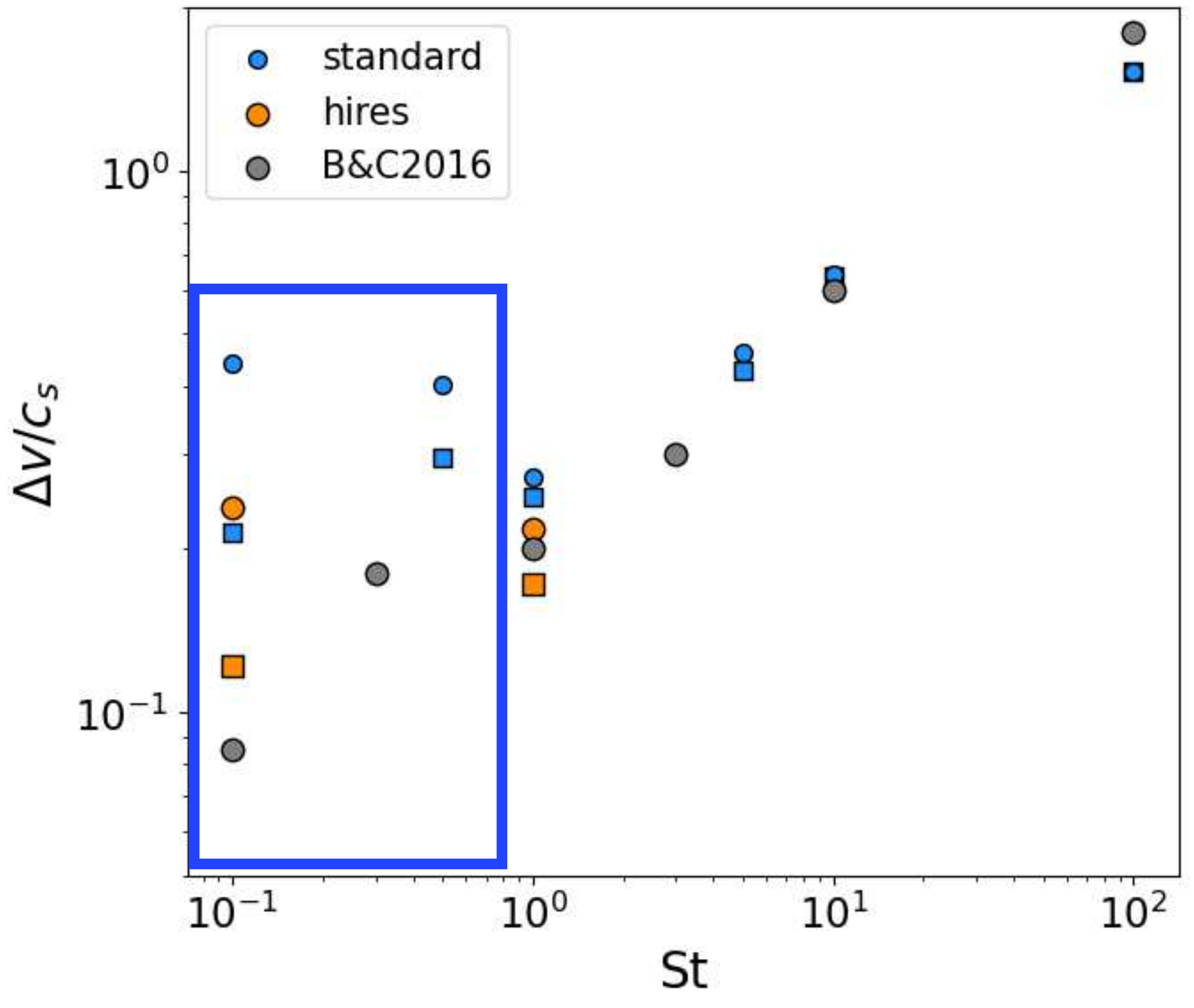
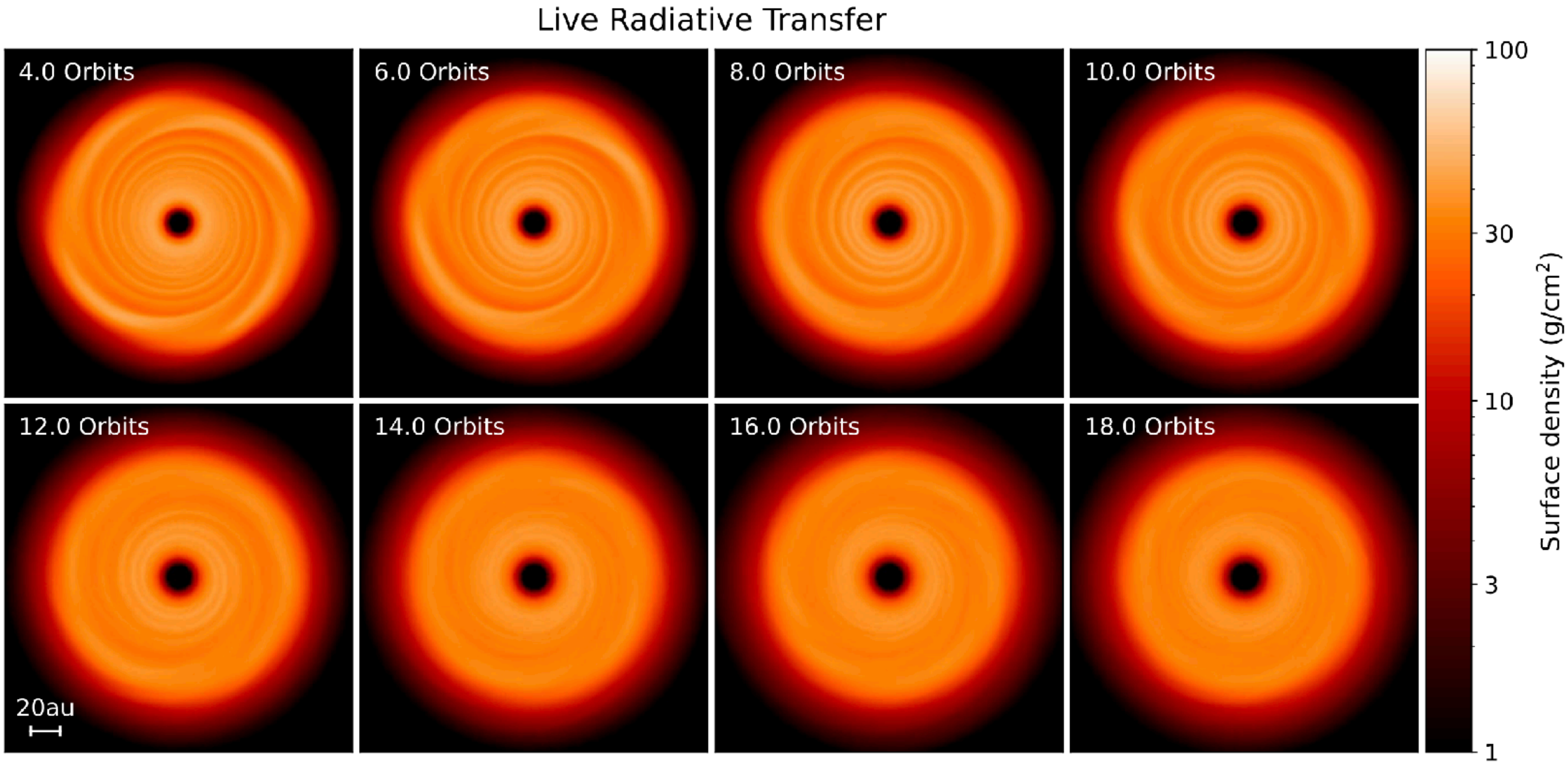


Longarini et al. in prep

Shamrock??

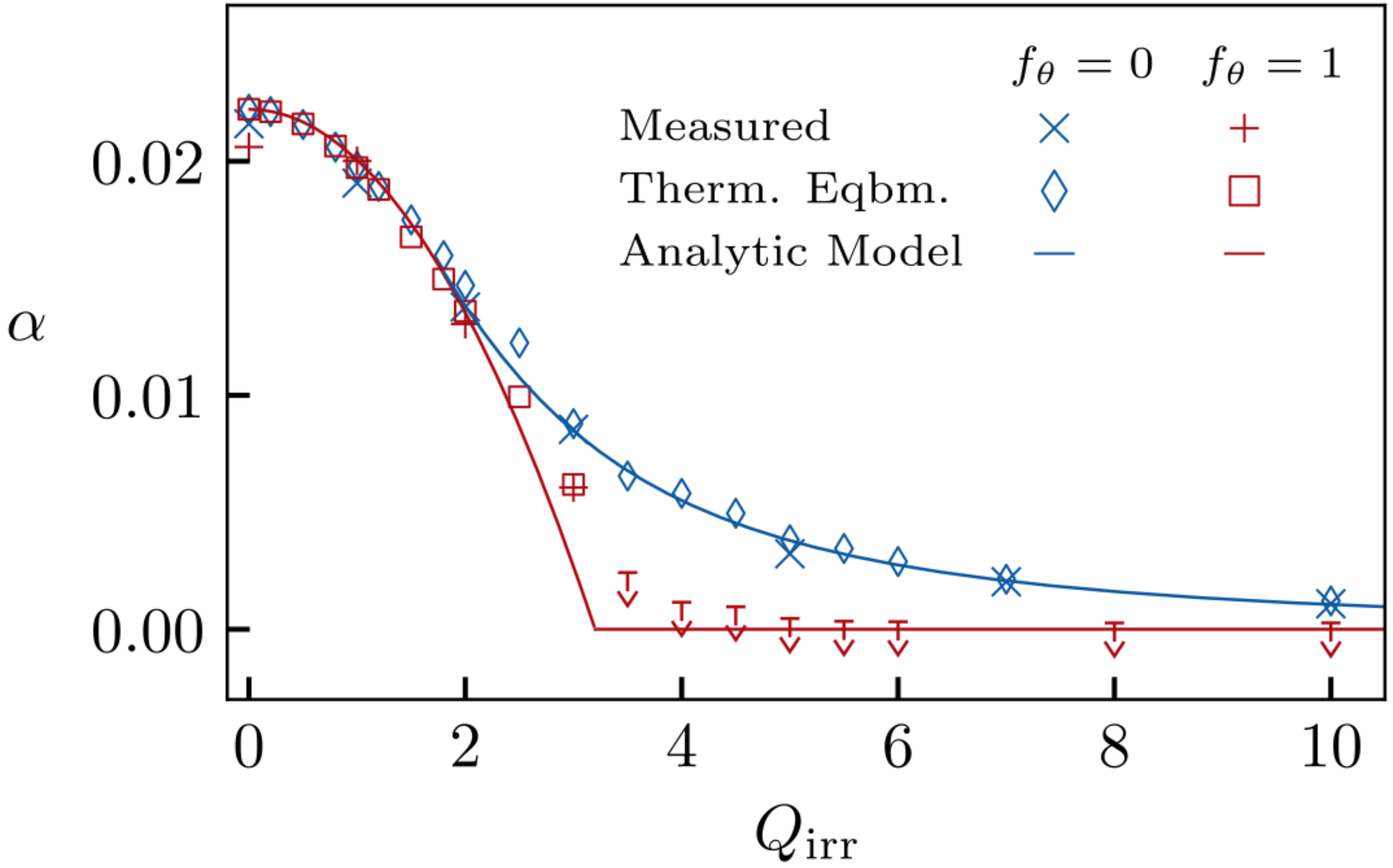
Cat Leedham

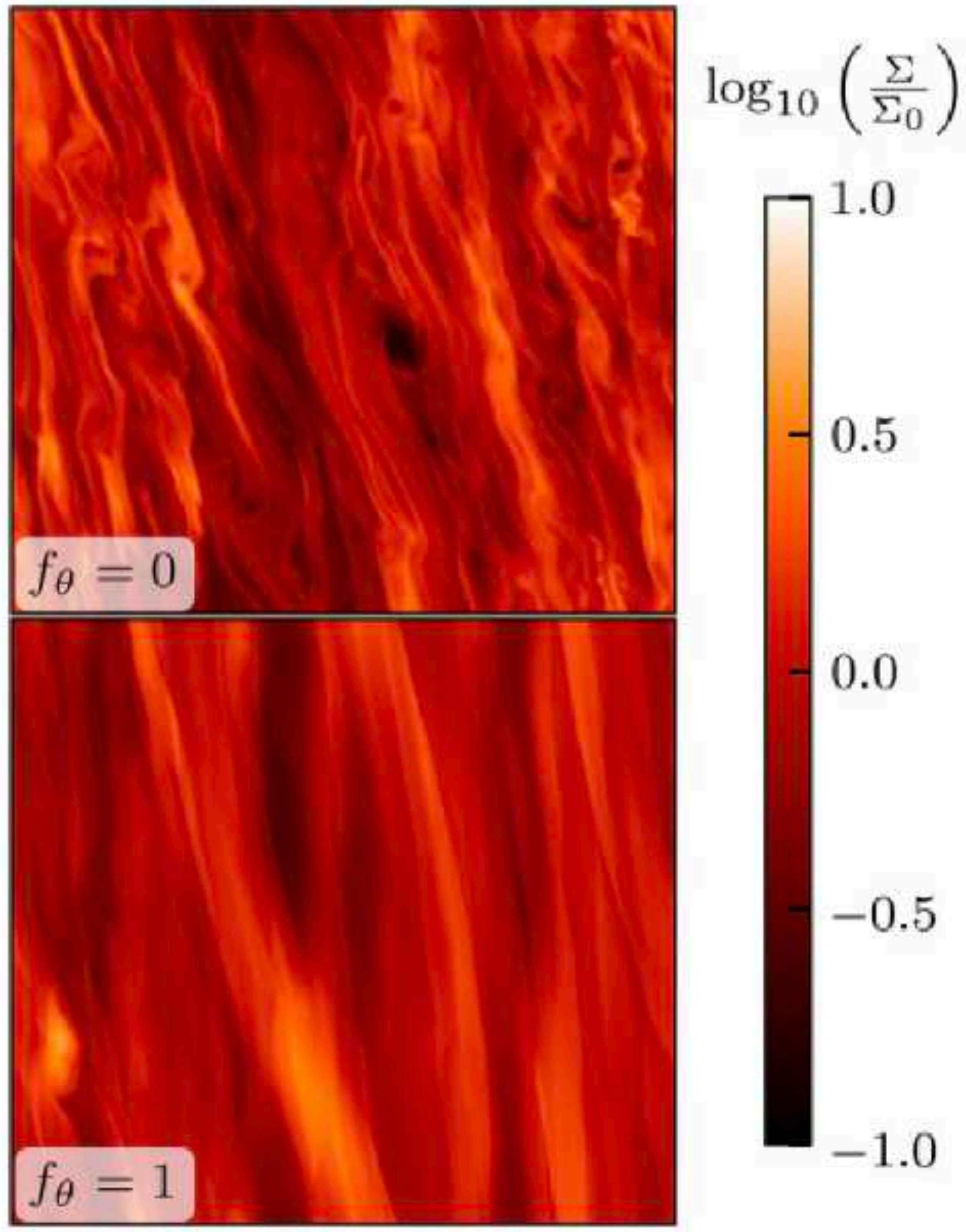
GI and heating



Rowther et al. 2024b

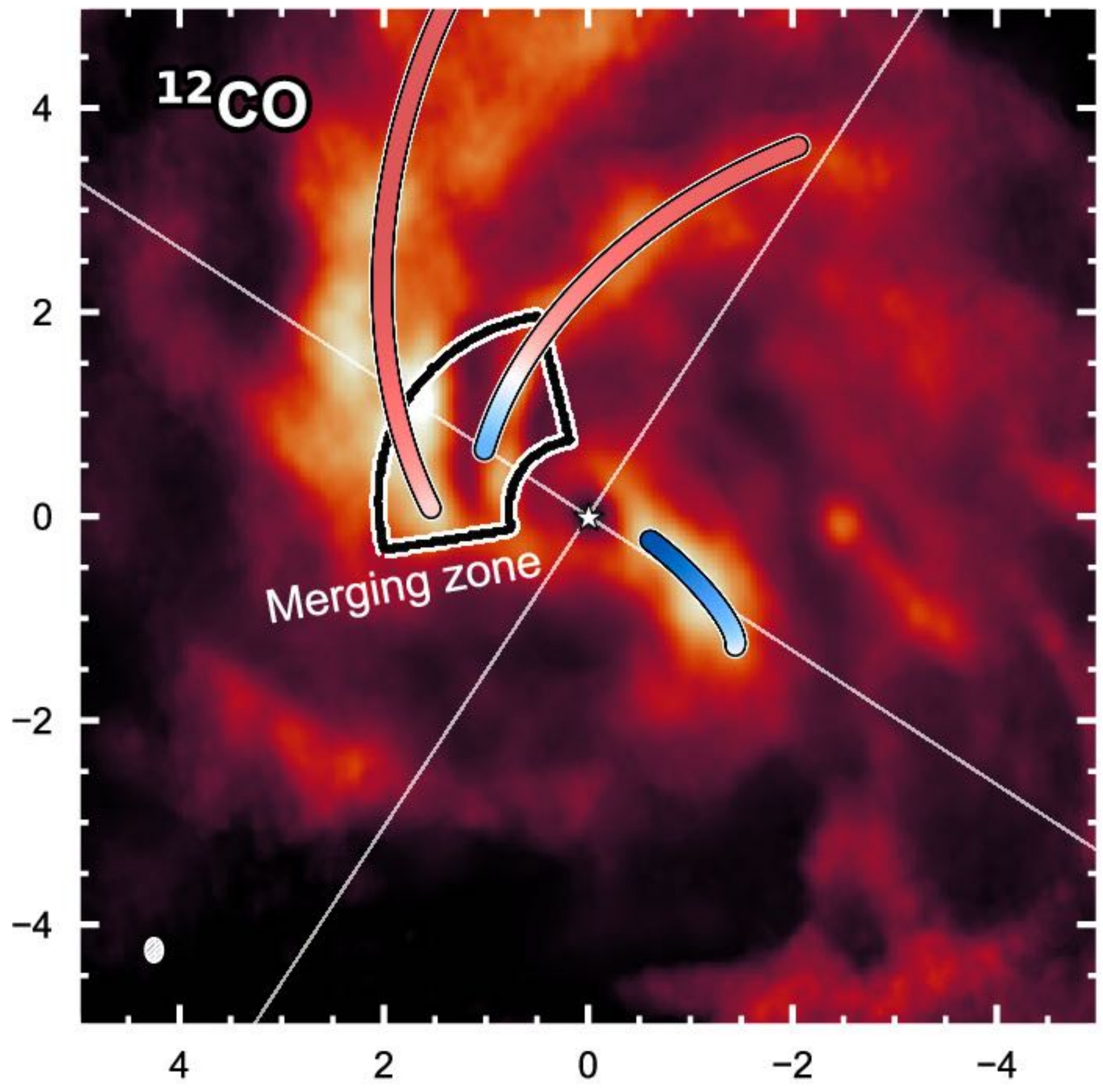
GI and heating



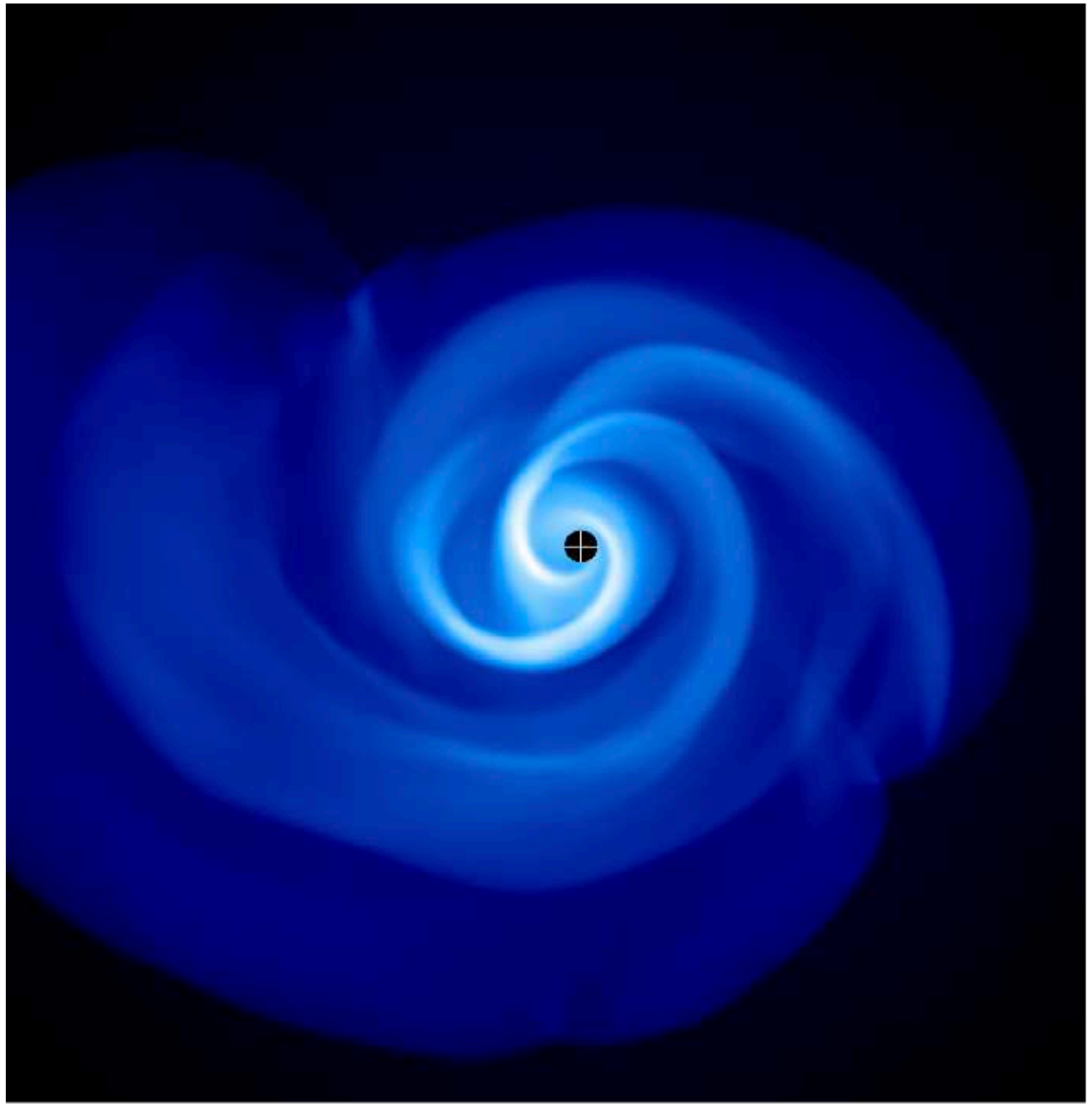


Leedham et al. 2025

GI and infall



Speedie et al. 2025



Kratter et al. 2010b

1D evolution code

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[\sqrt{R}\frac{\partial}{\partial R}\left(\sqrt{R}\Sigma\nu\right)\right] + \dot{\Sigma}_{\rm inj}$$

Diffusion equation for a Keplerian disc

Locally isothermal disc : H/R constant

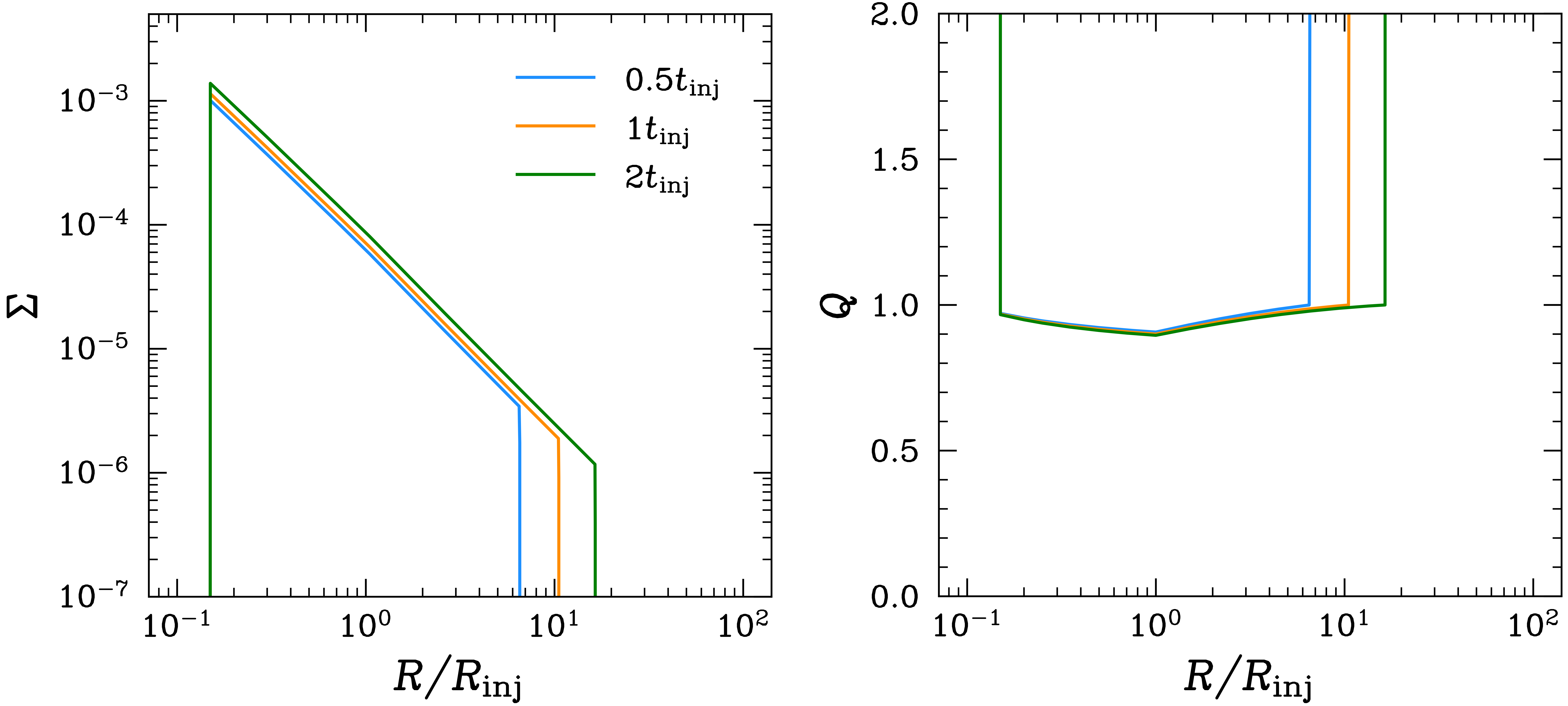
Viscosity prescription that mimics GI (Lin & Pringle 1987)

Activates only when $Q \simeq 1$

Injection term

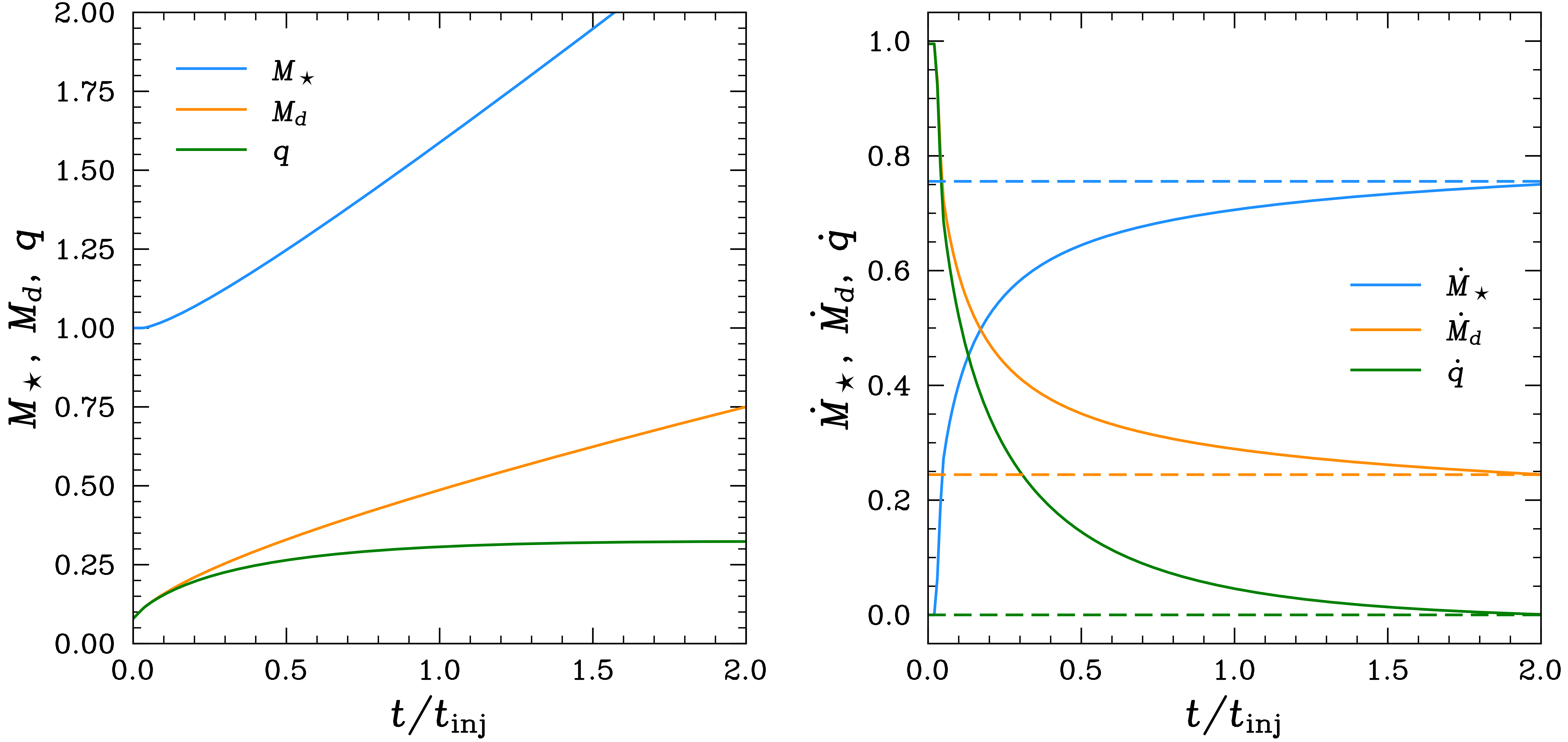
Mass is added with $\dot{M}_{\rm inj}$ at $R_{\rm inj}$

1D evolution code



Longarini et al. subm.

1D evolution code



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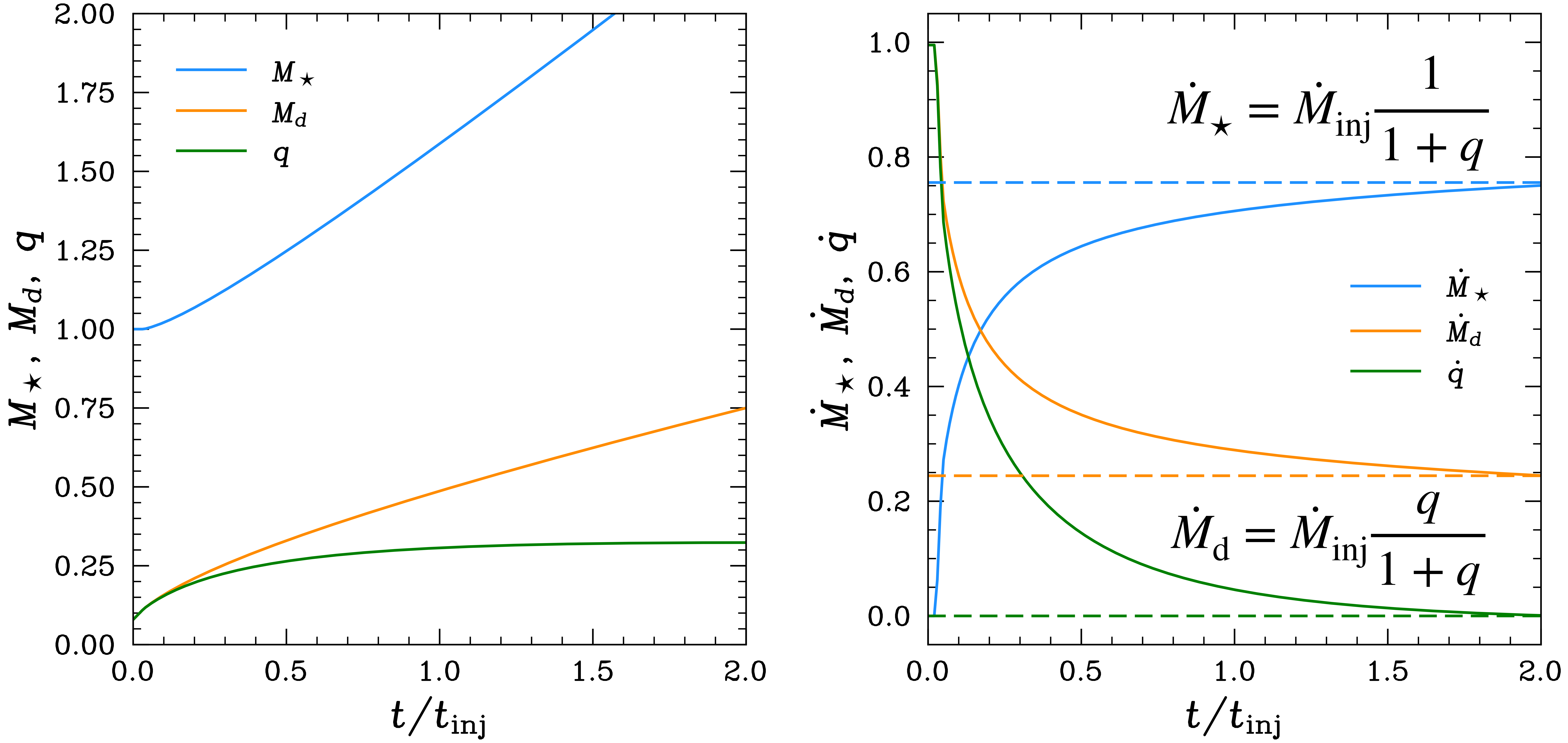


Longarini et al. subm.

3D PHANTOM simulations

inject_keplerian.f90

```
!--------------------------------------------------------------------------!
! The Phantom Smoothed Particle Hydrodynamics code, by Daniel Price et al. !
! Copyright (c) 2007-2025 The Authors (see AUTHORS)                        !
! See LICENCE file for usage and distribution conditions                   !
! http://phantomsph.github.io/                                             !
!--------------------------------------------------------------------------!
module inject
!
! Injection of material at keplerian speed in an accretion disc
!
! :References:
!
! :Owner: Cristiano Longarini
!
! :Runtime parameters:
!   - HonR_inj    : *aspect ratio to give temperature at rinj*
!   - follow_sink : *injection radius is relative to sink particle 1*
!   - mdot        : *mass injection rate [msun/yr]*
!   - rinj        : *injection radius*
!
! :Dependencies: eos, externalforces, infile_utils, io, options, part,
!   partinject, physcon, random, units
!
 implicit none
 character(len=*), parameter, public :: inject_type = 'keplerian'
```

setup (dusty)isosgdisc

PHANTOM > June 2024

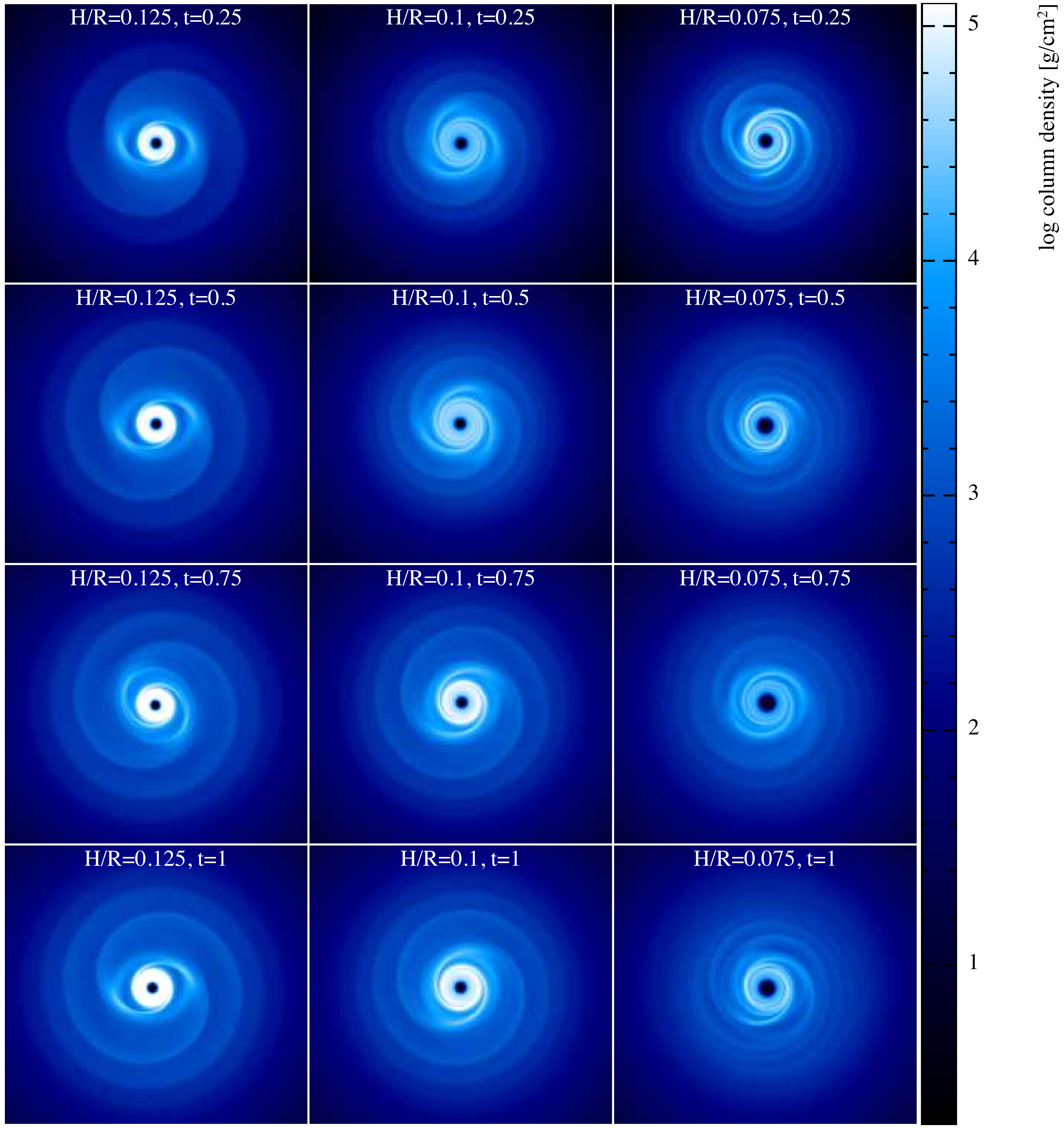
SPH particles injection at $R_{\rm inj}$ with $\dot{M}_{\rm inj}$ and Keplerian velocity
(Also possible gaussian injection TBP works also for distorted discs TBP)

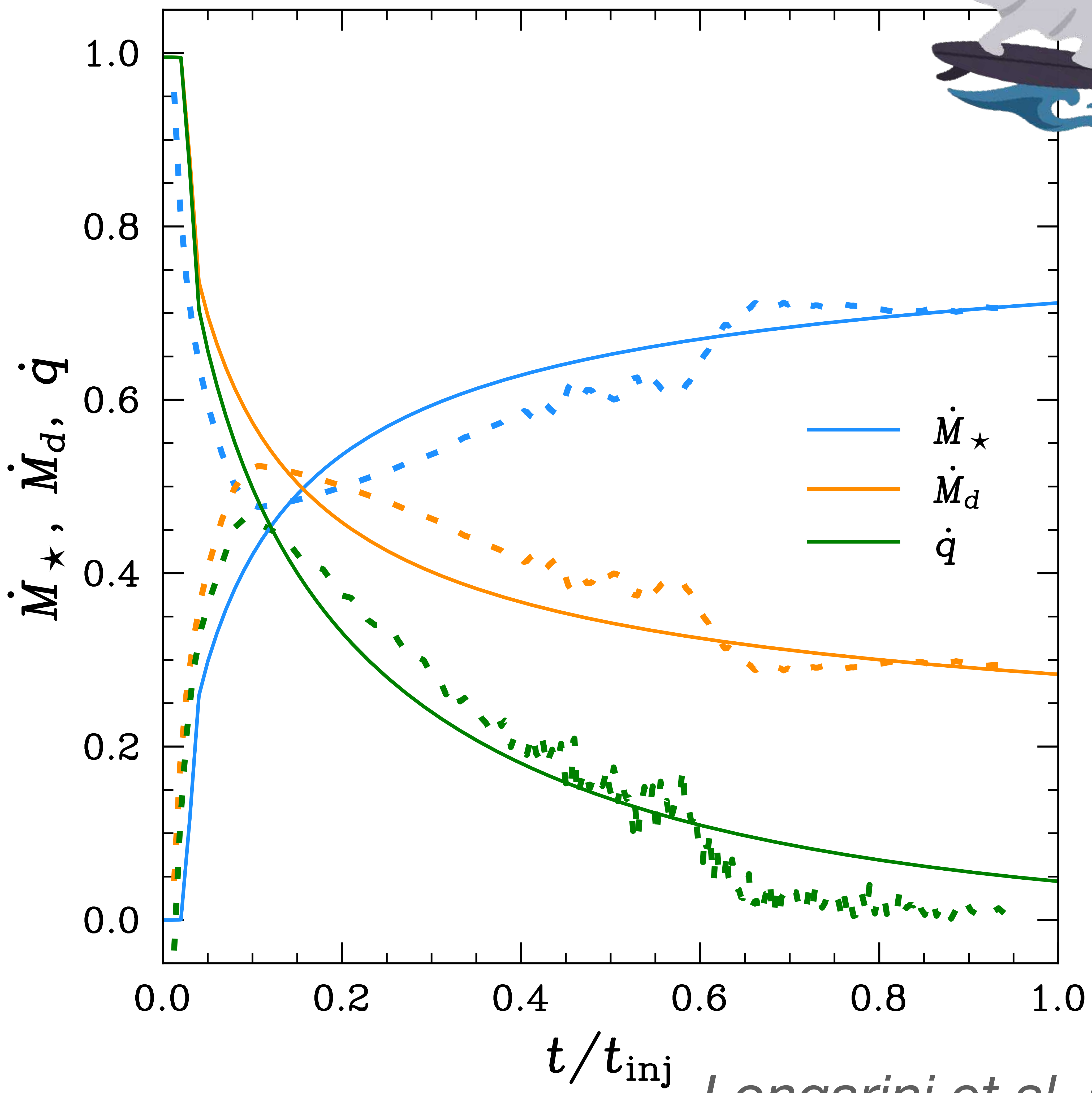
Locally isothermal disc:
c_s is fixed

Working on implementing injection for adiabatic discs - discs with cooling

Self regulation in 3D

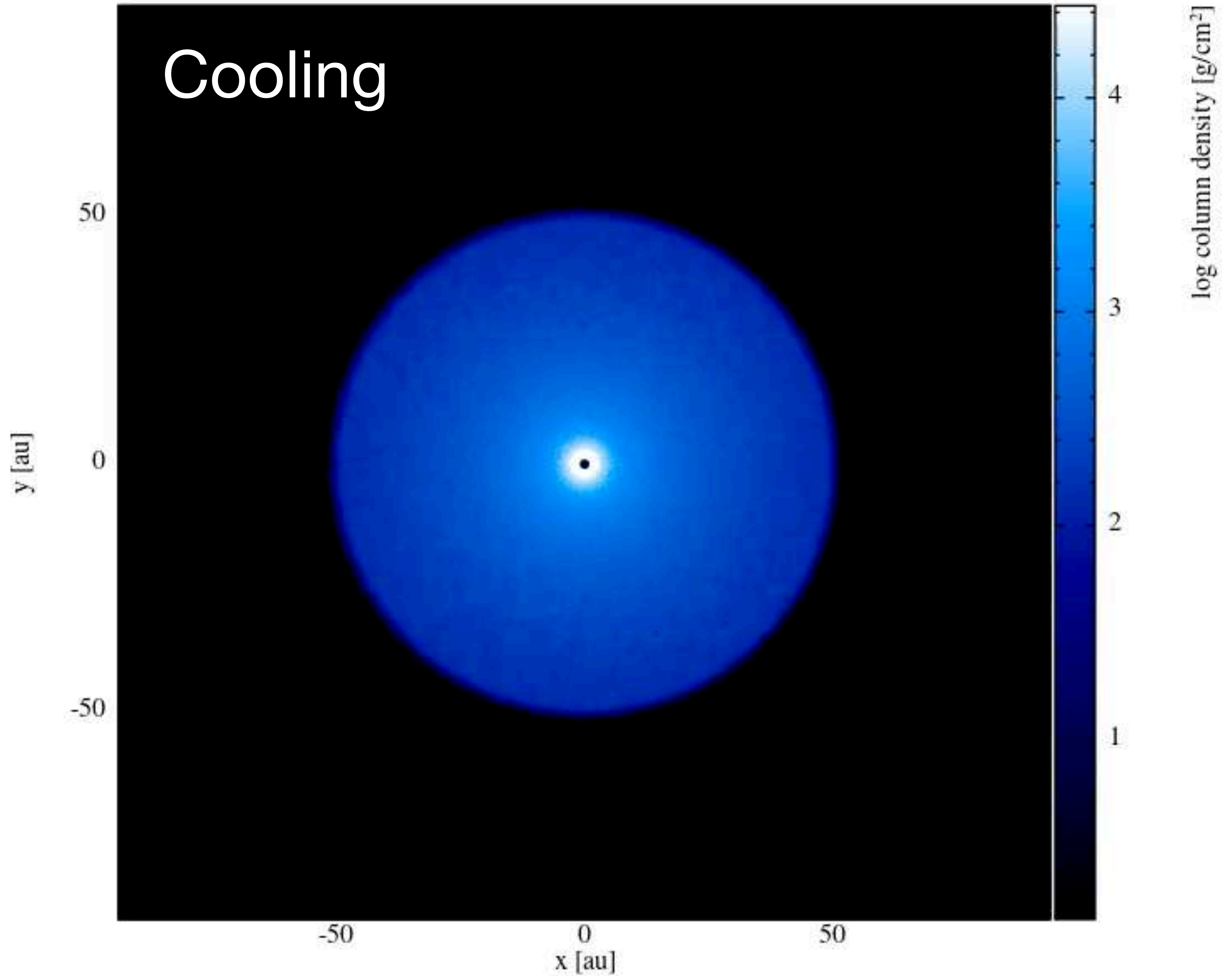
Simulation	$\dot{M}_{inj}$ [$M_\odot$/yr]	$\hat{R}_{inj}$	H/R
S3D_1 - reference	5×10^{-5}	1	0.1
S3D_2	1.25×10^{-5}	1	0.075
S3D_3	1×10^{-4}	1	0.125

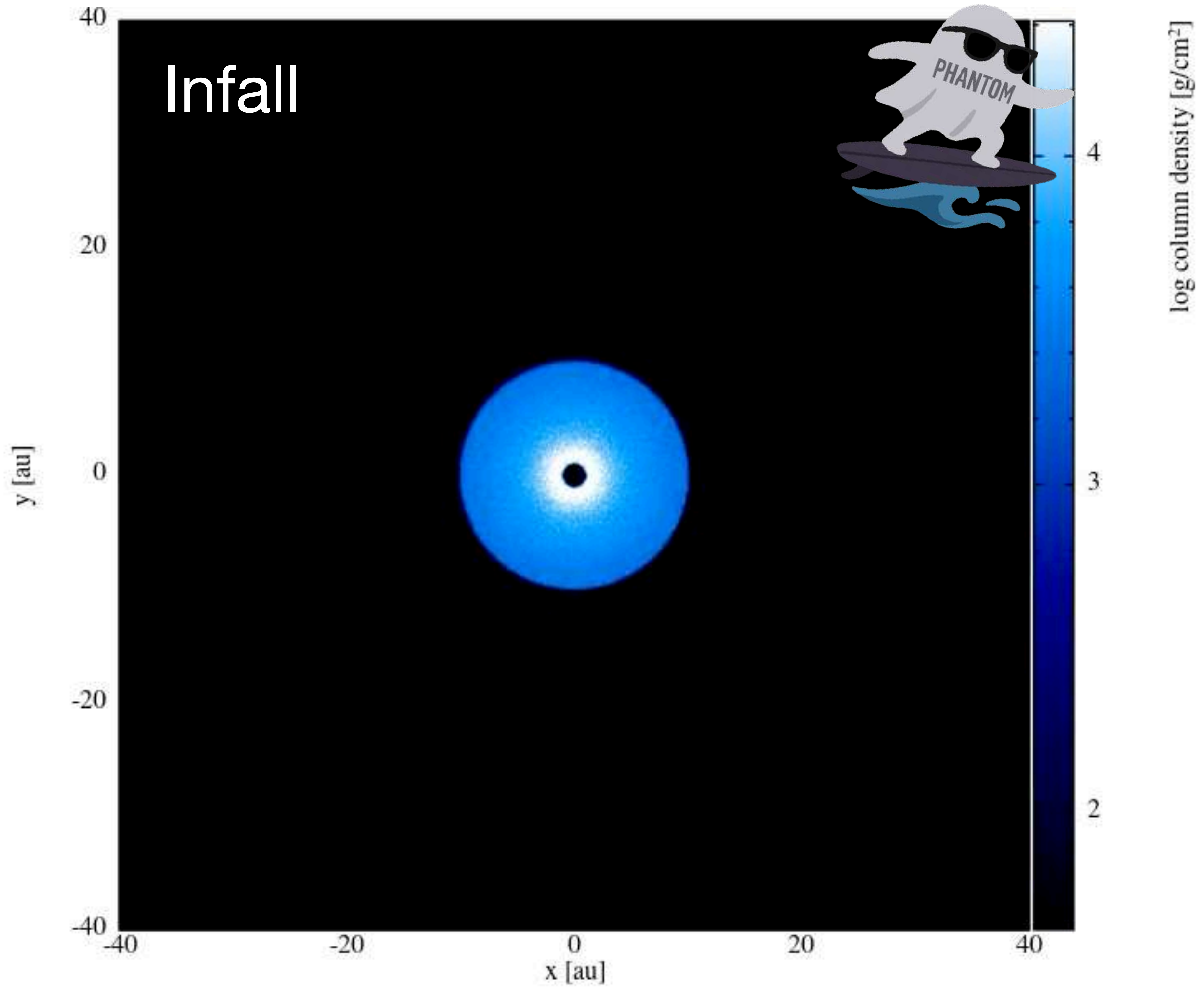




Longarini et al. subm.

Cooling vs infall





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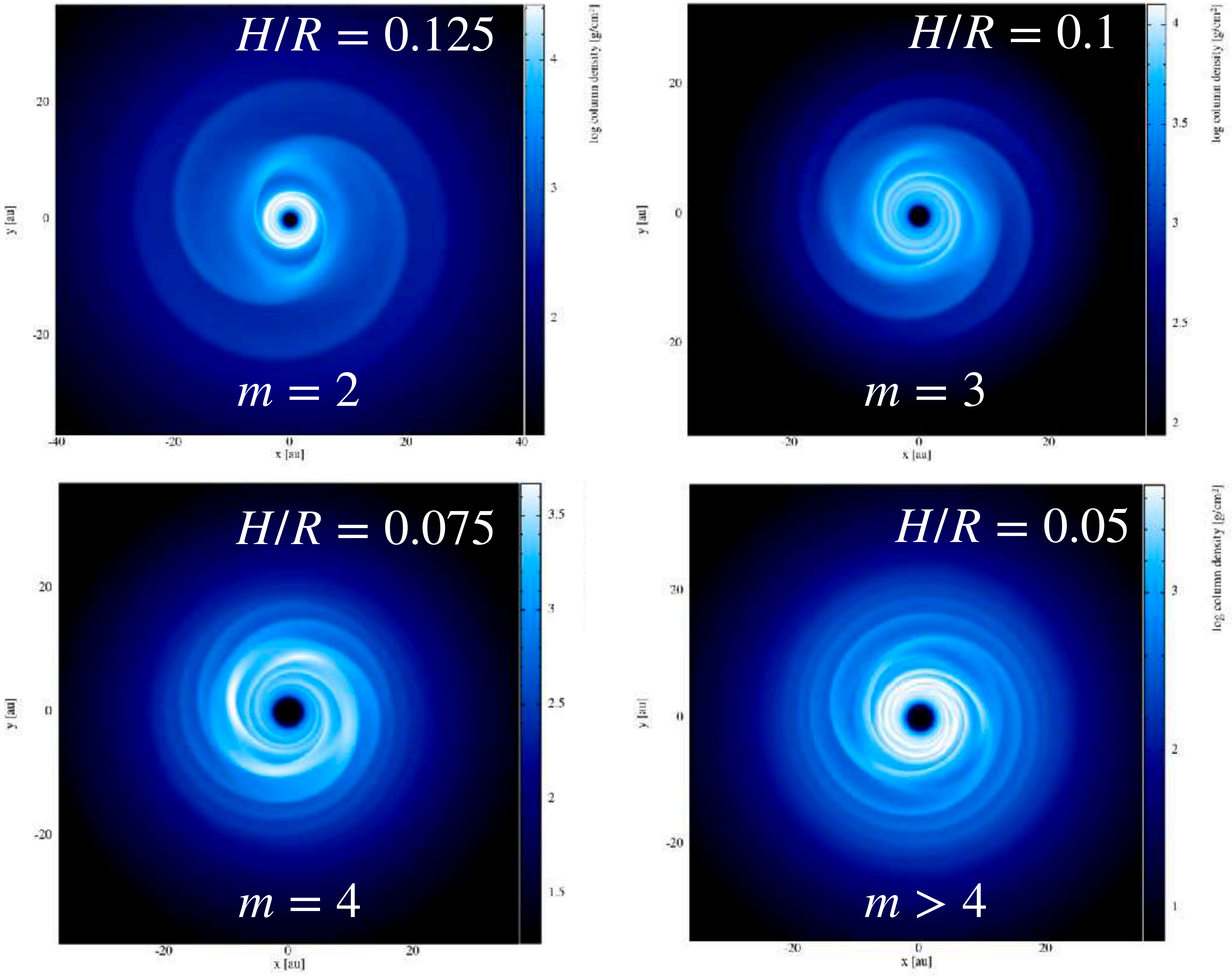
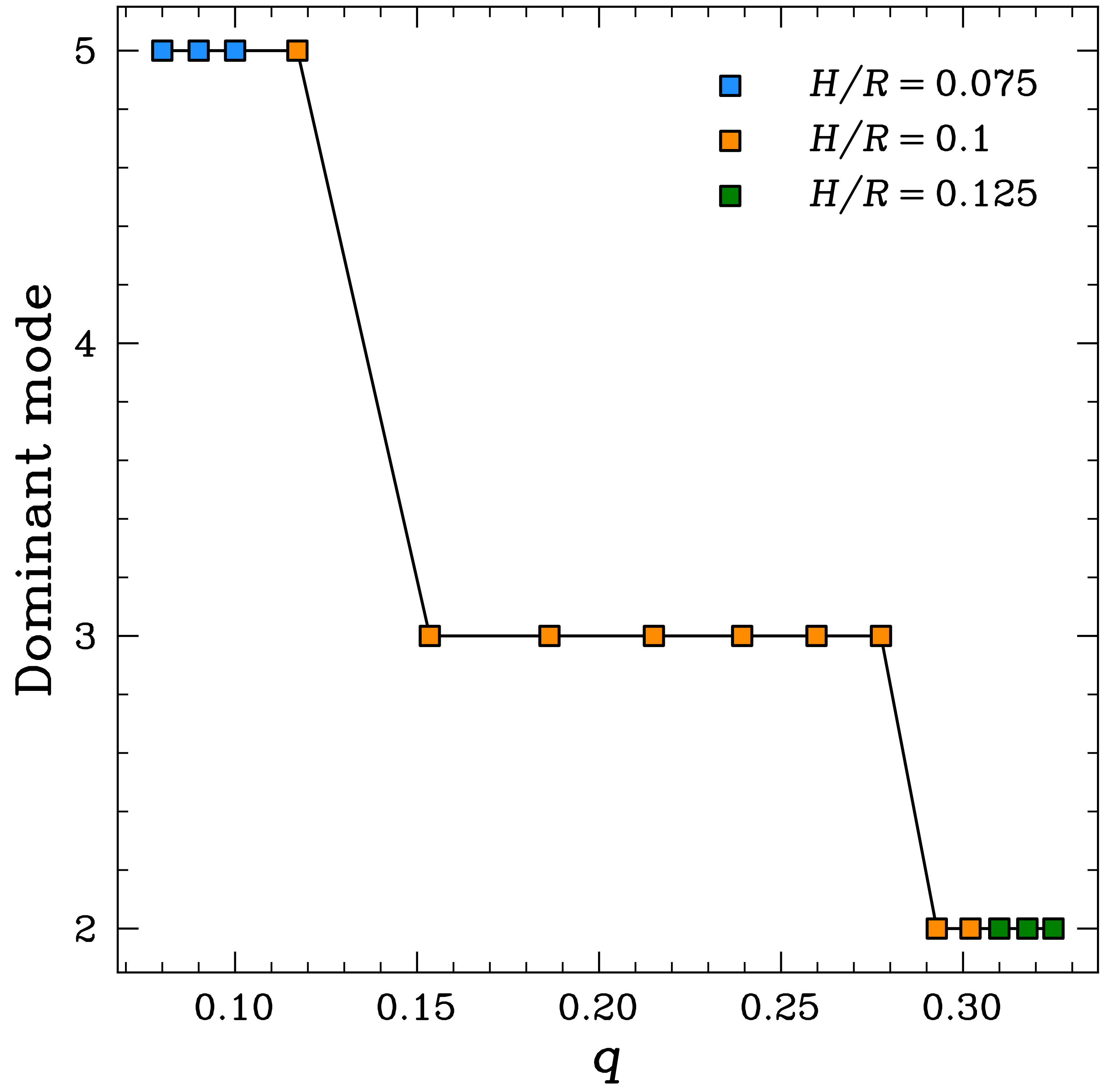
Morphological analogies / differences





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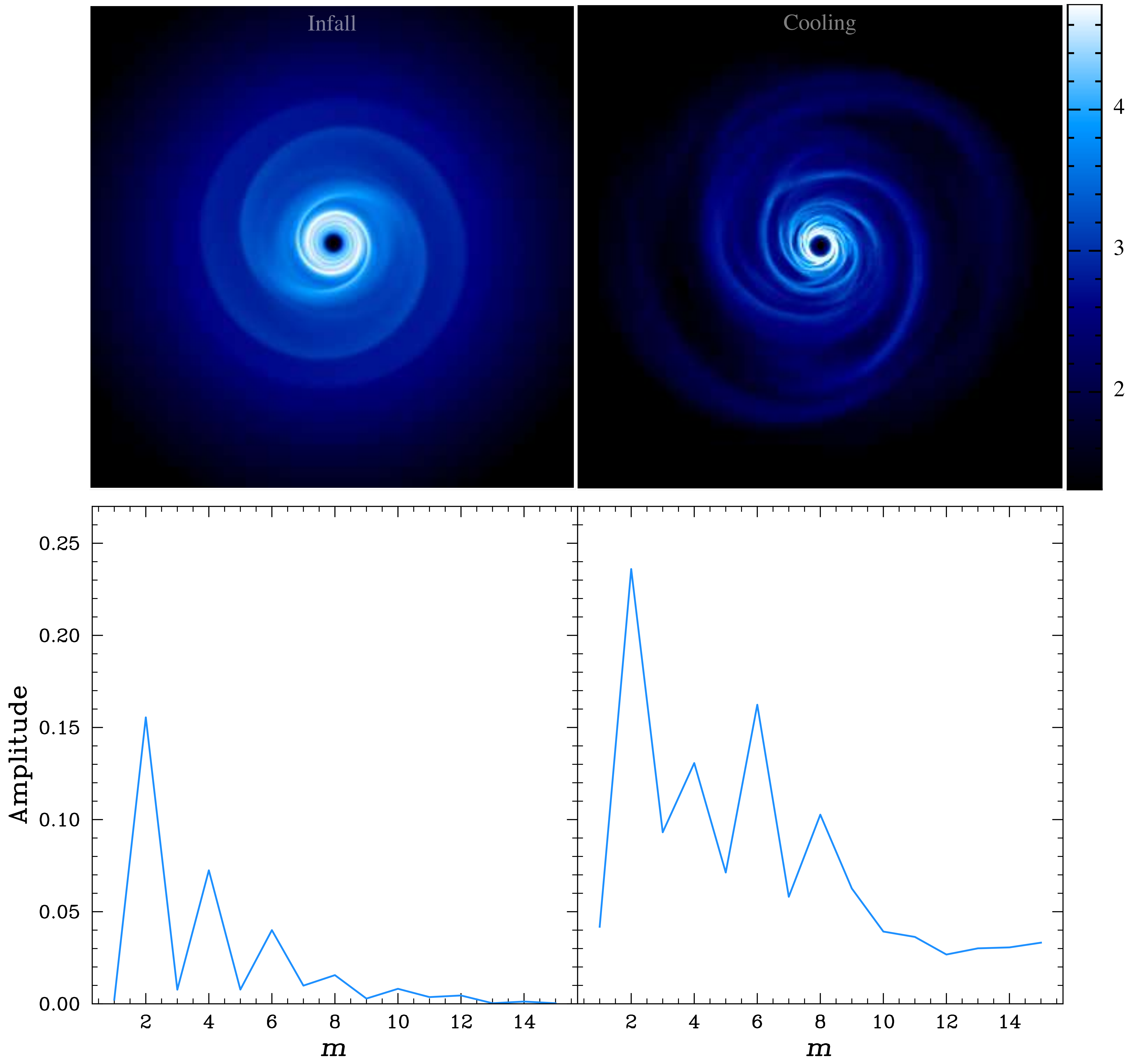

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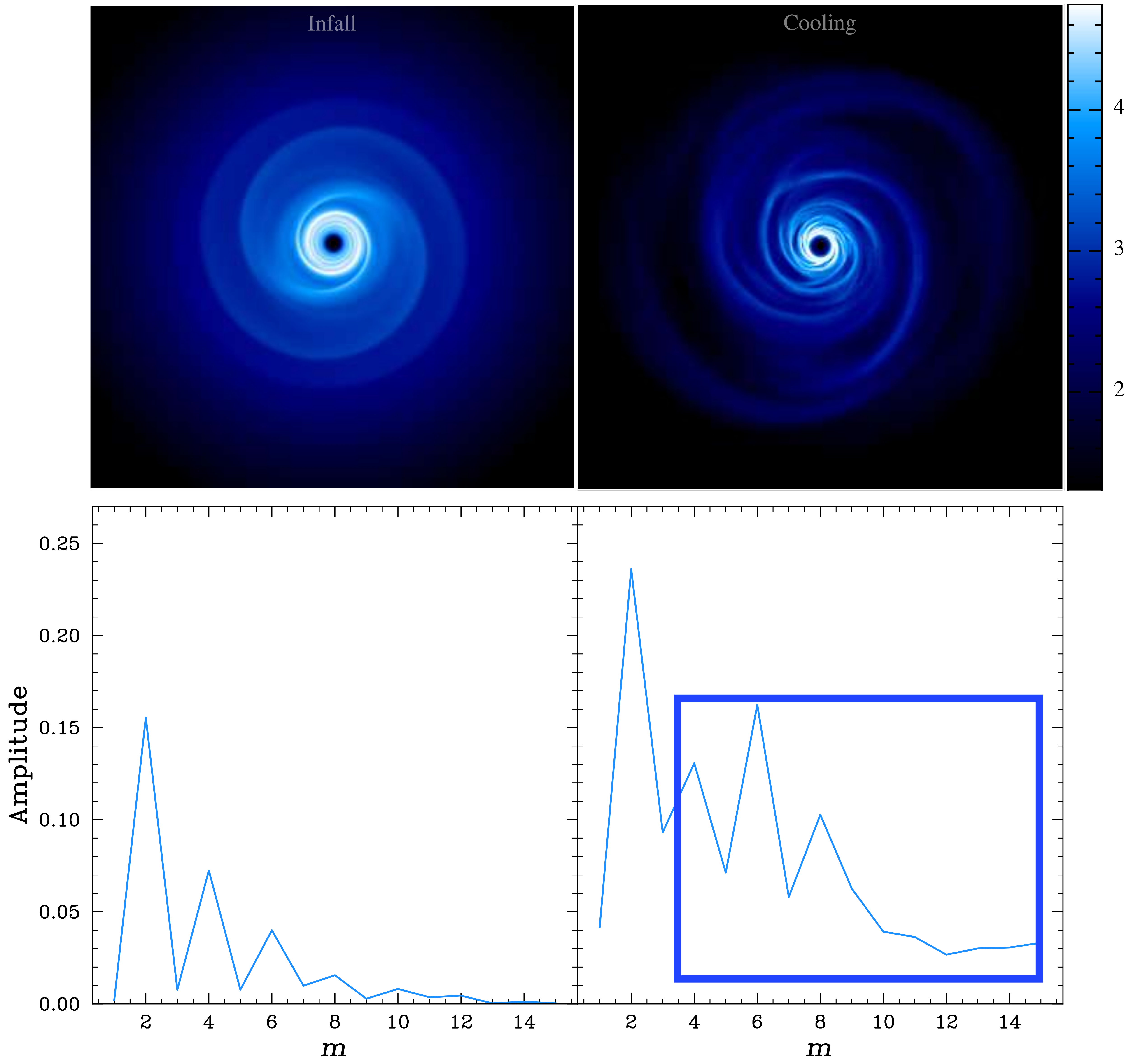
Morphological analogies / differences

Long-lived global spiral mode



Superposition of different short-lived modes

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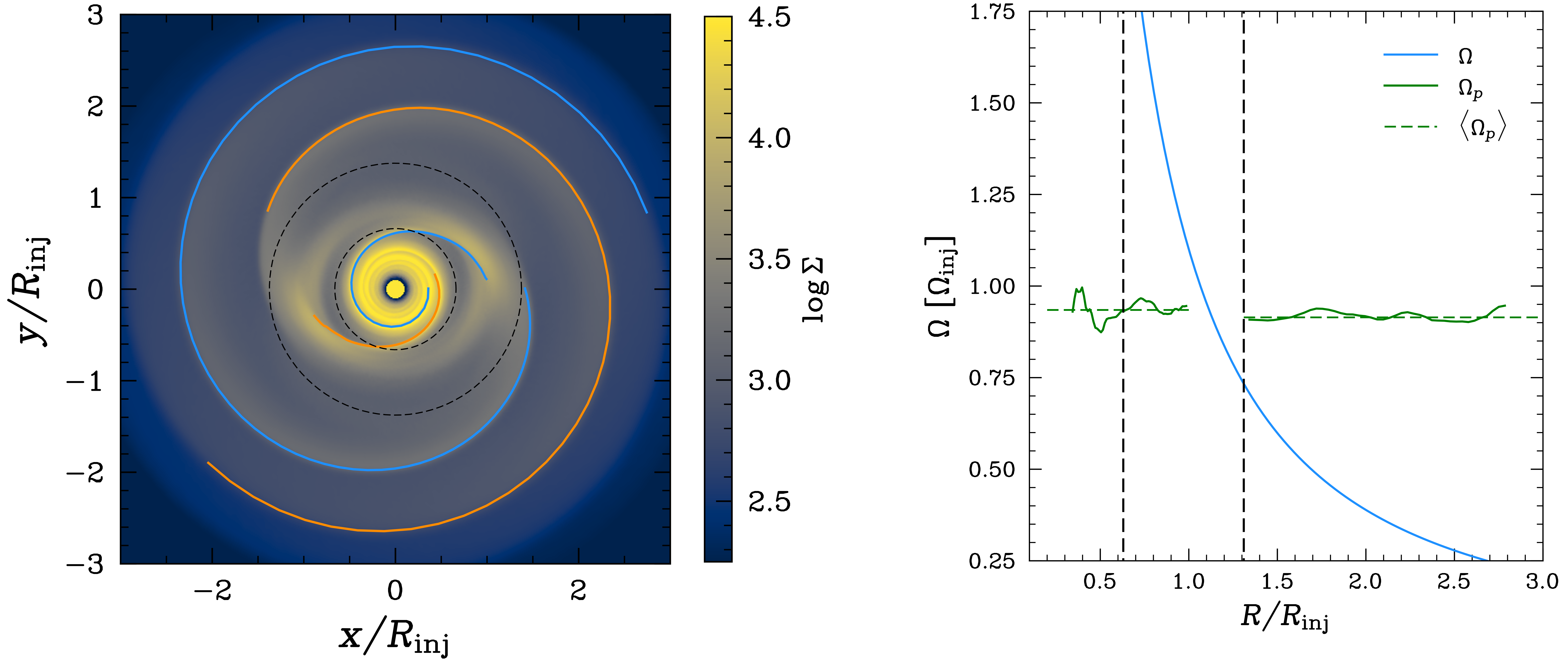
Spiral tracking



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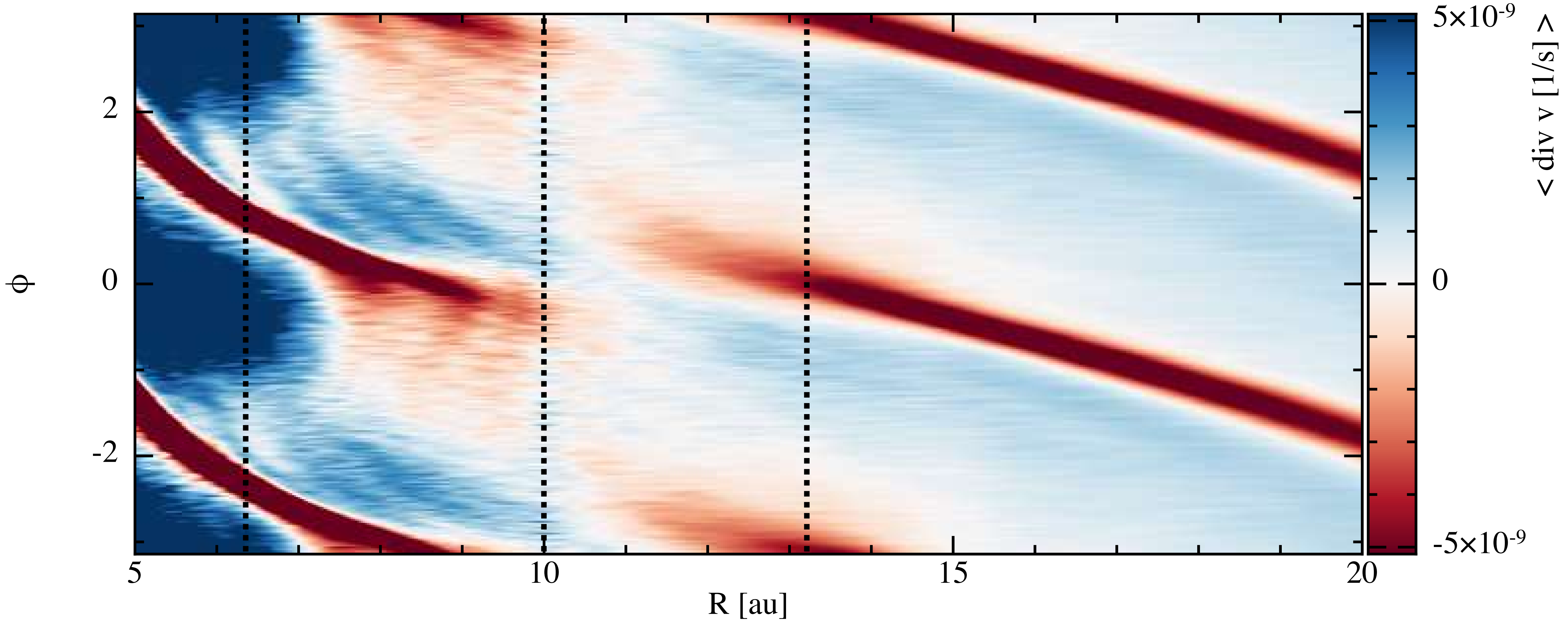
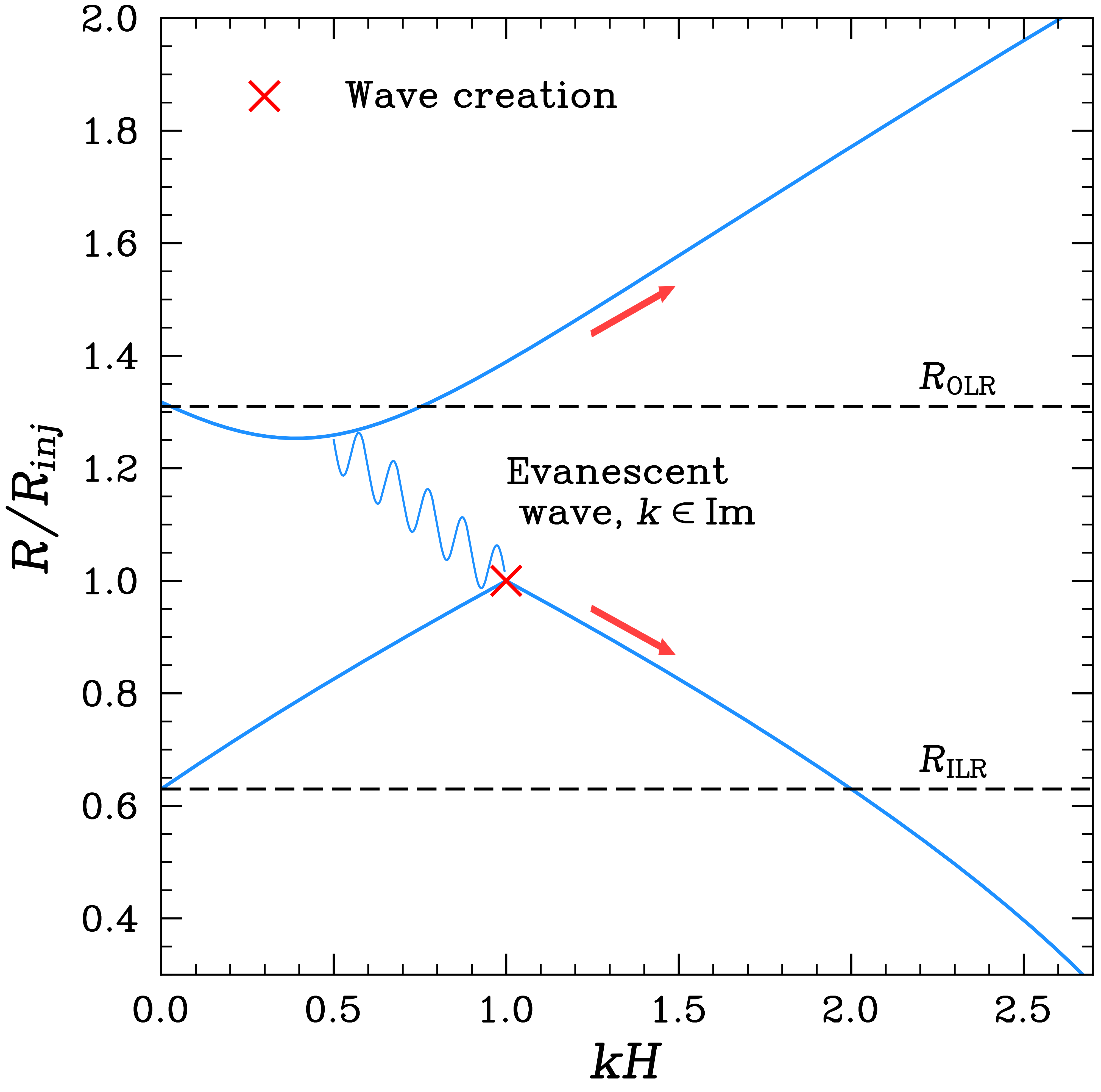
Wave propagation





$$s^2 = \frac{Q^2}{4}(kH)^2 - |kH| + 1$$

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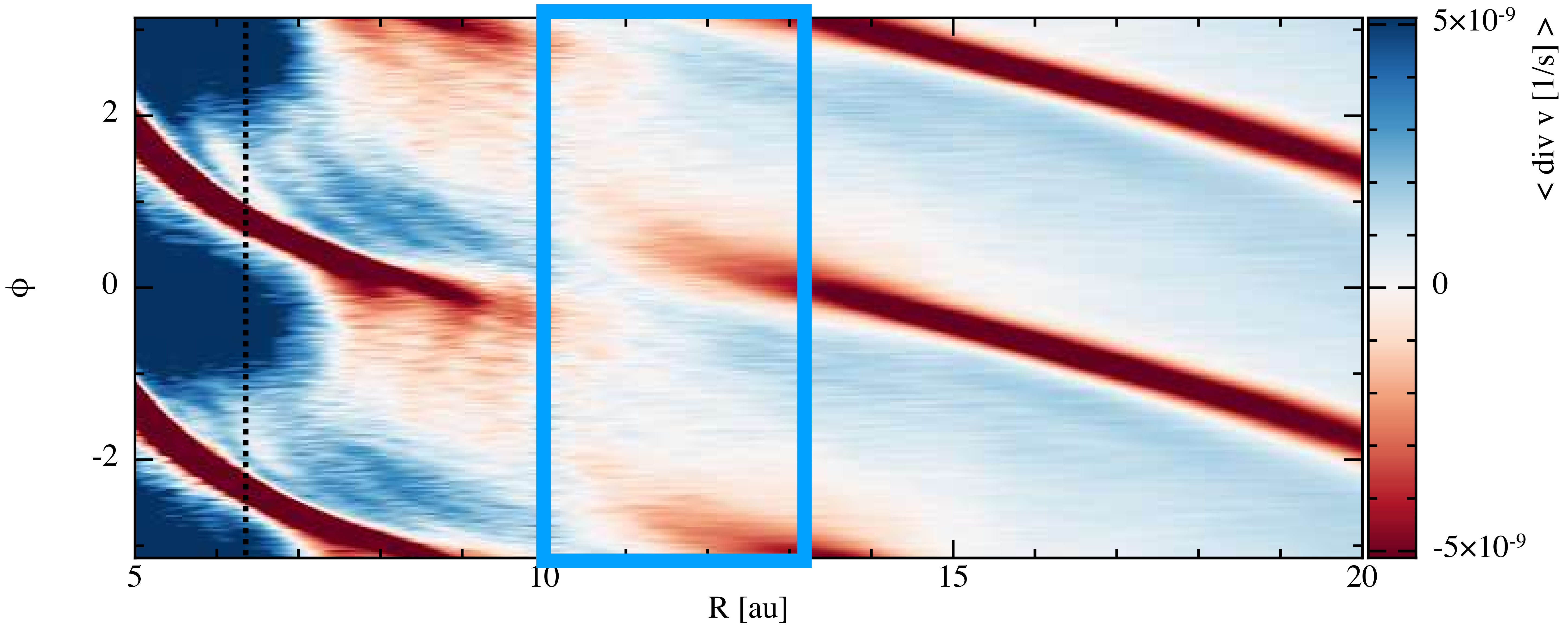
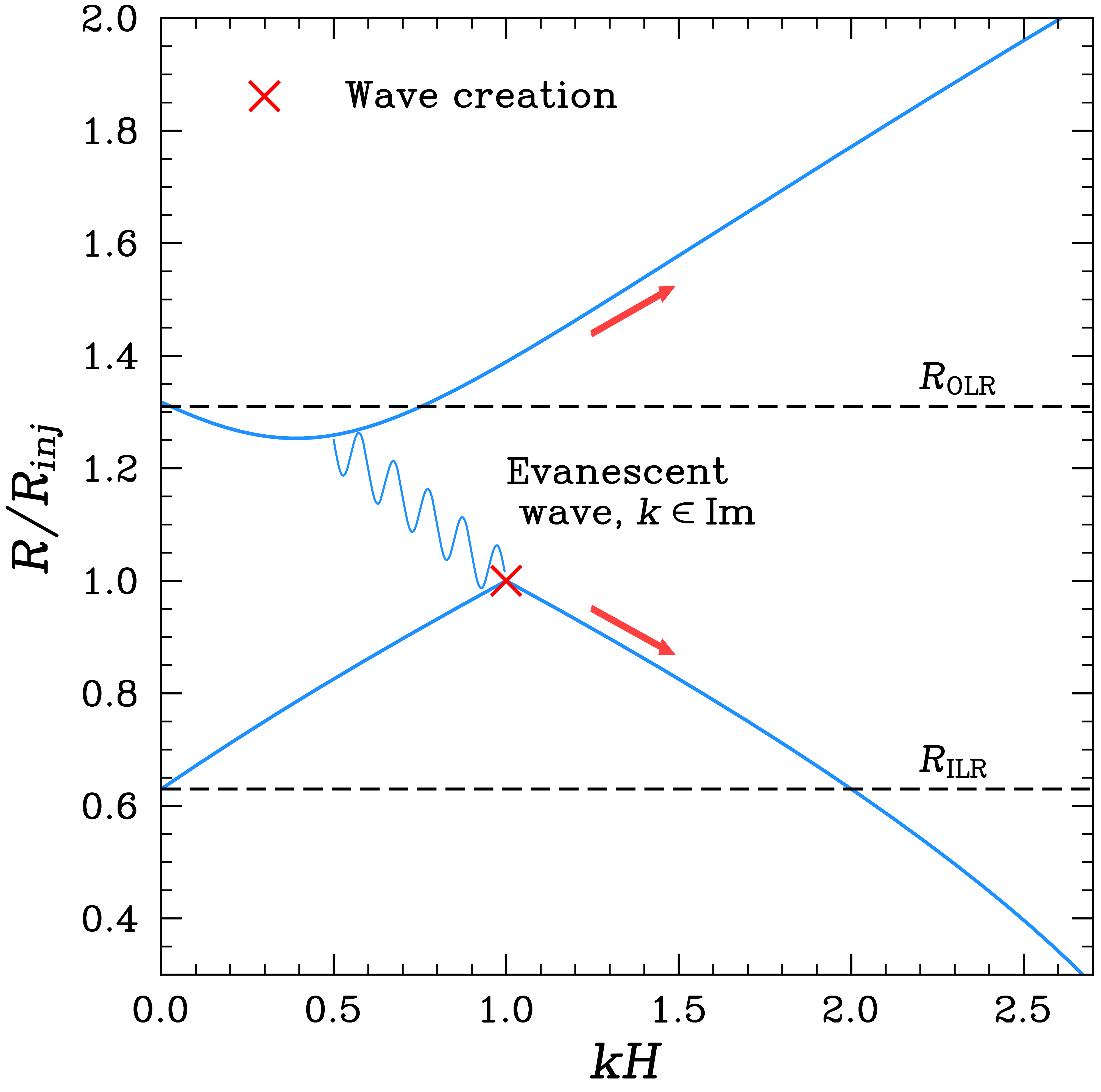
Wave propagation





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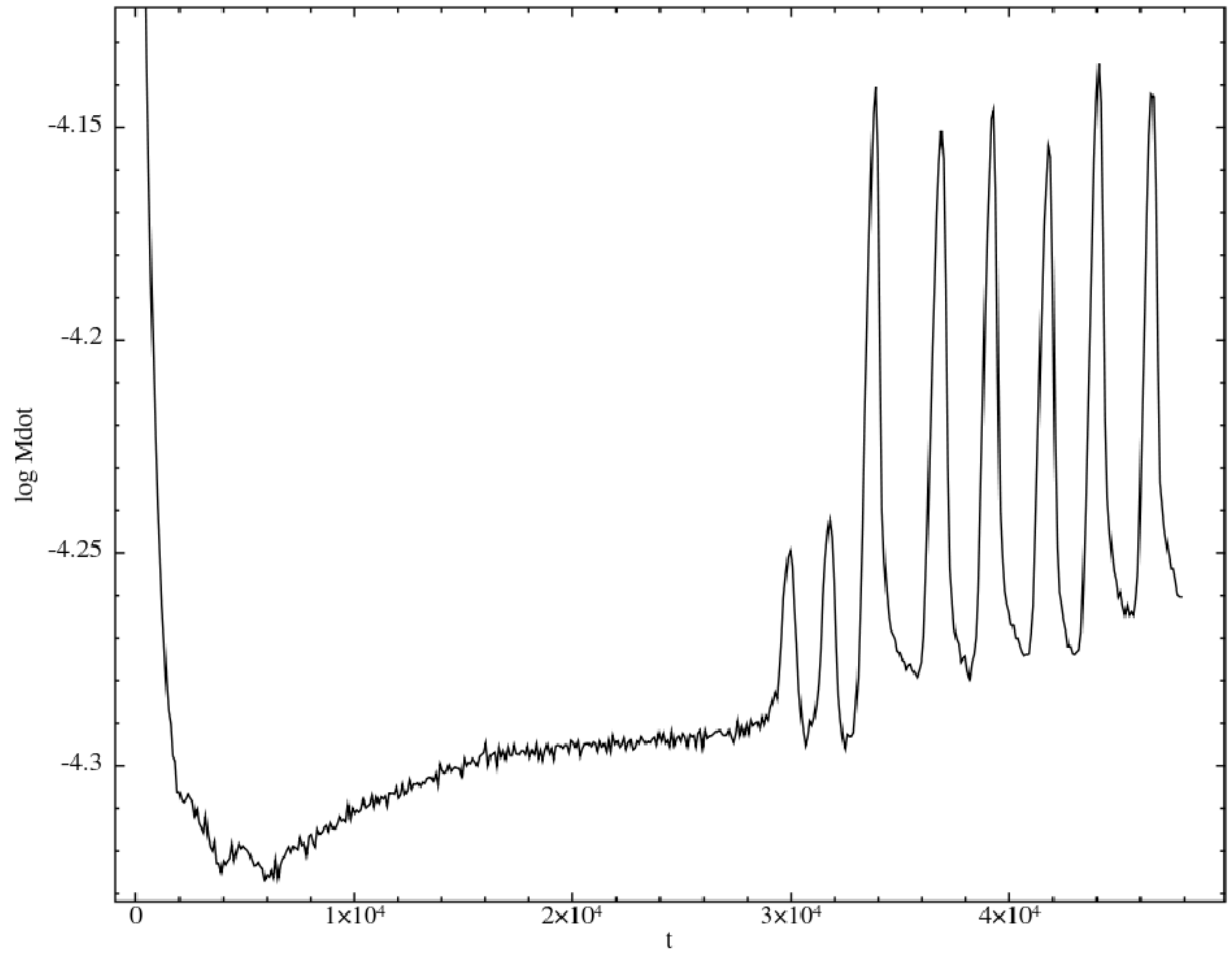
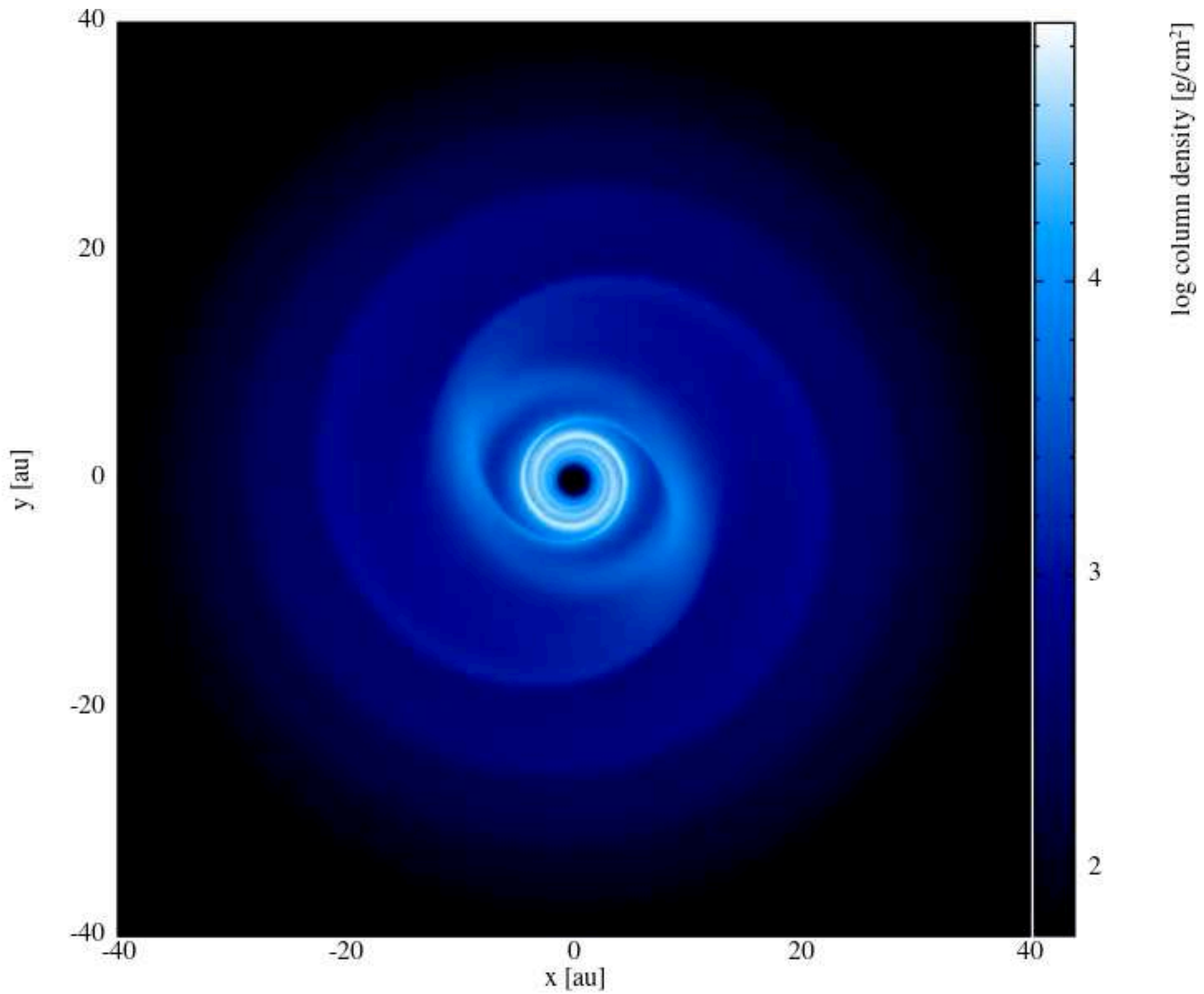
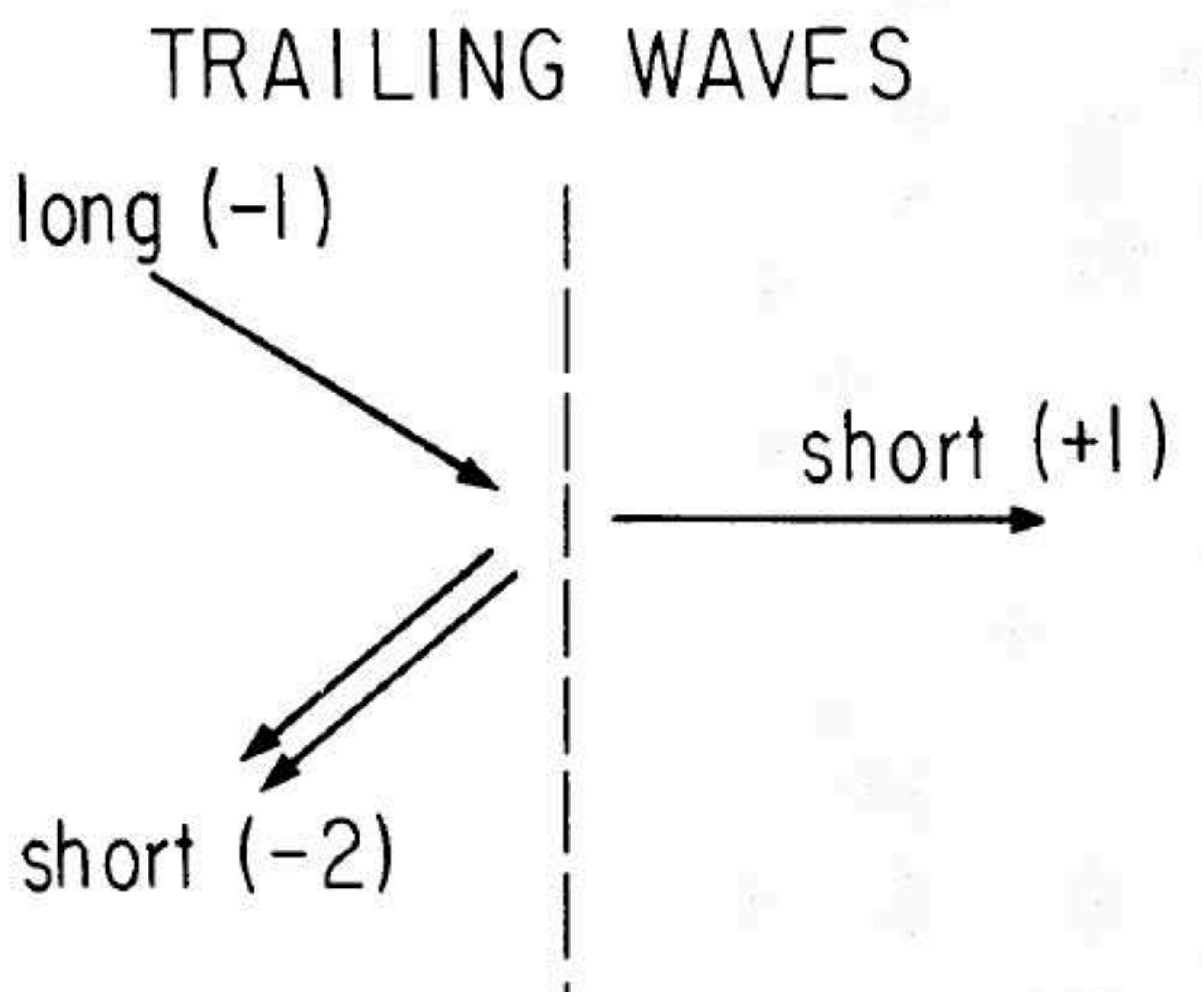
Variability and overreflection





Longarini et al. in prep.

Overreflection at corotation?



Corotation
(injection)

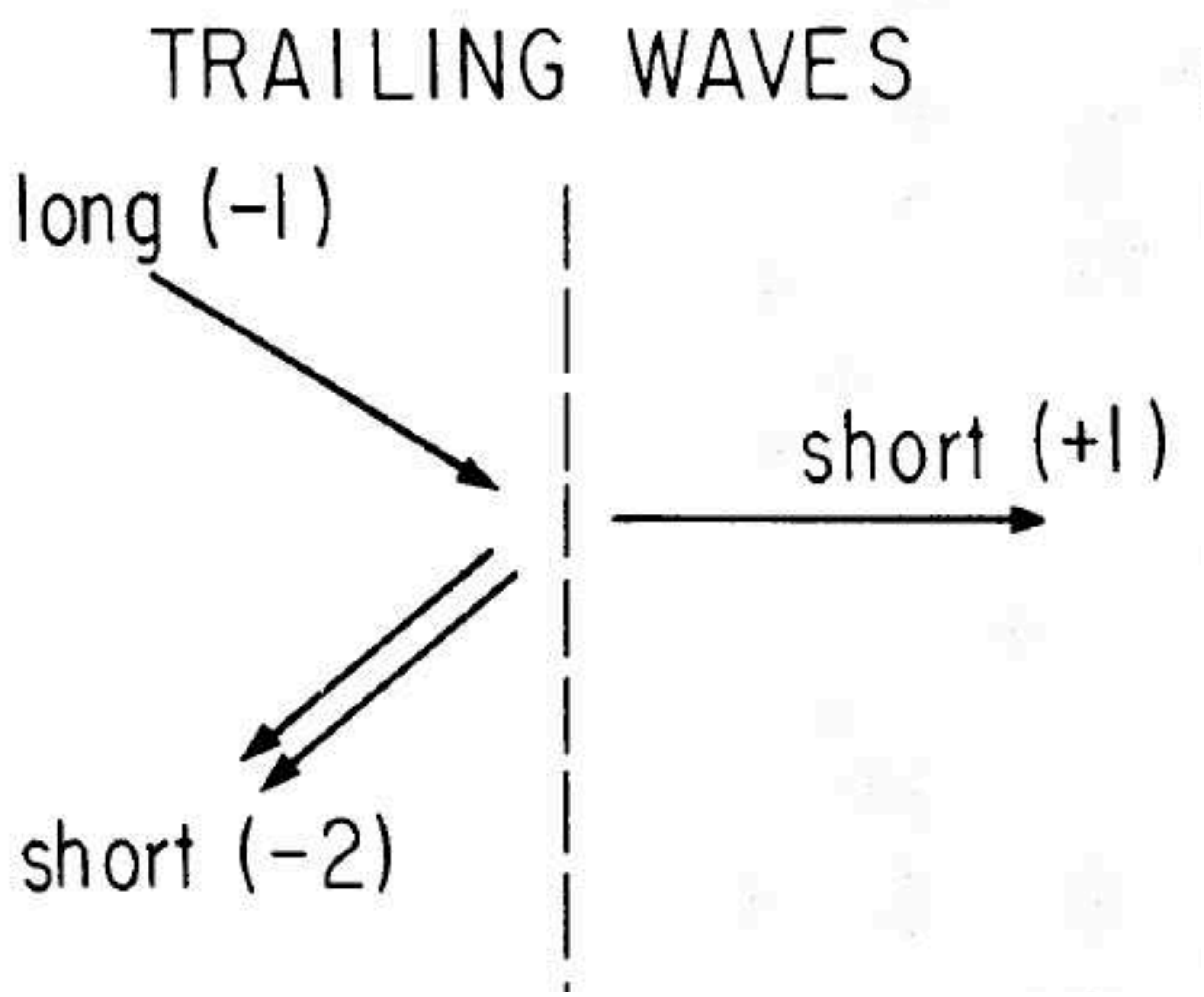
$c_g > 0$ $c_g < 0$

$$c_g = \frac{\partial \omega}{\partial k}$$

Mark 1975

Longarini et al. in prep.

Overreflection at corotation?



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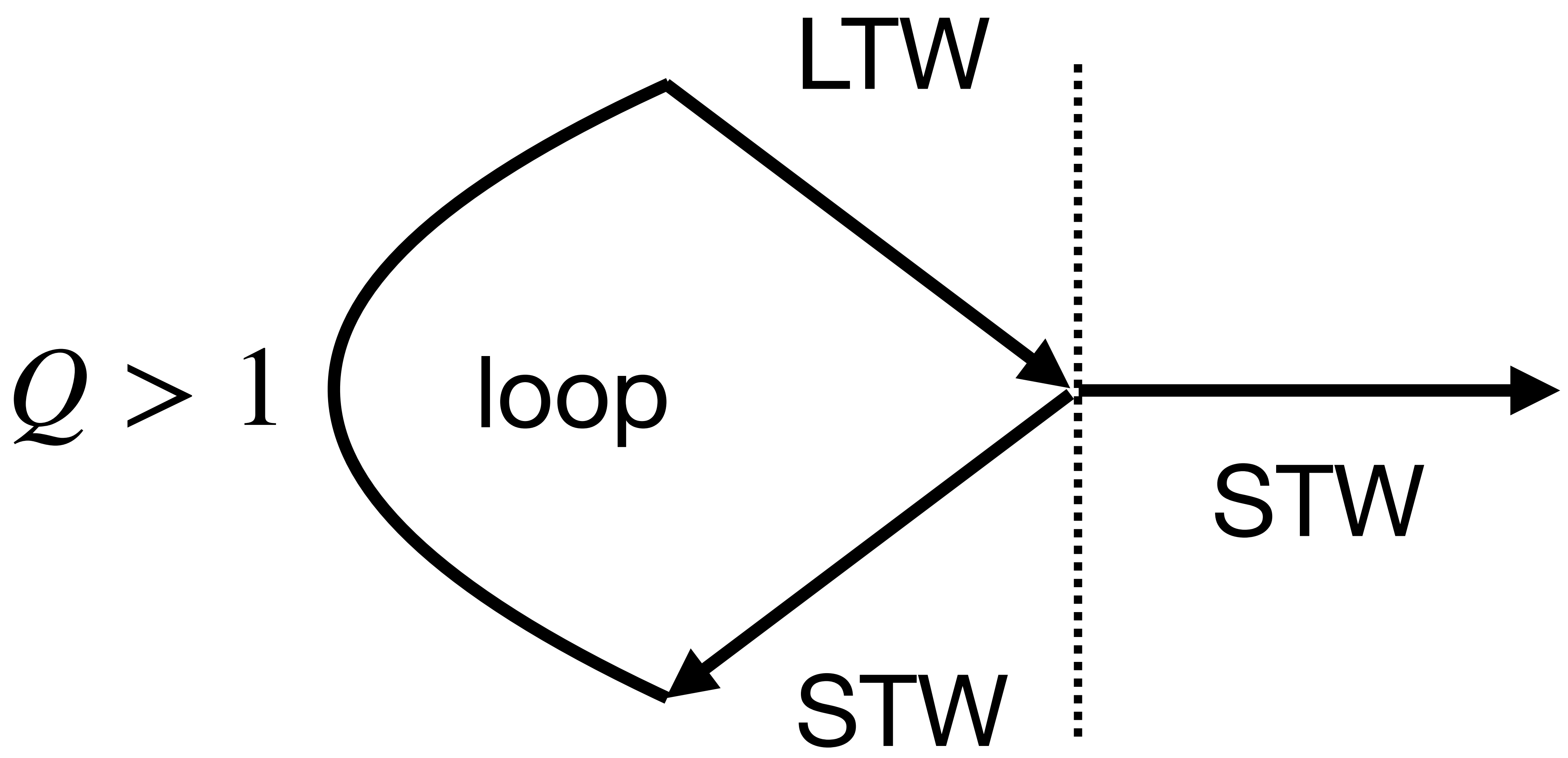

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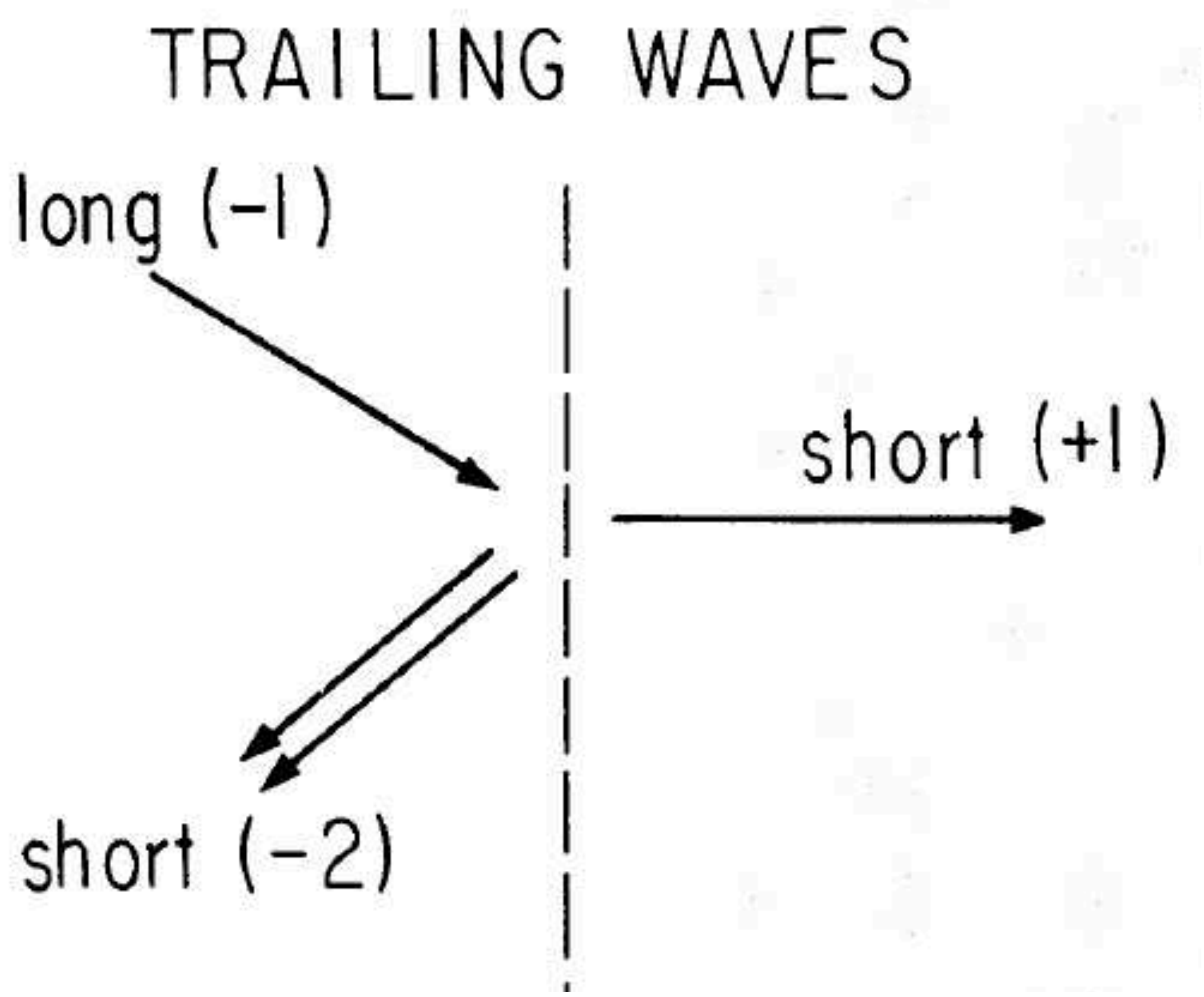

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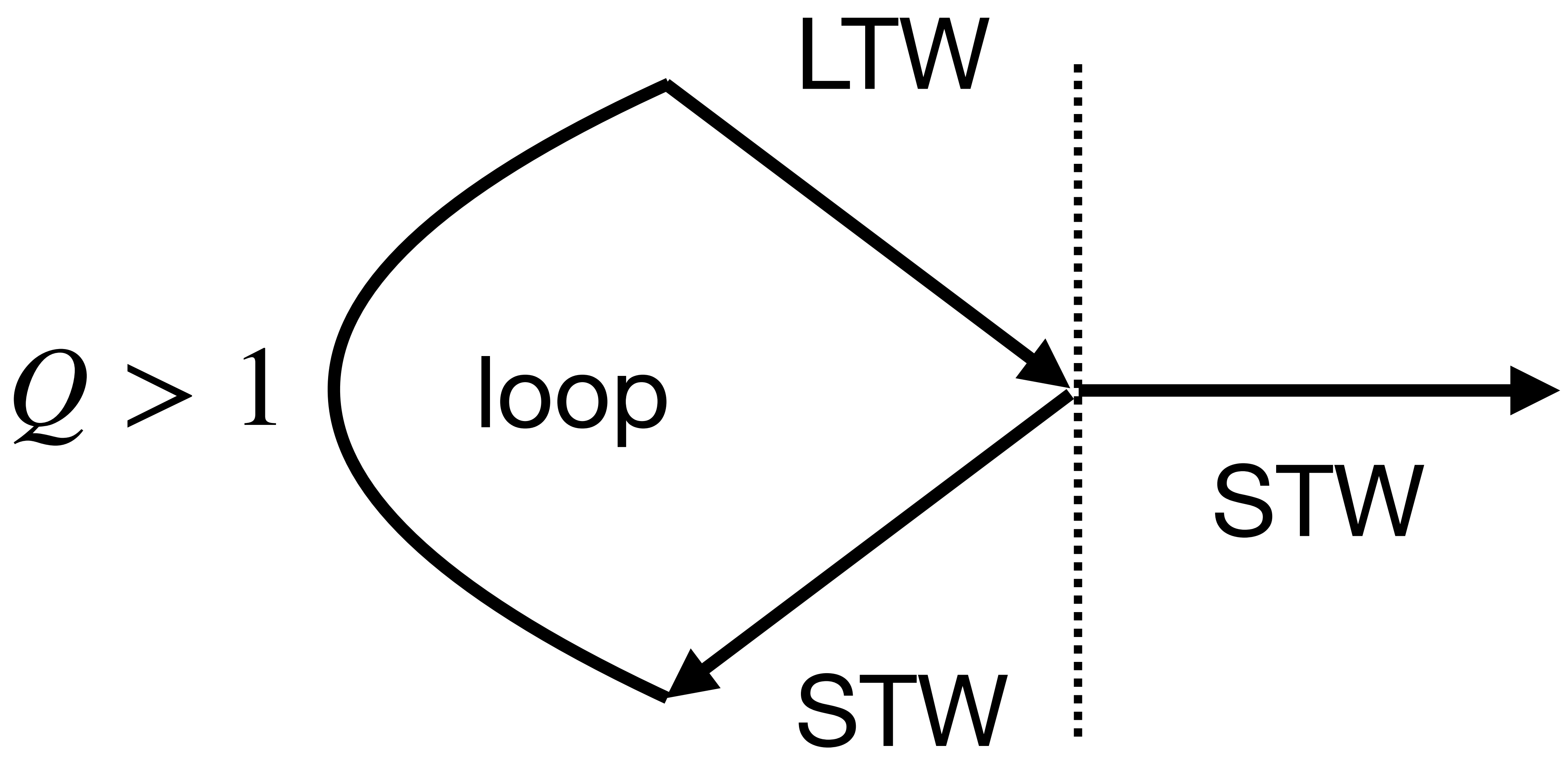
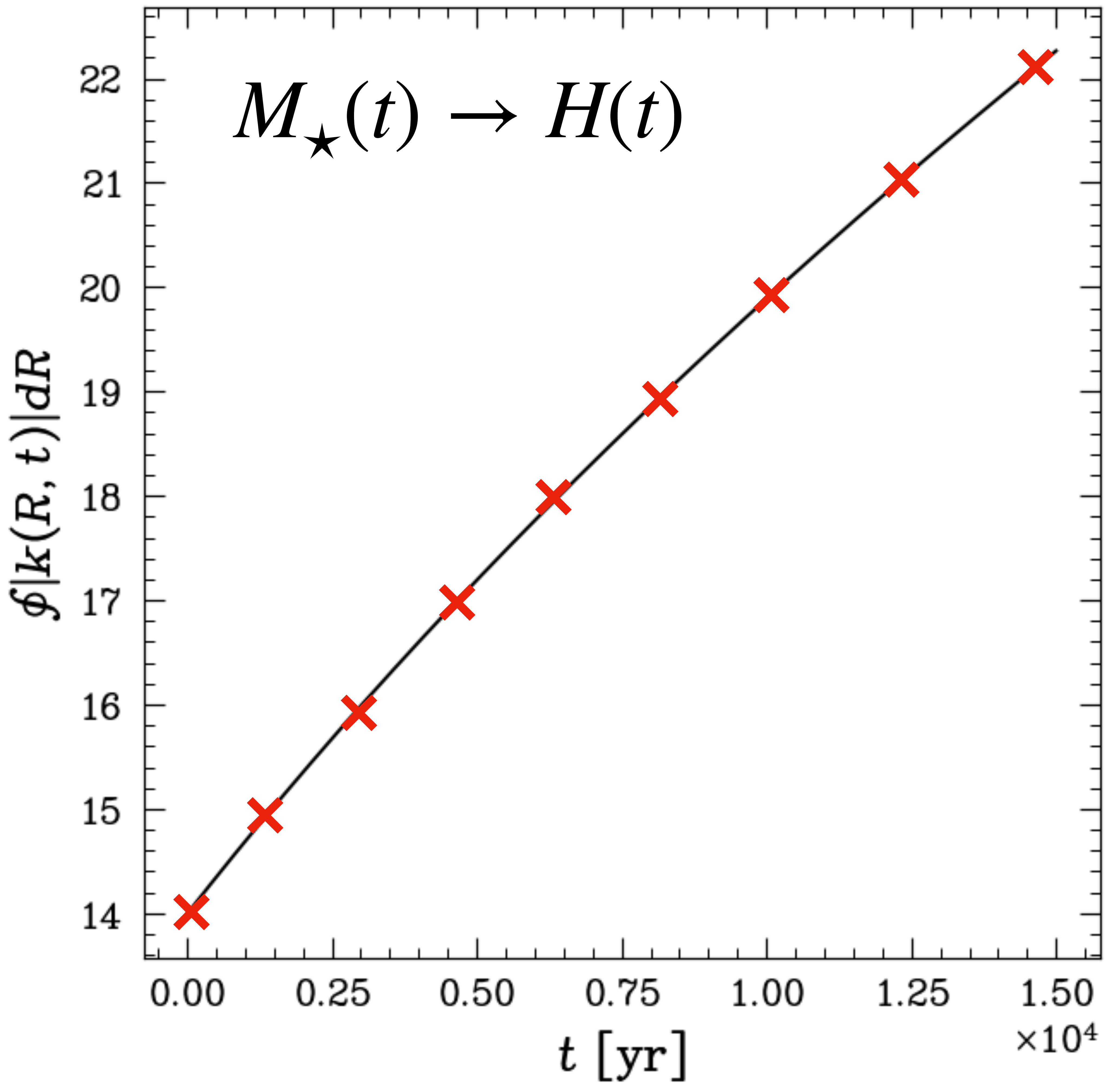


Constructive interference condition

$$\oint_{\text{loop}} dR\,|\,k(R)\,| = (2n+1)\pi$$

Longarini et al. in prep.

Overreflection at corotation?



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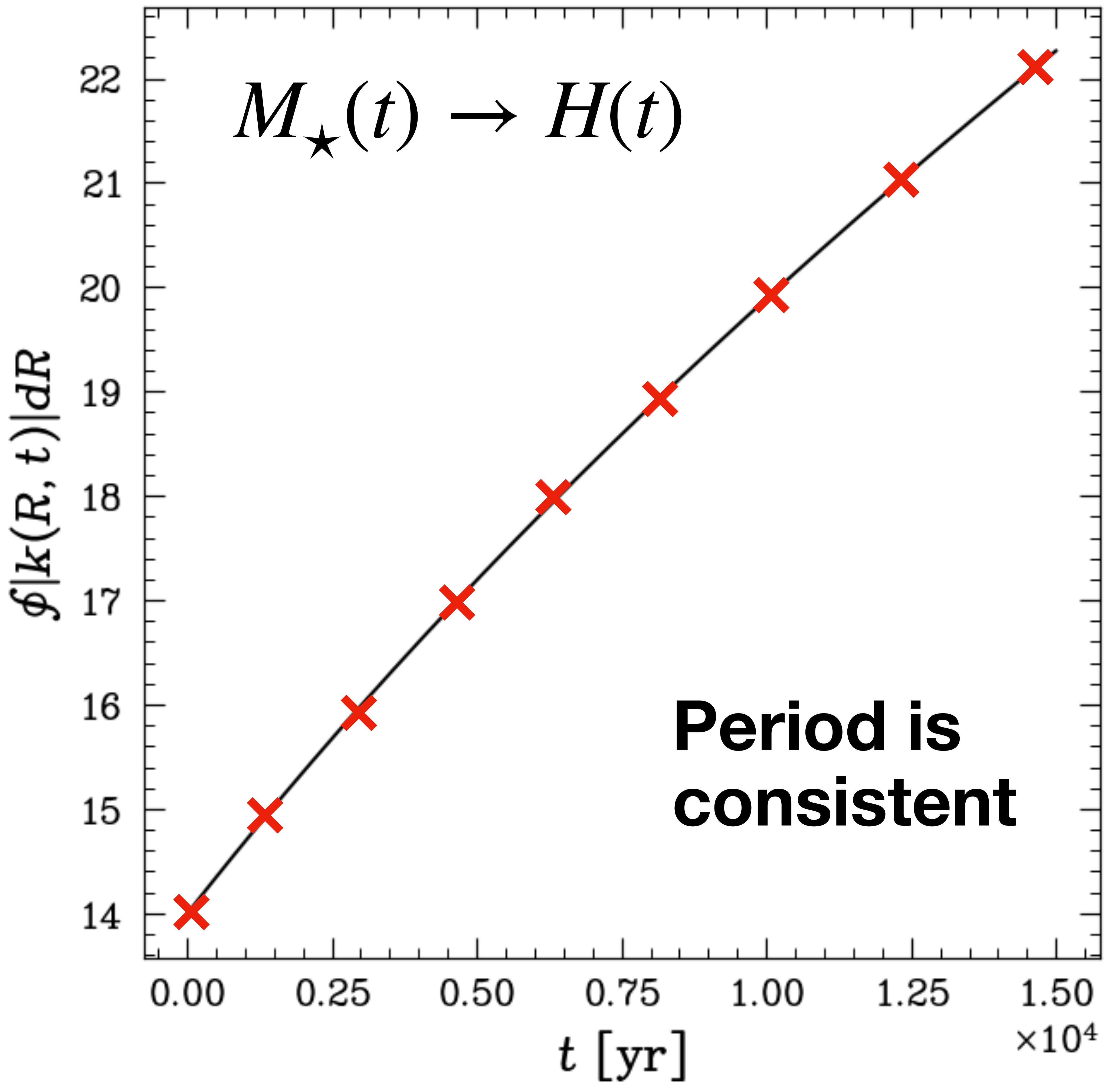

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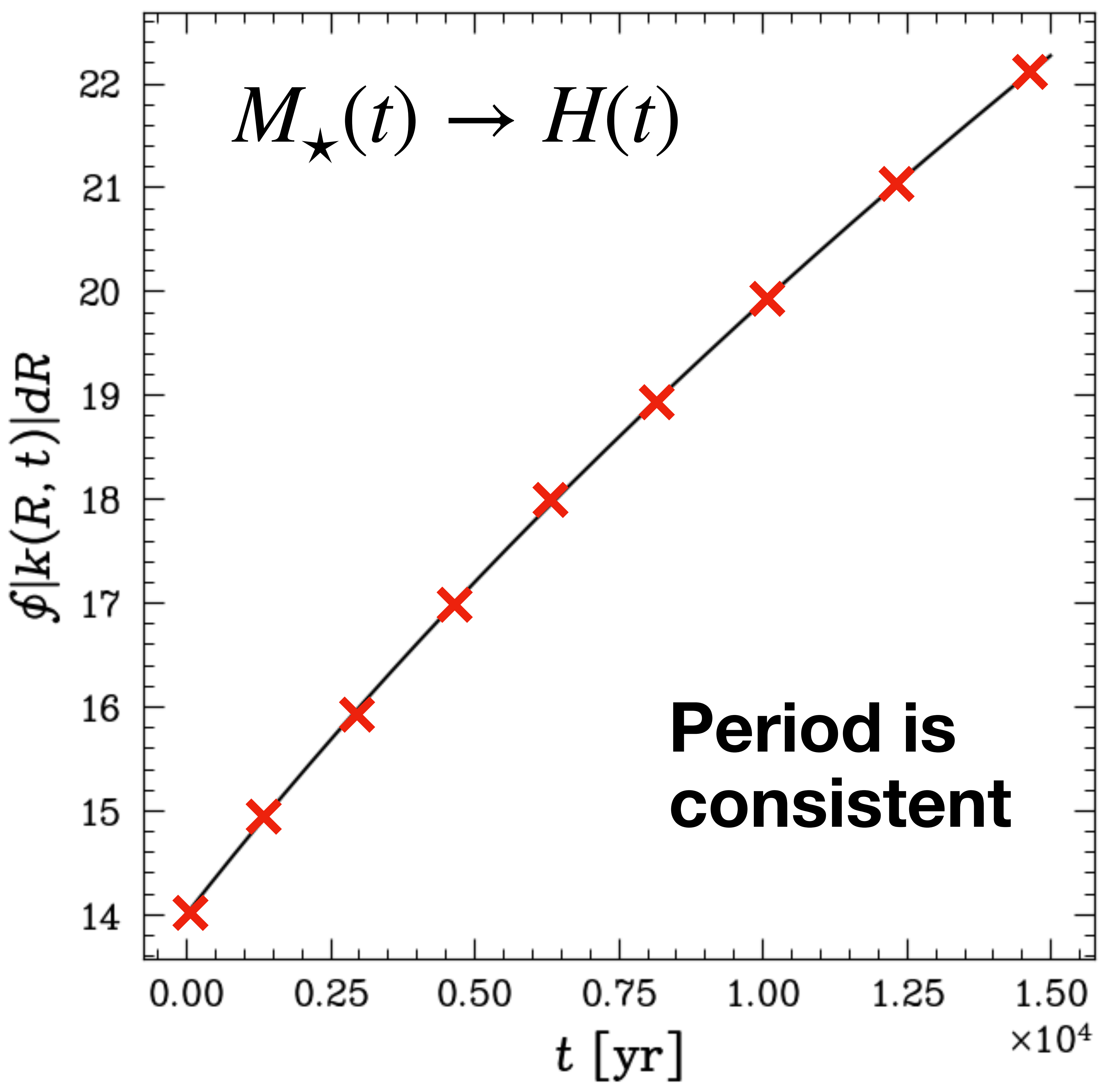

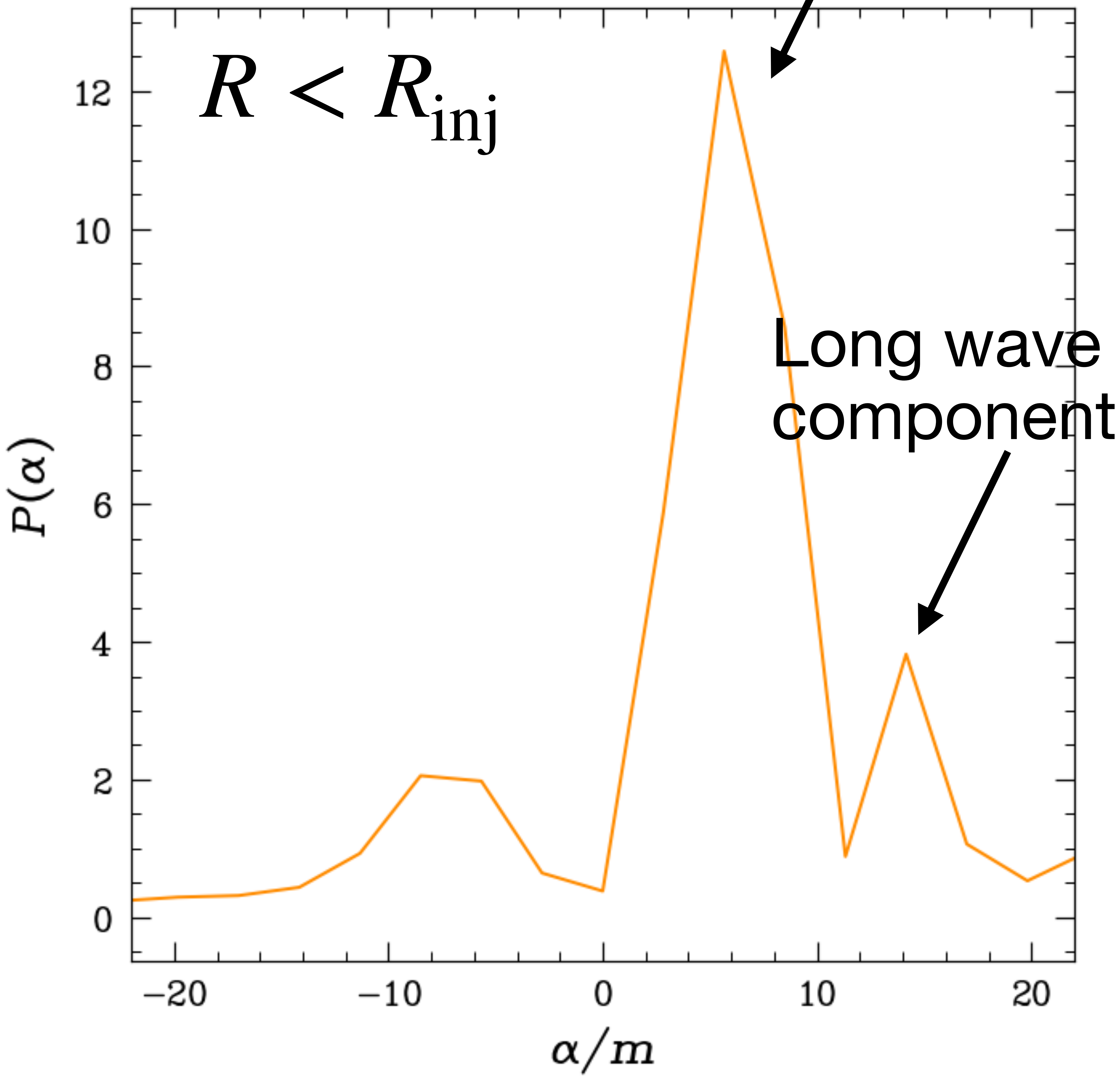

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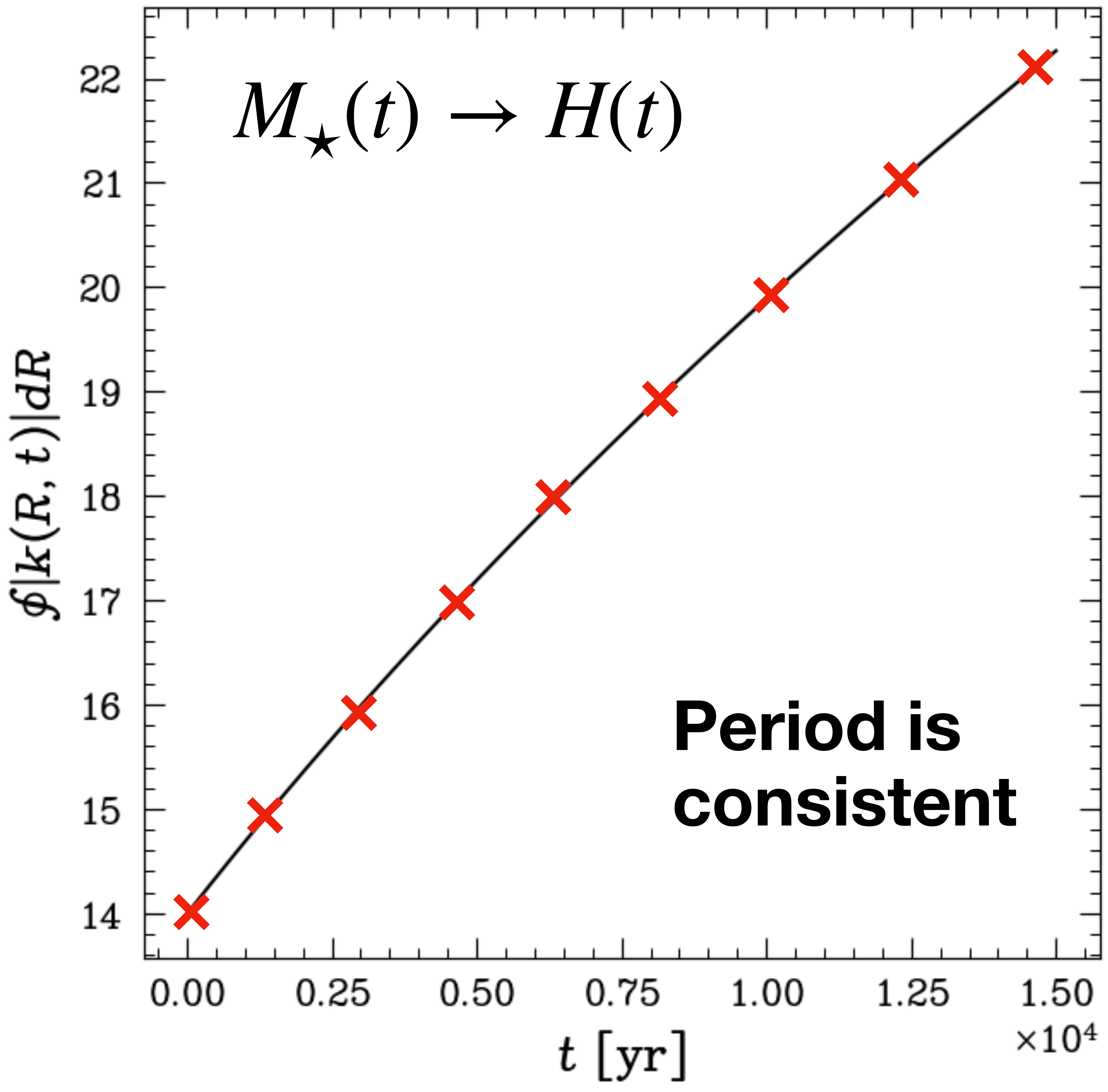
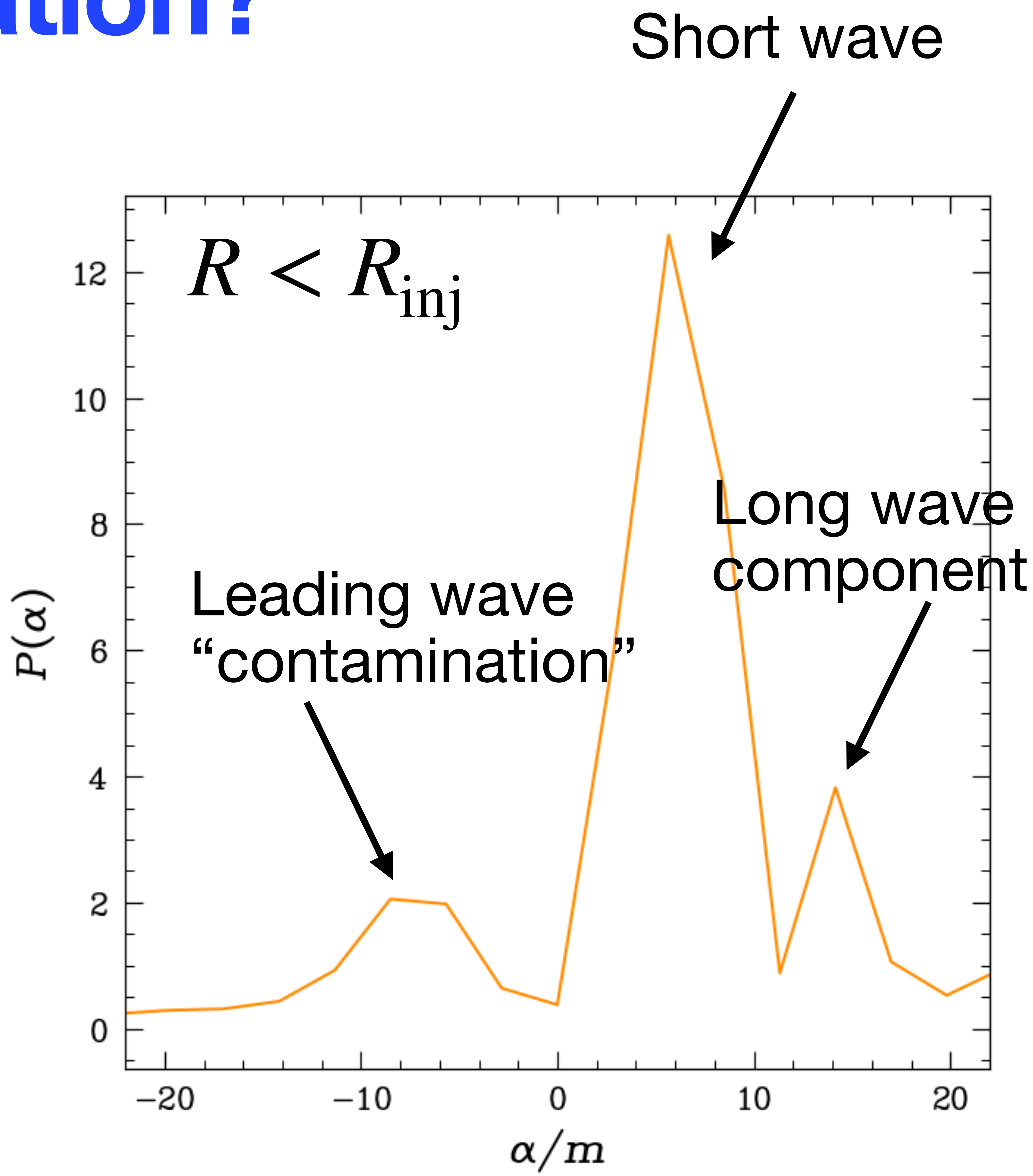
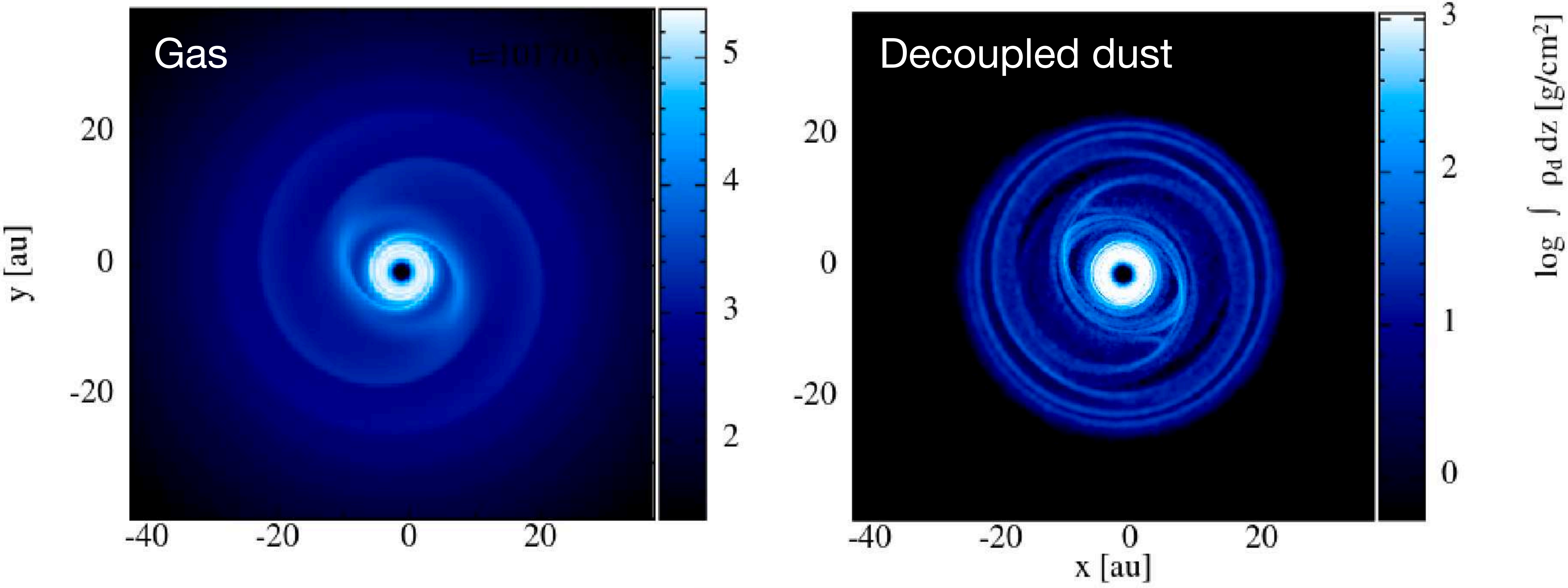




Longarini et al. in prep.

Infall and dust dynamics - planet formation?



Summary

SG is fundamental in astrophysics

GI and cooling: very successful so far for experiments
- Kinematics and angular momentum transport
- Dust dynamics and planet formation

GI and infall:
- Self regulation in terms of mass
- Good agreement between 1D and 3D

Infall vs Cooling
- Analogies: morphological scalings, dominant modes
- Differences: pattern speed, global modes
- Planet formation?