Why merging is really hard

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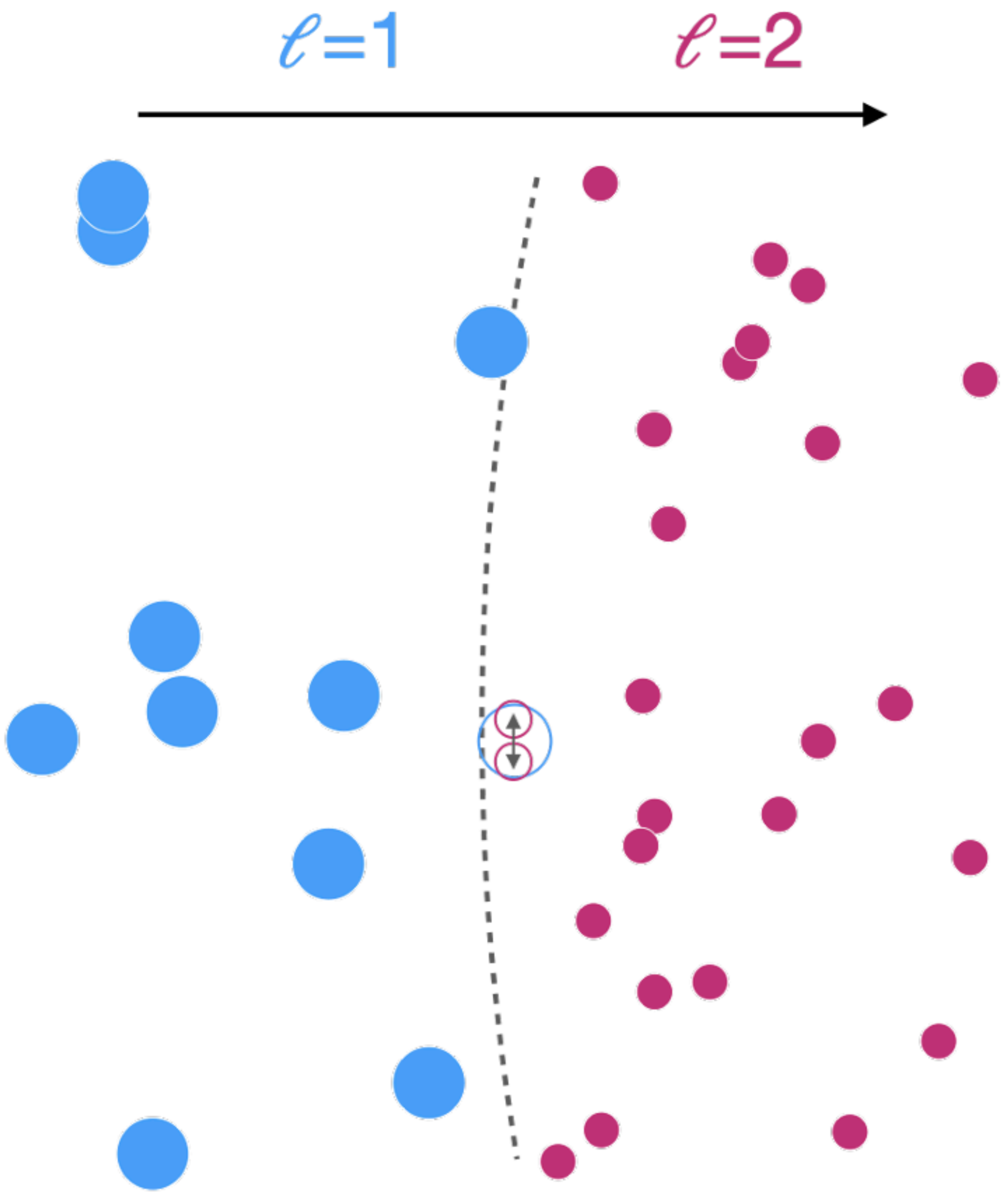
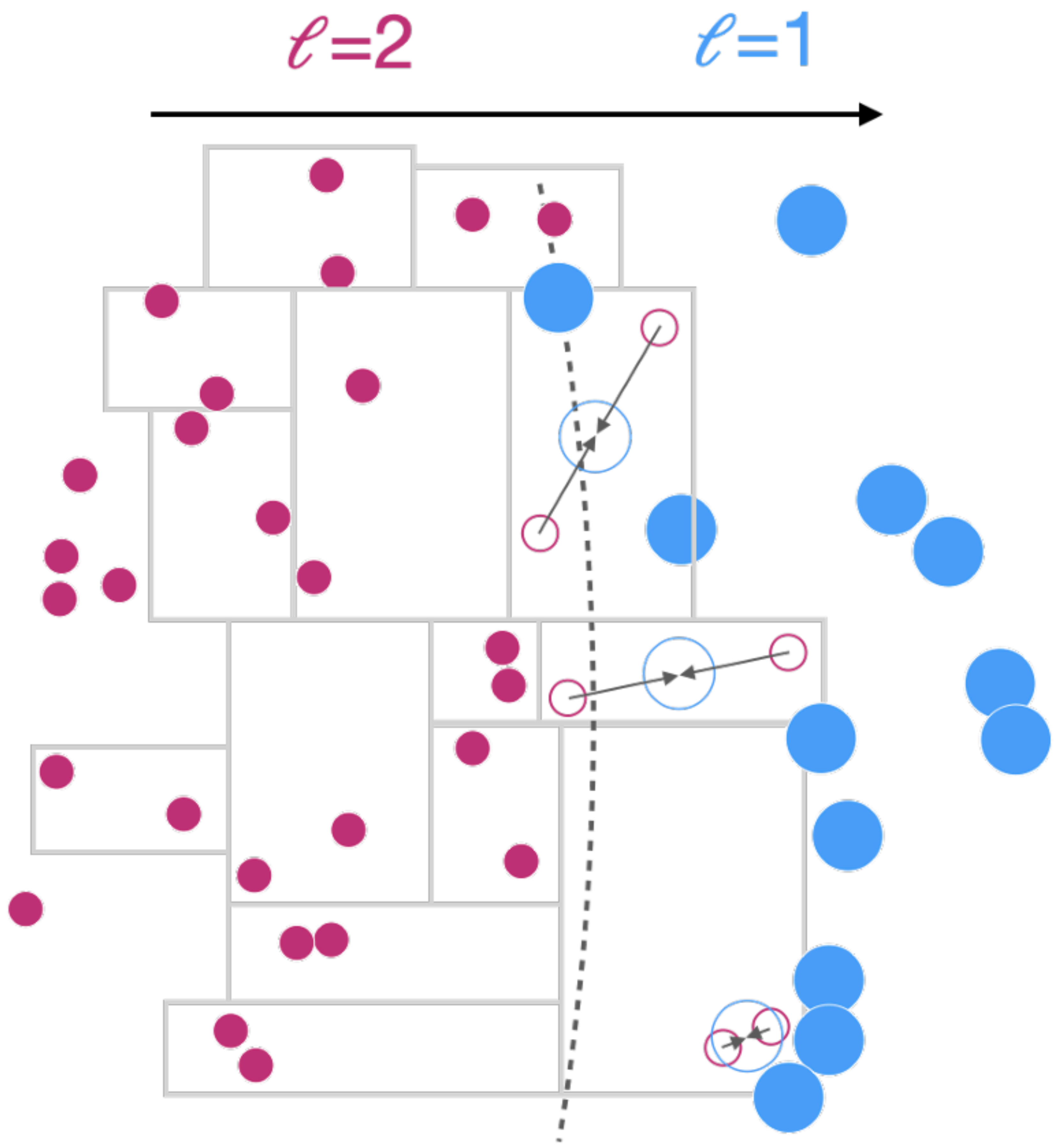
PHANTOM

Adaptive particle refinement



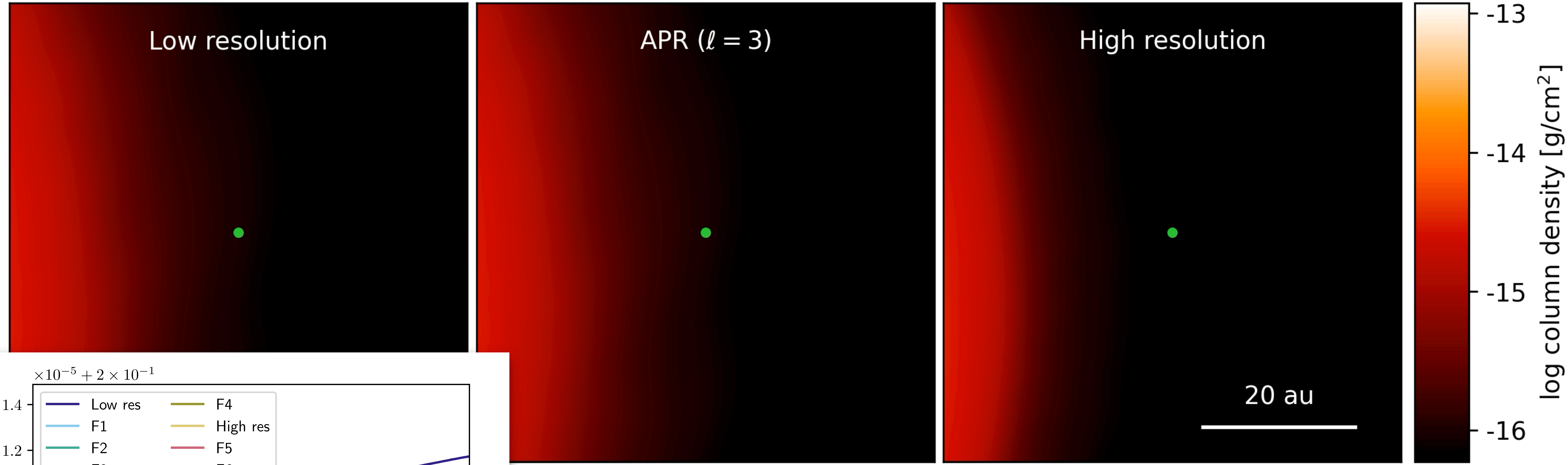
Refining

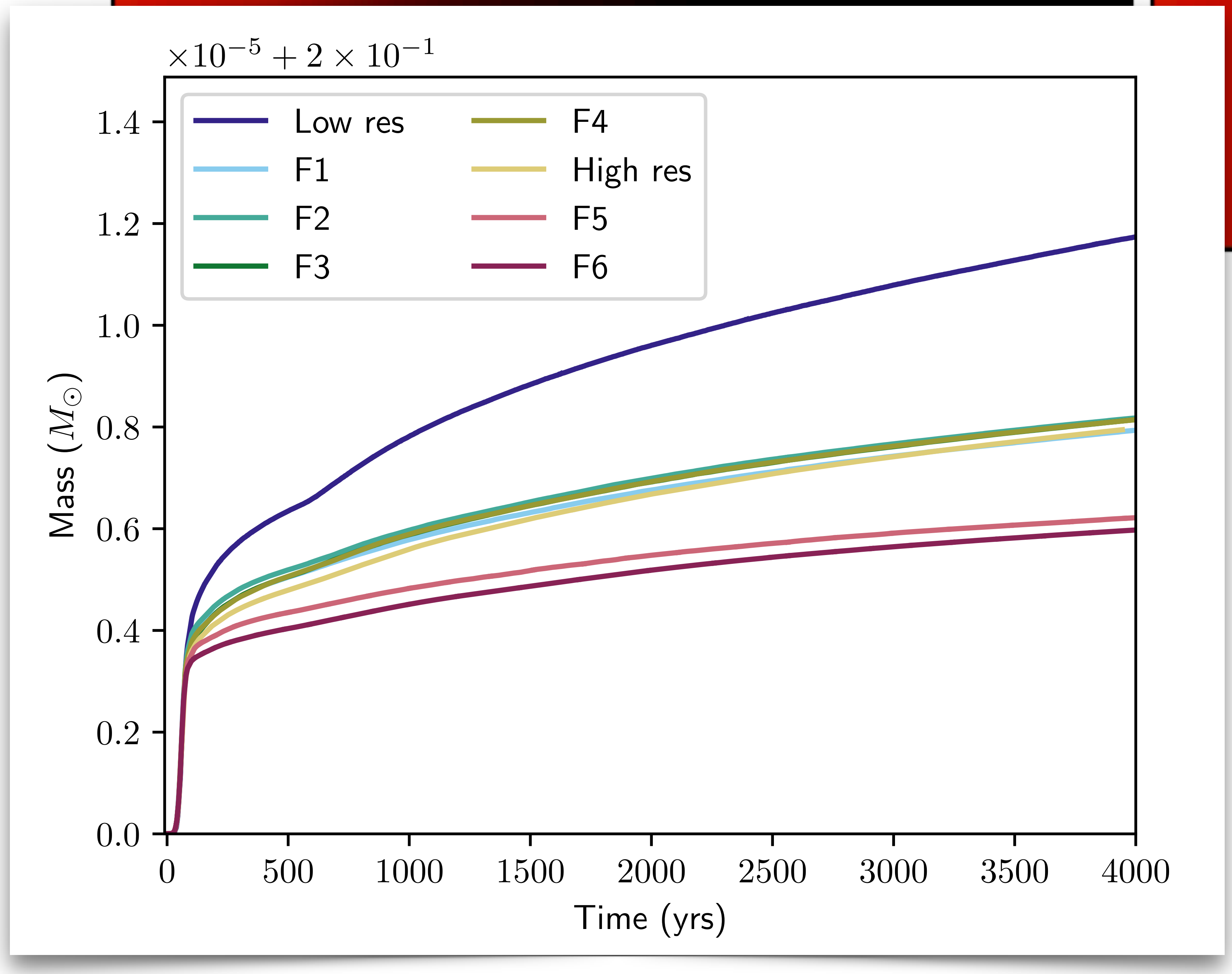


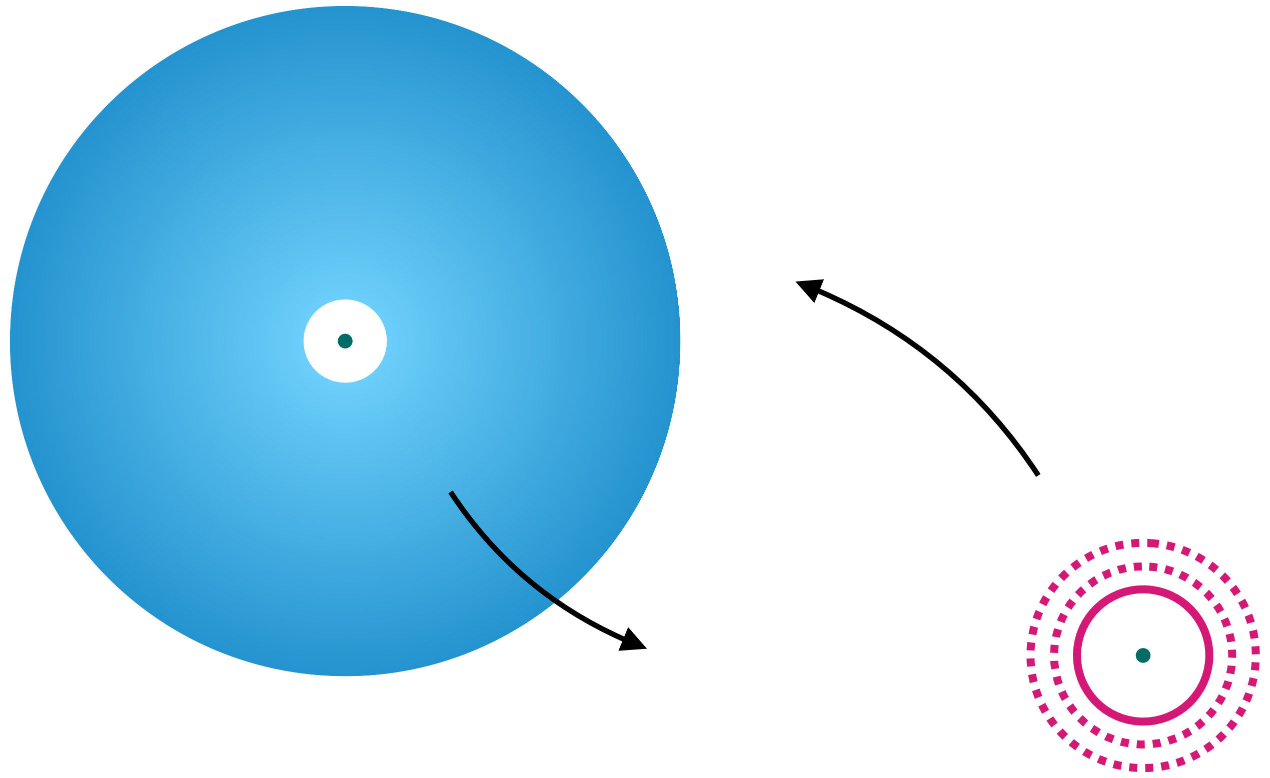
Derefining

Nealon & Price 2025, based on Smallwood et al. 2024

APR can work very well …

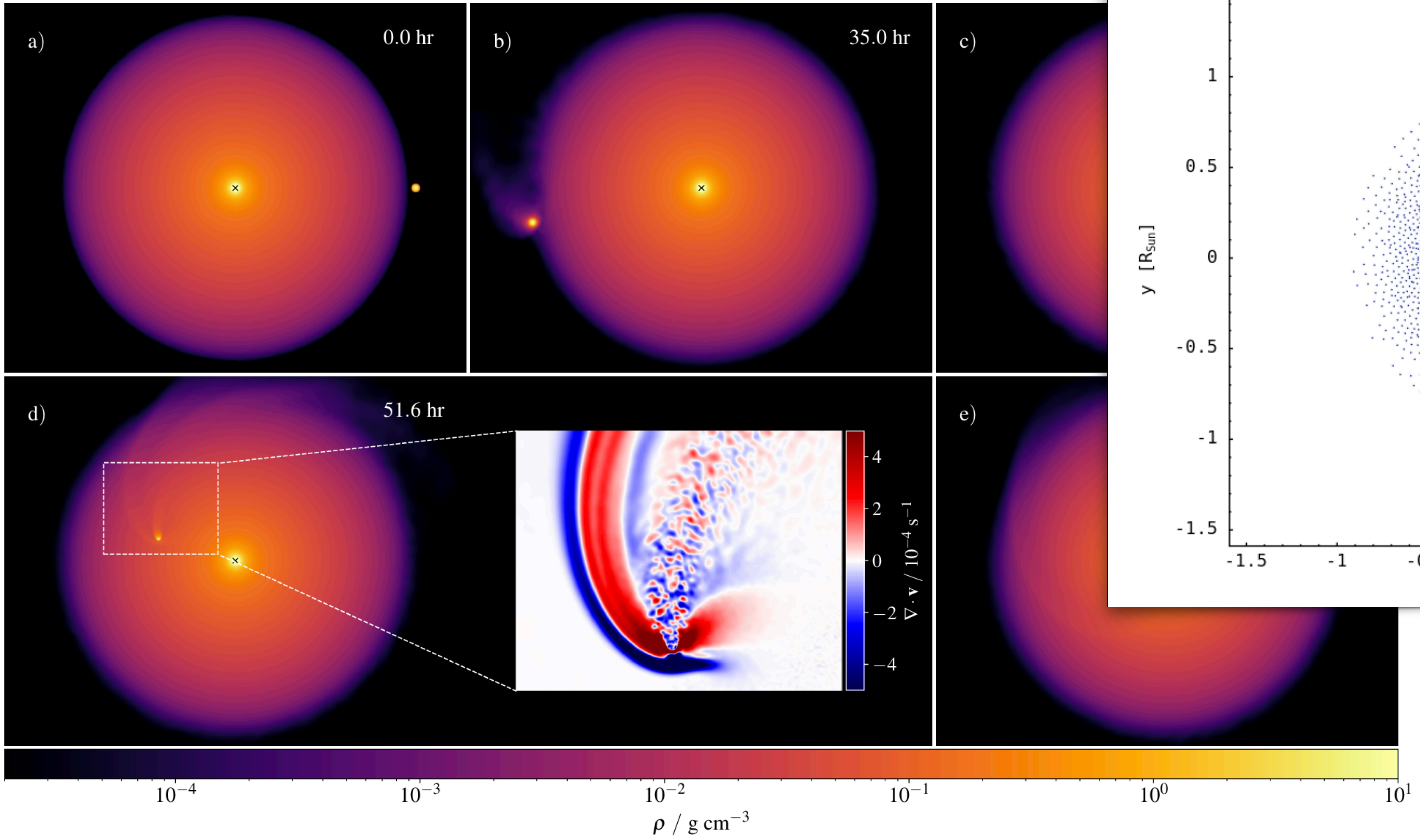


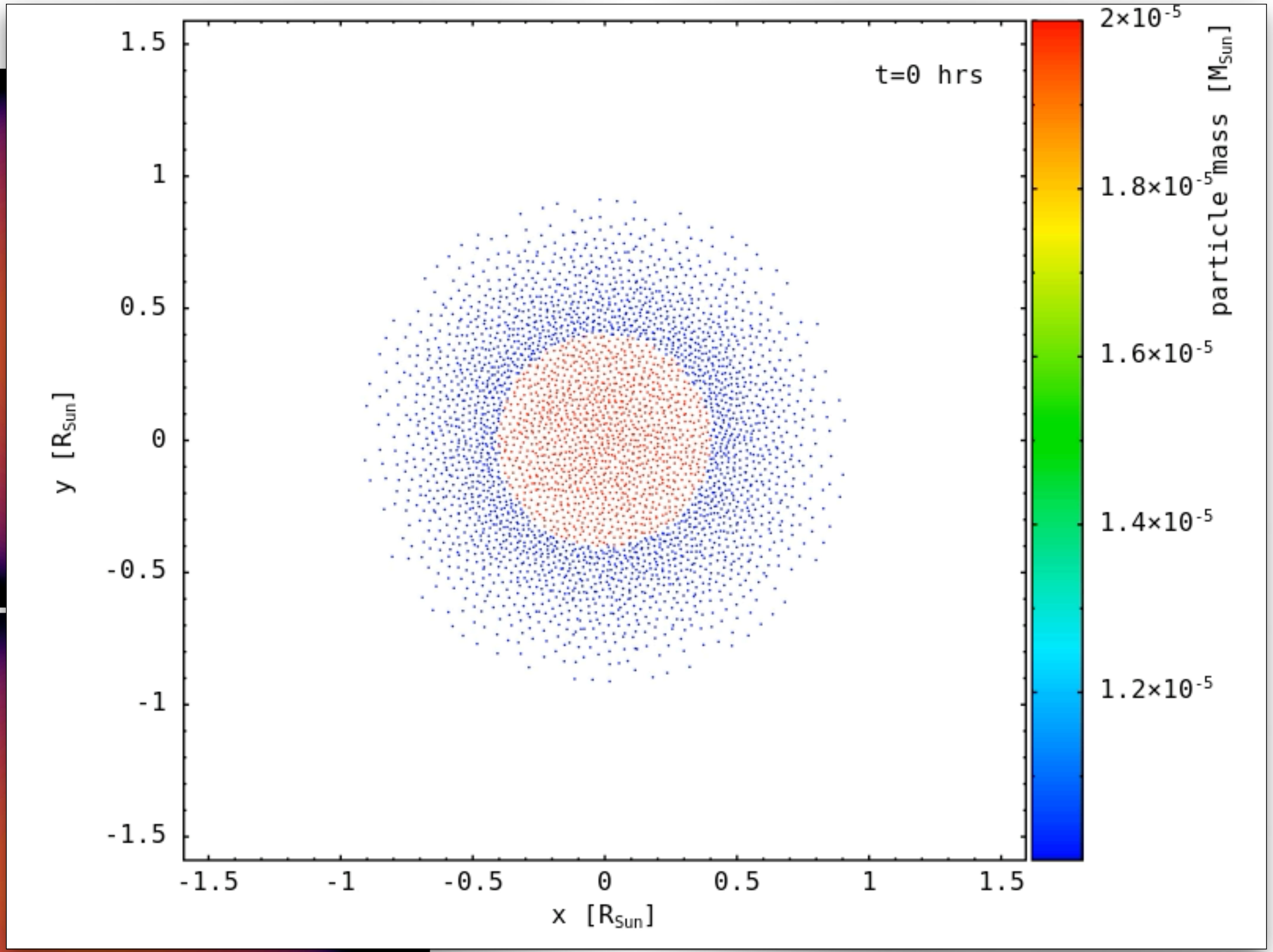




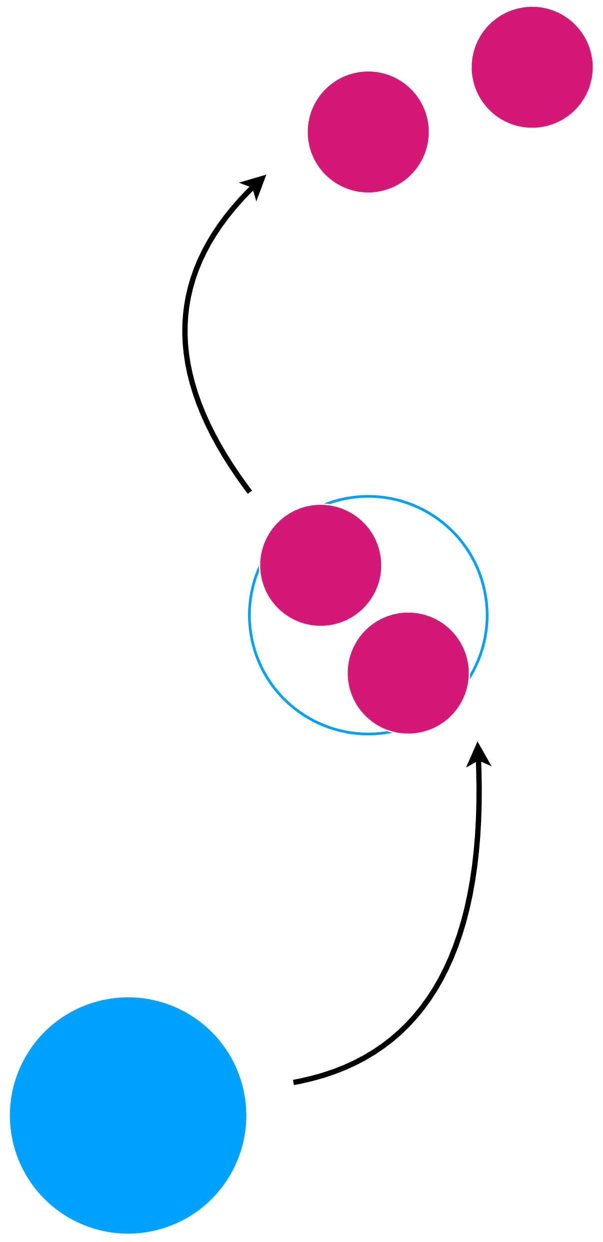
.... but it also sometimes doesn't.

Lau et al. 2025





During a split



Linear momentum

$$m_p\mathbf{v}_p = \frac{1}{2}m_p\mathbf{v}_p + \frac{1}{2}m_p\mathbf{v}_p$$
$$= m_c\mathbf{v}_p + m_c\mathbf{v}_p$$
$$\to m_c\mathbf{v}_1 + m_c\mathbf{v}_2$$

Internal energy

$$u_p = u_1 = u_2$$

Kinetic energy

$$\frac{1}{2}m_p v_p^2 = \frac{1}{2}\left(\frac{1}{2}m_p v_p^2 + \frac{1}{2}m_p v_p^2\right)$$
$$= \frac{1}{2}m_c v_p^2 + \frac{1}{2}m_c v_p^2$$
$$\to \frac{1}{2}m_c v_1^2 + \frac{1}{2}m_c v_2^2$$

Angular momentum

$$m_p\mathbf{r}_p \times \mathbf{v}_p = \frac{1}{2}m_p\mathbf{r}_p \times \mathbf{v}_p + \frac{1}{2}m_p\mathbf{r}_p \times \mathbf{v}_p$$
$$= m_c\mathbf{r}_p \times \mathbf{v}_p + m_c\mathbf{r}_p \times \mathbf{v}_p$$
$$= m_c\mathbf{r}_p \times \mathbf{v}_1 + m_c\mathbf{r}_p \times \mathbf{v}_2$$
$$\to m_c\mathbf{r}_1 \times \mathbf{v}_1 + m_c\mathbf{r}_2 \times \mathbf{v}_2$$

… we conserve all of the above.

During a merge

Linear momentum

$$m_c\mathbf{v}_1 + m_c\mathbf{v}_2 = m_c(\mathbf{v}_1 + \mathbf{v}_2)$$
$$= m_p\frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2)$$
$$\rightarrow m_p\mathbf{v}_p$$

Kinetic energy

$$\frac{1}{2}m_c v_1^2 + \frac{1}{2}m_c v_2^2 = \frac{1}{2}m_c\left(v_1^2 + v_2^2\right)$$
$$= \frac{1}{2}m_p\frac{\left(v_1^2 + v_2^2\right)}{2}$$
$$\rightarrow \frac{1}{2}m_p v_p^2$$

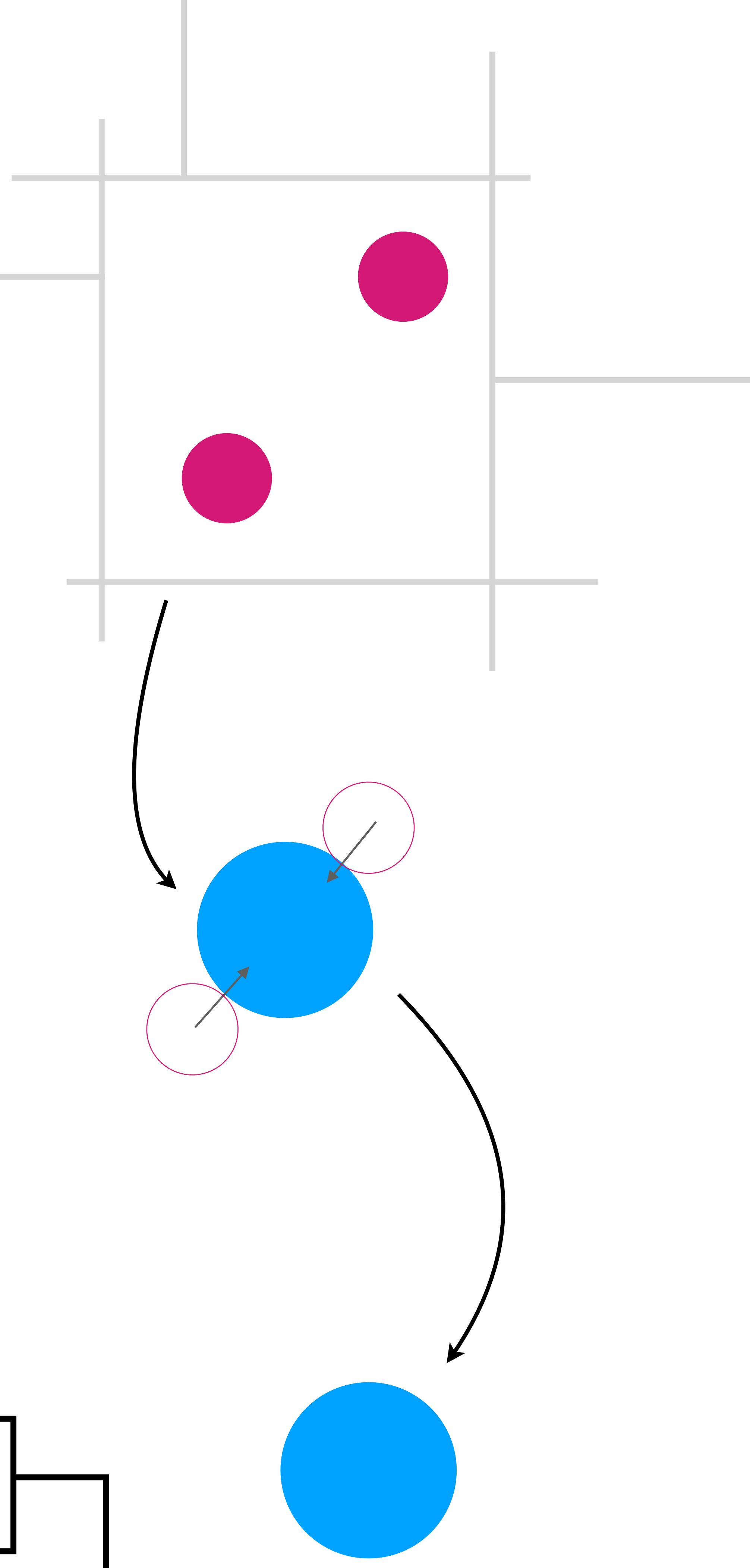
$$\mathbf{v}_1 \neq \mathbf{v}_2$$

Angular momentum

$$m_c\mathbf{r}_1 \times \mathbf{v}_1 + m_c\mathbf{r}_2 \times \mathbf{v}_2 = m_c\left(\mathbf{r}_1 \times \mathbf{v}_1 + \mathbf{r}_2 \times \mathbf{v}_2\right)$$
$$= m_p\frac{1}{2}\left(\mathbf{r}_1 \times \mathbf{v}_1 + \mathbf{r}_2 \times \mathbf{v}_2\right)$$
$$\rightarrow m_p\mathbf{r}_p \times \mathbf{v}_p$$

Internal energy

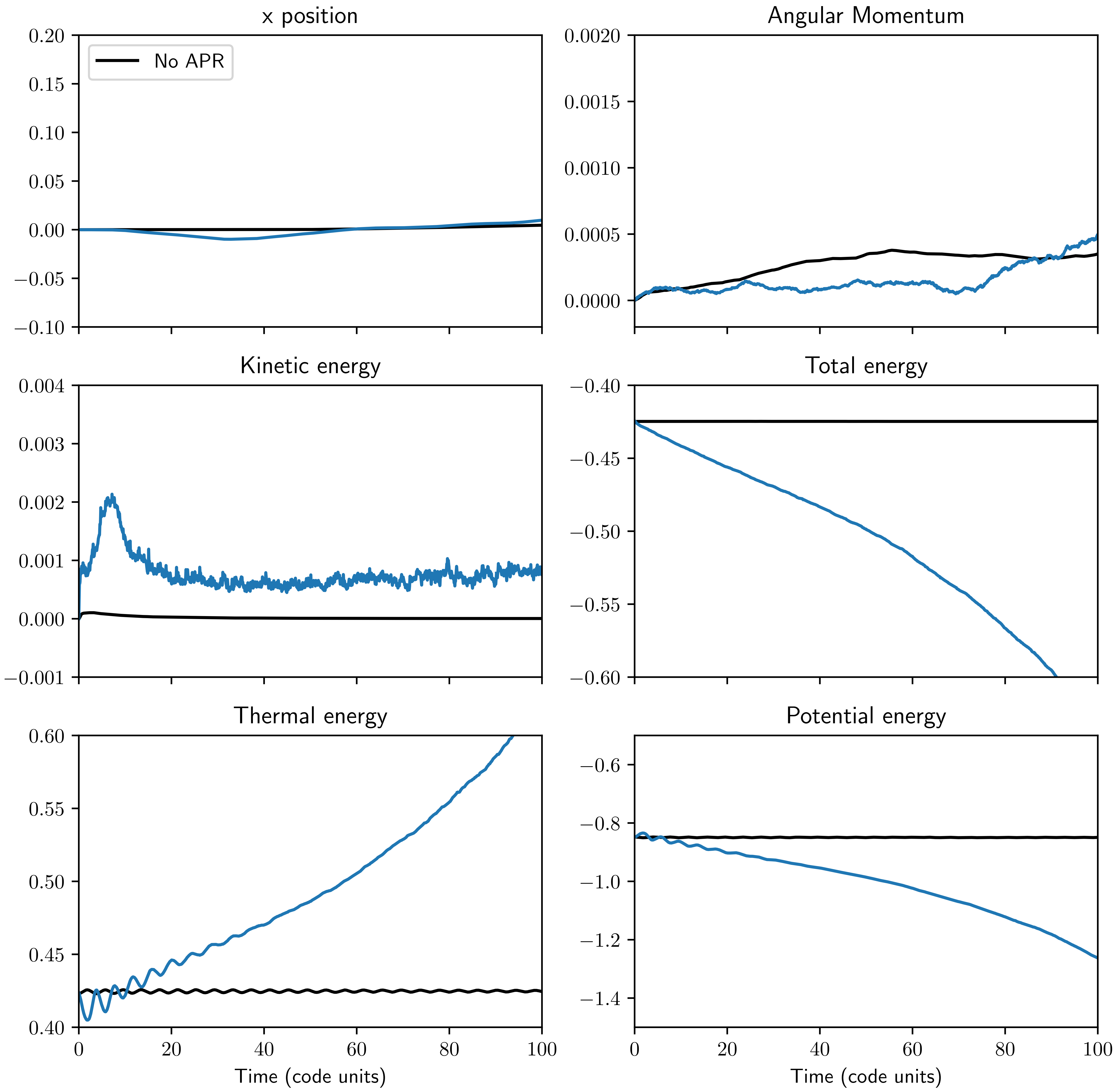
$$\frac{1}{2}(u_1 + u_2) = u_p$$



… not so much.

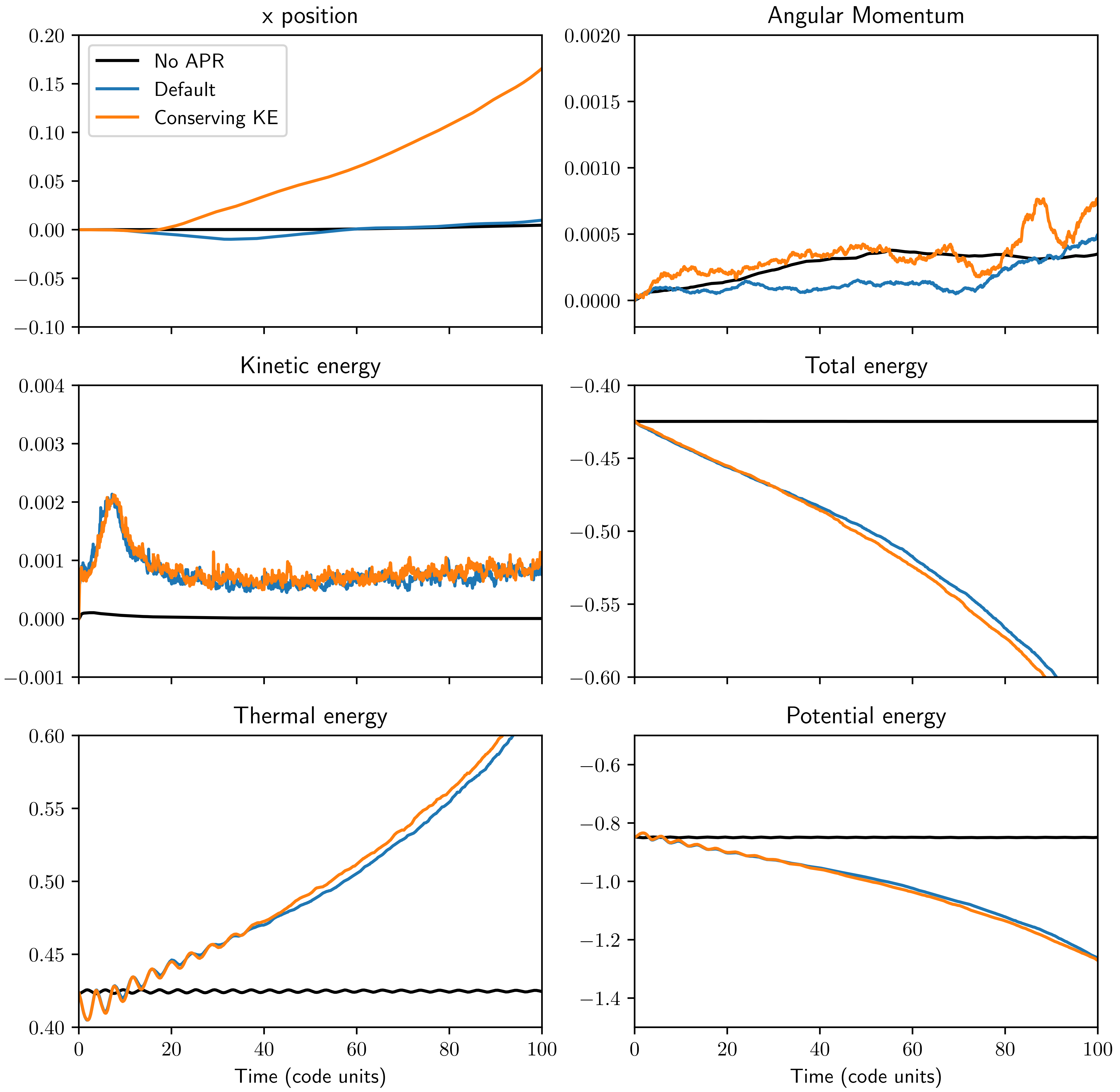
2. Weighted average for everything (current default)

i) Parent properties are just the average of the children



3. Scaling the magnitude of the velocity to conserve kinetic energy

i) Parent velocity is the quadratic average of the children



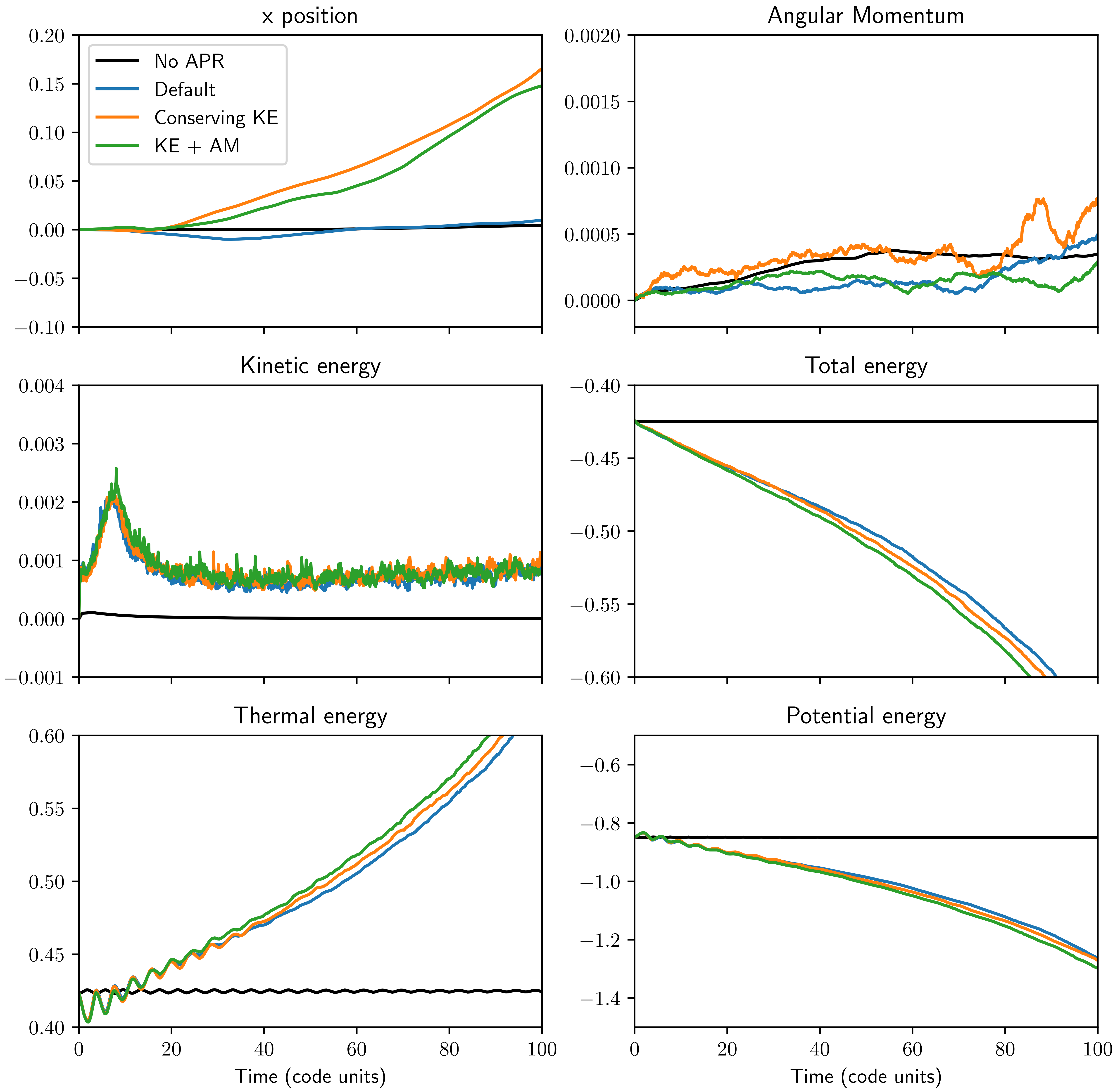
4. Scale magnitude of velocity for kinetic energy, direction for angular momentum

i) Magnitude of total velocity set from total energy

ii) Perpendicular component from inverting cross product of $L_{\rm tot}$

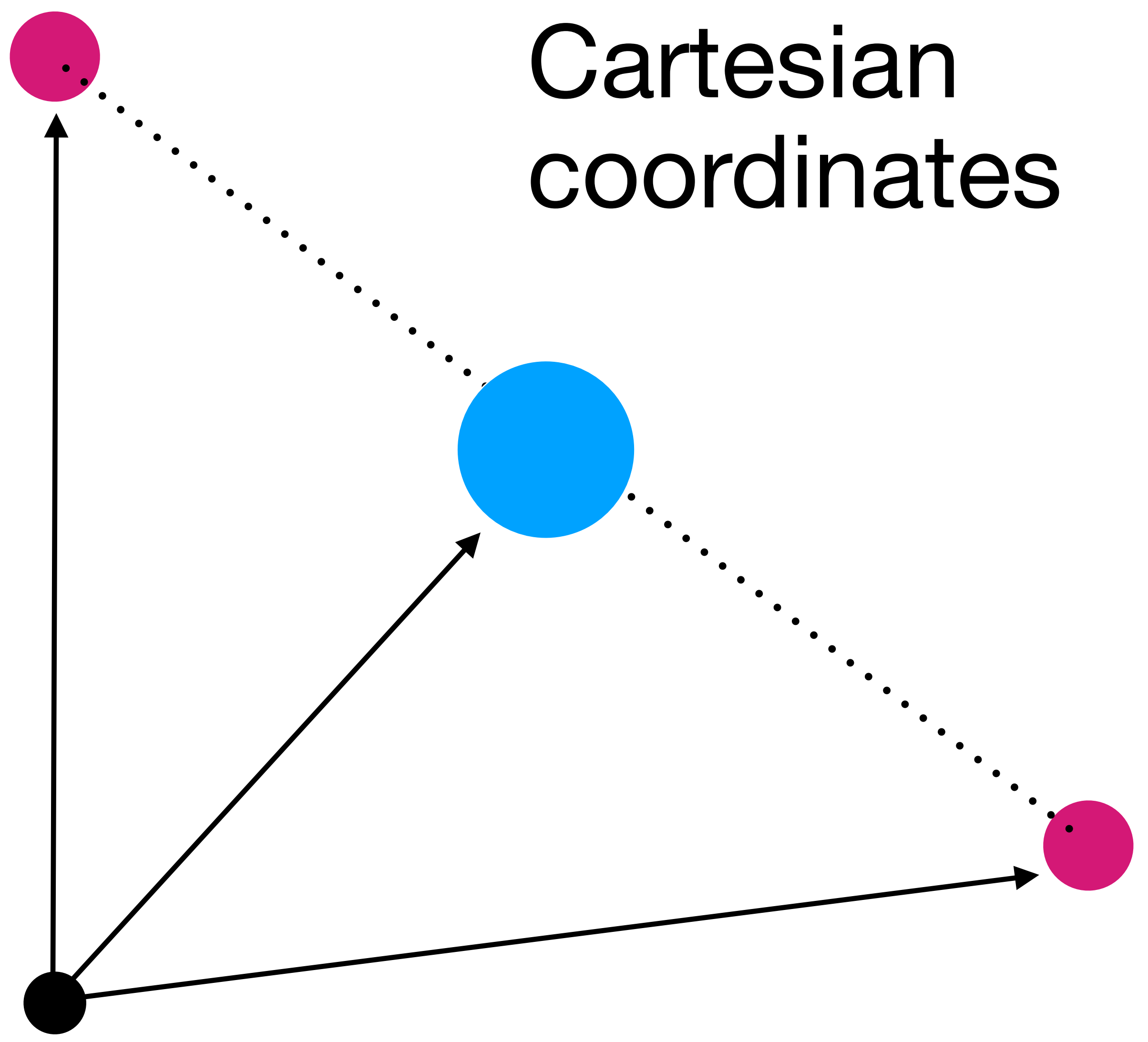
iii) Magnitude of parallel component from i) - ii)

iv) Total velocity is combination of both: $\mathbf{v}_p = \mathbf{v}_{\rm perp} + \mathbf{v}_{\rm para}\hat{\mathbf{r}}$

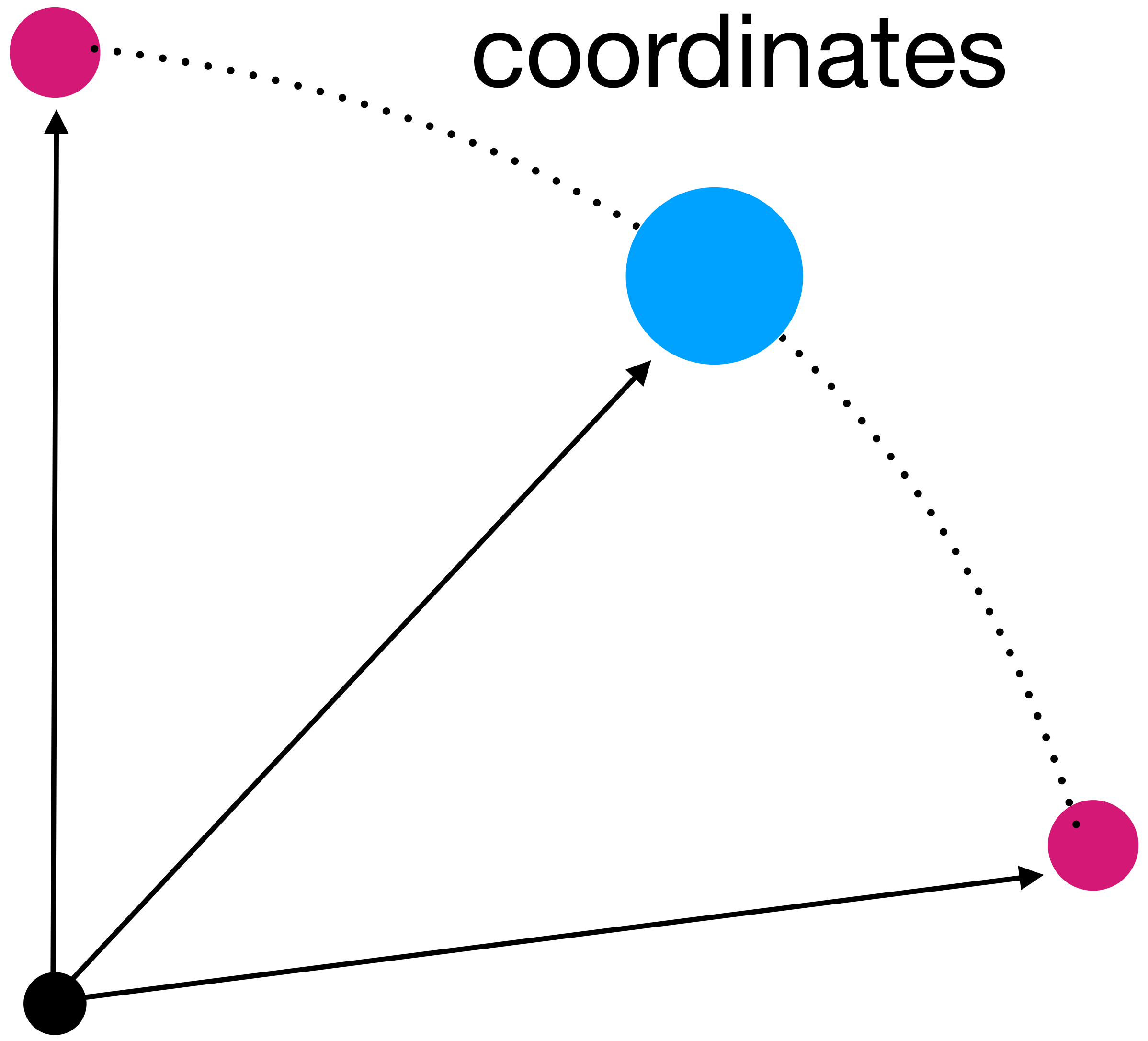


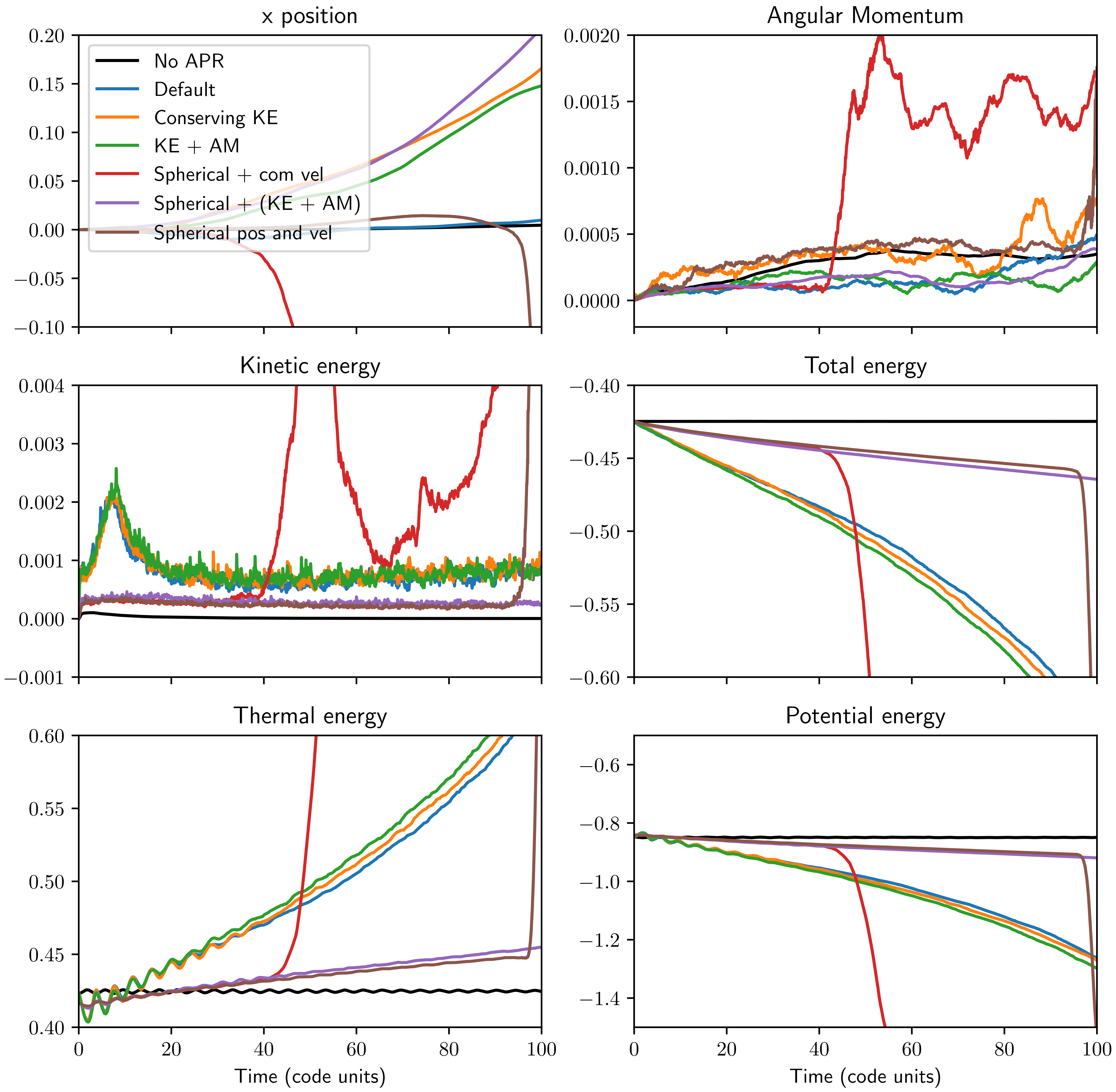
5. Spherical averaging

Cartesian coordinates

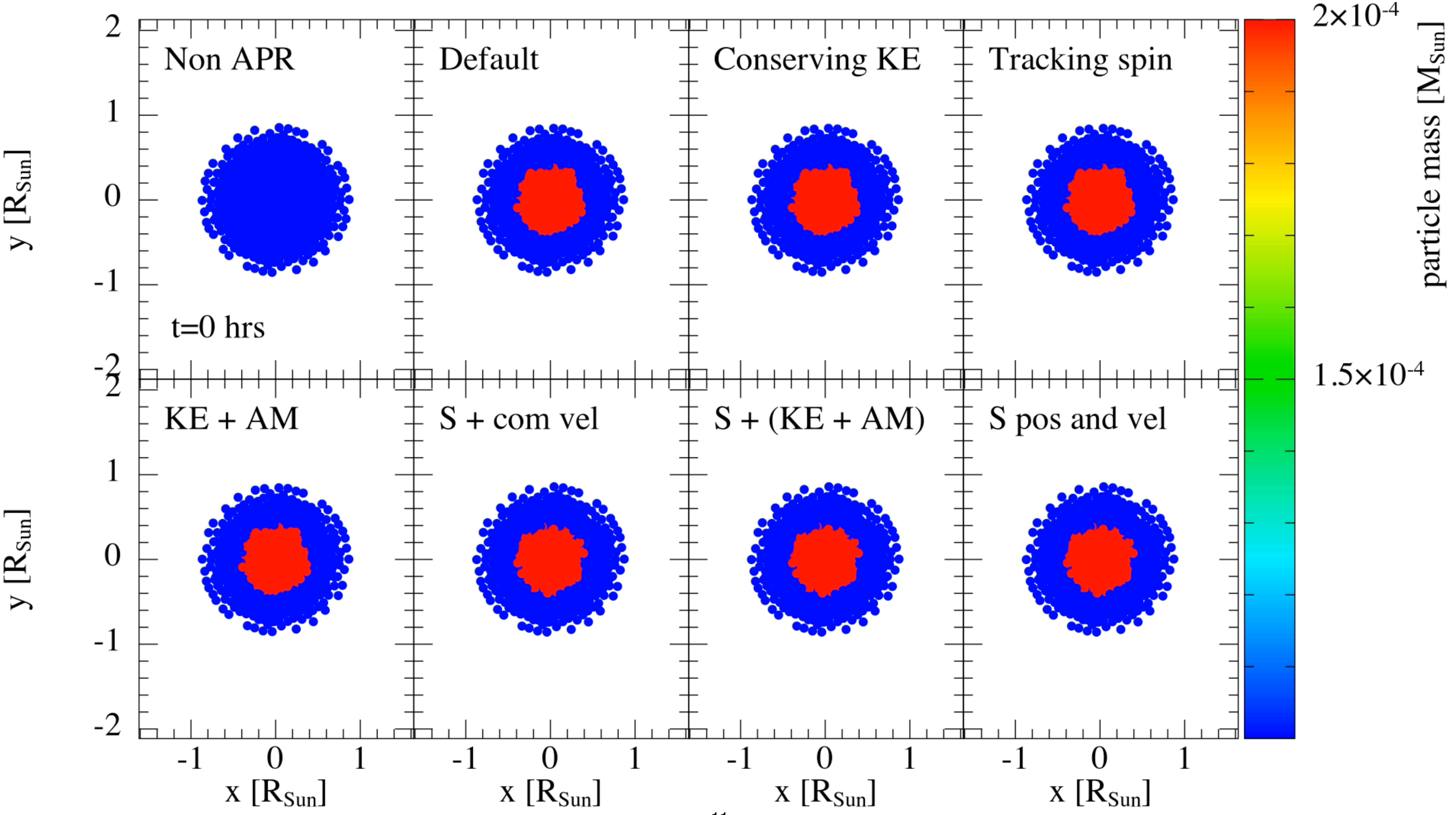


Spherical coordinates

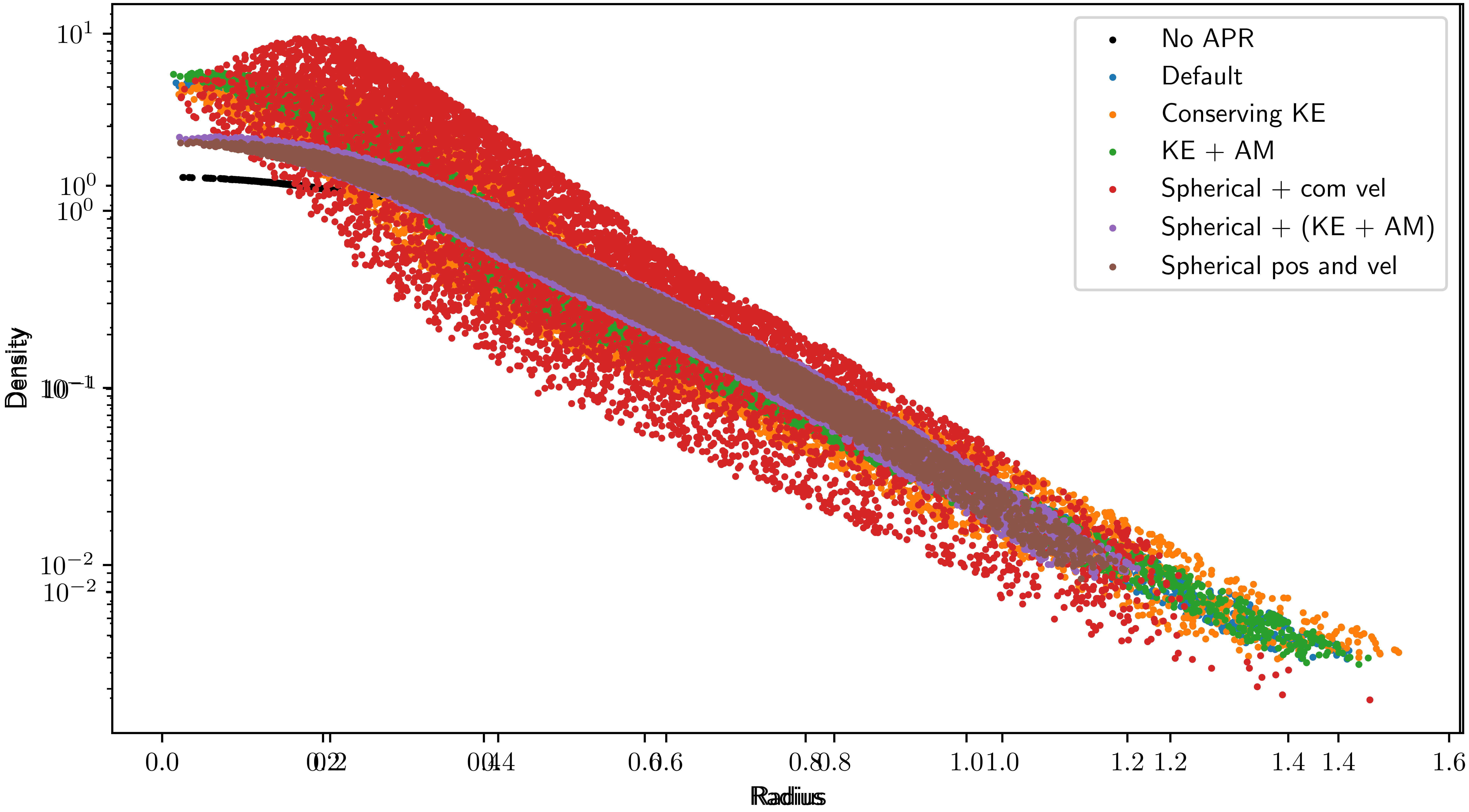




All together now …



All together now …



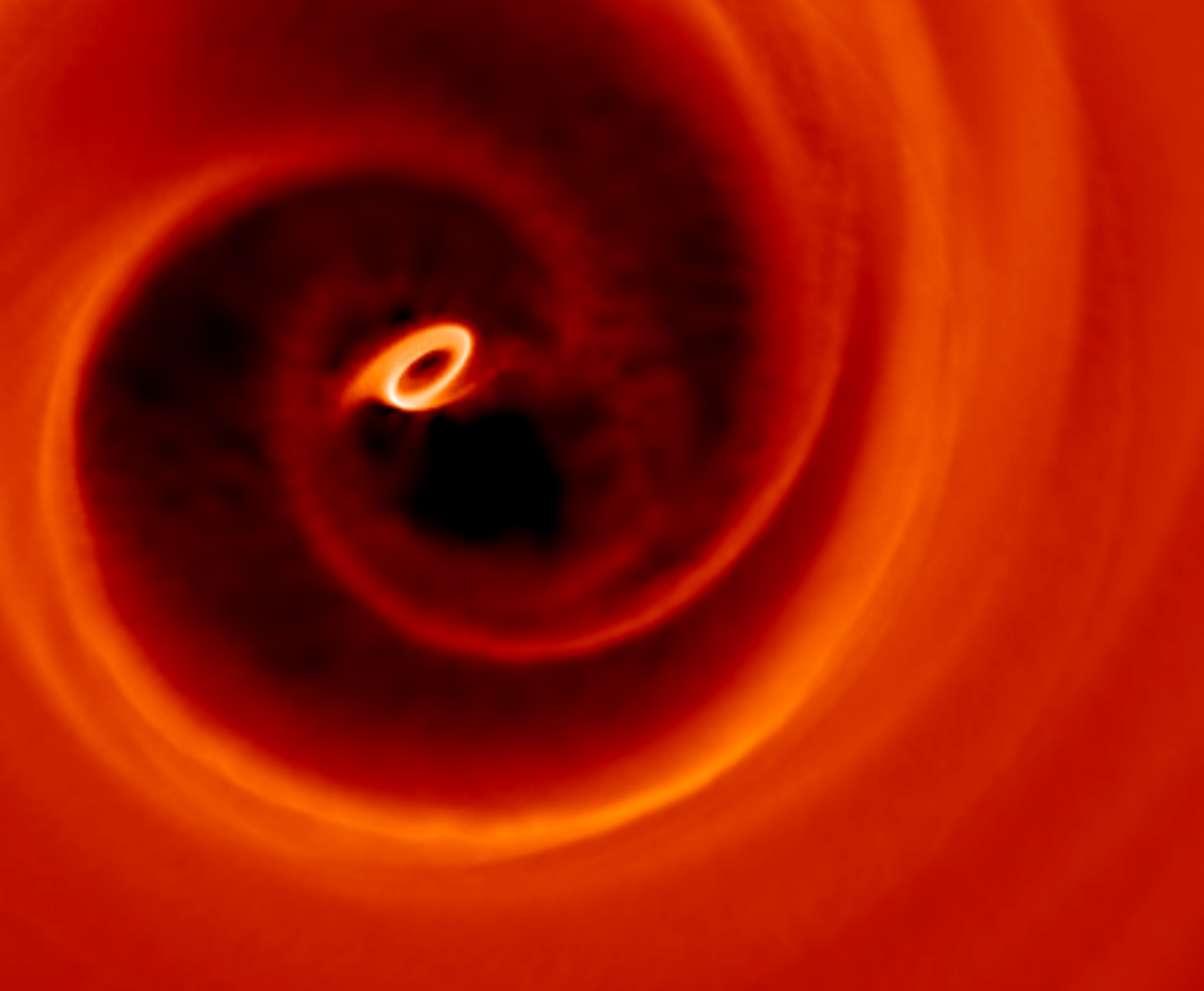
Summary

'Splitting can be made kinetic energy and angular momentum conservative … the same cannot be said for merging' Villodi & Ramachandran 2025

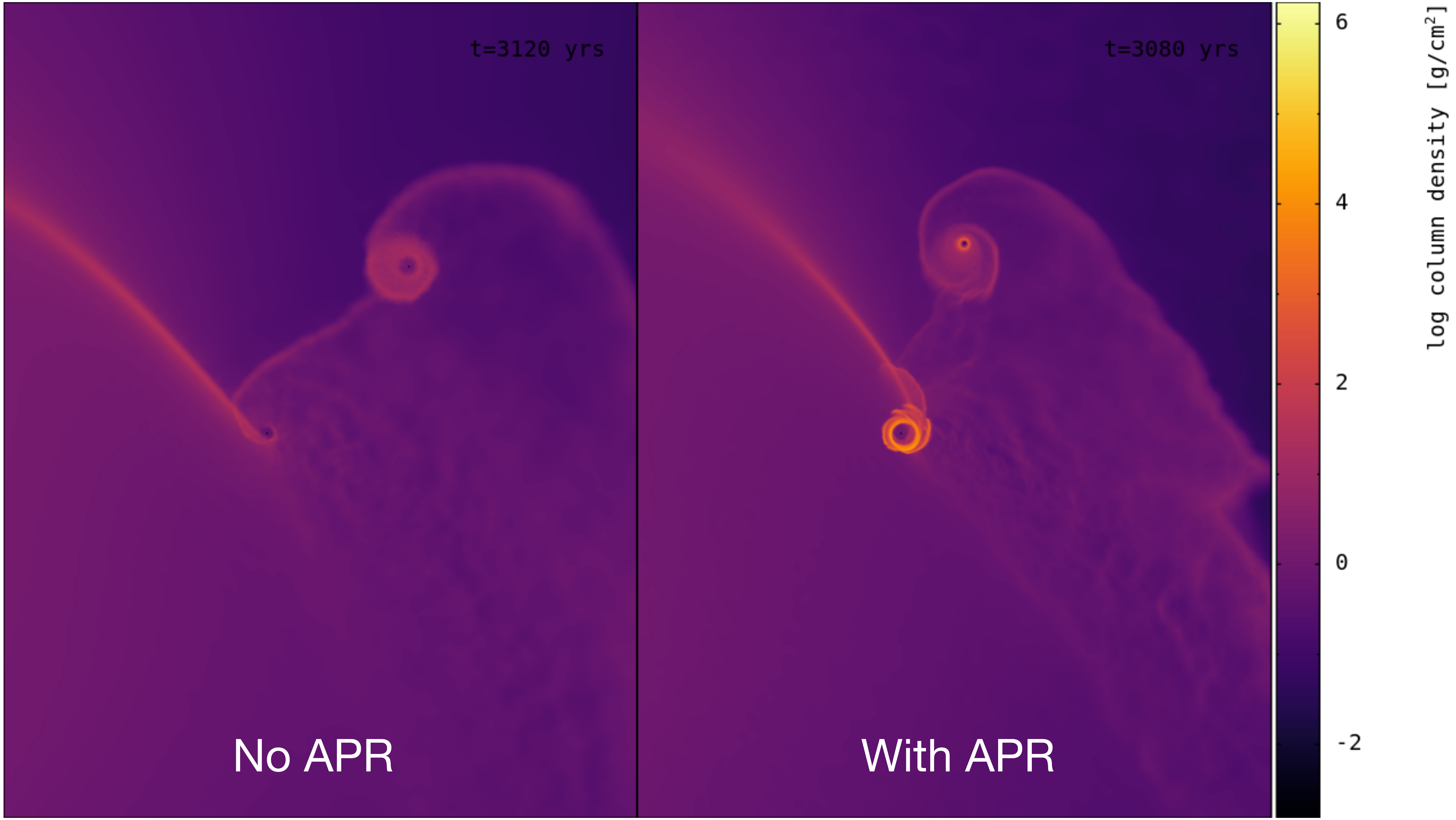
- Splitting is fine, merging sucks
- Maybe it's slightly better if you do a spherical average and set the velocities to conserve kinetic energy and angular momentum
- We can definitely do better here

Once this is solved, things to unlock with APR:

- Common envelope simulations
- Fragments in self-gravitating discs
- Uniform vertical resolution of a disc
- MCFOST



It does work well sometimes!



Toci et al. (in prep)